Understanding students' poor performance on mathematical problem solving in physics

Jonathan Tuminaro
Edward F. Redish
Physics Education Research Group
University of Maryland

Supported in part by NSF grant
REC-008 7519

Madison AAPT Meeting
August 4, 2003

Motivation

Most instructors in physics agree that introductory students have difficulty on mathematical problem solving tasks. Two possible reasons:

- Students simply lack the mathematical knowledge needed to solve problems in physics.
- Students have the relevant mathematical knowledge, but do not always use it appropriately.

Goals for this talk

- Present evidence suggesting that students have the relevant mathematical resources.
- Present an instructional strategy that can help students use their mathematical resources.

Data for this talk

Student Population

- Introductory algebra-based physics course at UMD
- >95% of the students have completed two semesters of calculus

Problem-solving Sessions

- Video-taped students solving homework problems
- TA present to assist students

You are driving on the New Jersey Turnpike at 65 mi/hr. You pass a sign that says "Lane ends 500 feet." How much time do you have in order to change lanes?

The Episode – *Mary*

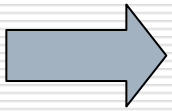
…alright if I convert 65 mph to feet per second, which is the other thing that's given in feet… So then I got 95 feet per second is what you're moving, so in 500 feet like how long? So, I was trying to do a proportion, but that doesn't work. I was like 95 feet per second…oh wait…yeah in 500 feet, like, x would be like the time…that doesn't—I get like this huge number and that doesn't make any sense.

The Episode – *What's her problem?*

A possible interpretation:

She knows that a proportion could help her on this problem, but…

1. doesn't have the mathematical skills to set up the proportion correctly, or worse…
2. doesn't even know how to divide reliably!!



<u>Pedagogical Approach</u>:

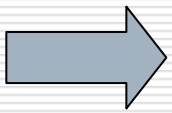
She needs practice on proportion and/or division exercises.

The Episode – *What's her problem?*

An alternative interpretation:

She has the relevant mathematical resources, but…

1. doesn't know how to use these resources.
2. her problem solving strategy restricts her from using these resources.
3. the resources she uses restricts her from using certain strategies.



<u>Pedagogical Approach</u>:

She needs help *reframing* this

We need some theoretical background.

Theoretical Background

Knowledge Elements

- Intuitive Mathematics Knowledge
- Phenomenological Primitives (*diSessa*)
- Symbolic Forms (*Sherin*)
- Mathematical Ontology (*Sfard*)

Procedural Knowledge

- Interpretive Devices (*Sherin*)
- Formal Mathematics Knowledge
- Epistemic Games (*Collins & Ferguson*)
- Frames (*Tannen*)

Theoretical Background

- **Intuitive Mathematics Knowledge**: innate or learned at an early age; *e.g. Counting* or *Subitizing.*
- **Formal Mathematics Knowledge**: learned during formal education; *e.g. Multiplication tables* or *proportions*.
- **Epistemic Games**: "general purpose strategies used to achieve a particular target structure;" *e.g. List Making*.

*Collins, A. & Ferguson, W. (1993). *Educational Psychologist, 28(1),* 25-42.

Epistemic Game: *Pattern Matching*

Reframing?

S: So, I was trying to do a proportion, but that doesn't work. I was like 95 feet per second...oh wait...yeah in 500 feet, like, x would be like the time.

- FMK: *Proportions*

  Syntax: $\frac{\square}{\square} = \frac{\square}{\square}$

- Epistemic Game: *Pattern Matching*

Matches quantities to syntax of a proportion.

Reframing - *Video*

TA: So what if I said something like...if you're traveling 8 feet per second and you go 16 feet, how long would that take you?

Student: 2 seconds.

Reframing - *Video*

TA: So, how did you do that? Can you generalize that?

Mary: Well, like, OK. Divide--the total by like how fast you're moving. Or, like how far you went by your speed will give you the time.

Epistemic Game: *Touchstone Example*

TA: So what if I said something like...if you're traveling 2 feet per second and you go 4 feet, how long would that take you?

S: 2 seconds.

- Epistemic Game: *Touchstone Example*
- Intuitive Mathematics Knowledge

New epistemic game
→ new set of resources.

Epistemic Game: *Mapping to Mathematics*

TA: So, how did you do that? Can you generalize that?

S: Well, like, OK. Divide--the total by like how fast you're moving. Or, like how far you went by your speed will give you the time.

- Epistemic Game: *Mapping to Mathematics*
- Formal Mathematics Knowledge: *Division*

Syntax: $\frac{\square}{\square} = \square$

'Derives' formal relation from conceptual knowledge of quantities.

Discussion

1. Students have productive mathematical resources.

2. Reframing (the activation of different sets of resources and/or epistemic games) can be an effective pedagogical strategy.