A Framework for Understanding the Role of Mathematics in Physics

Jonathan Tuminaro
Edward F. Redish
Physics Education Research Group
University of Maryland

Supported in part by NSF grant
REC-008 7519

Austin AAPT Meeting
January 14, 2003

Motivations

- Detailed description of how students use mathematics in physics would help in diagnosis and instruction.
- There exist different models of student thinking in terms of *resources*, can they be used to understand student thinking about mathematics in physics?

Goals of this talk

1. Outline a theoretical framework that attempts to understand the role of mathematical thinking in physics.
2. Show how this framework ties together other research that views student thinking in terms of *resources*.
3. Hint at how to use this framework to understand students' "errors" in mathematics.

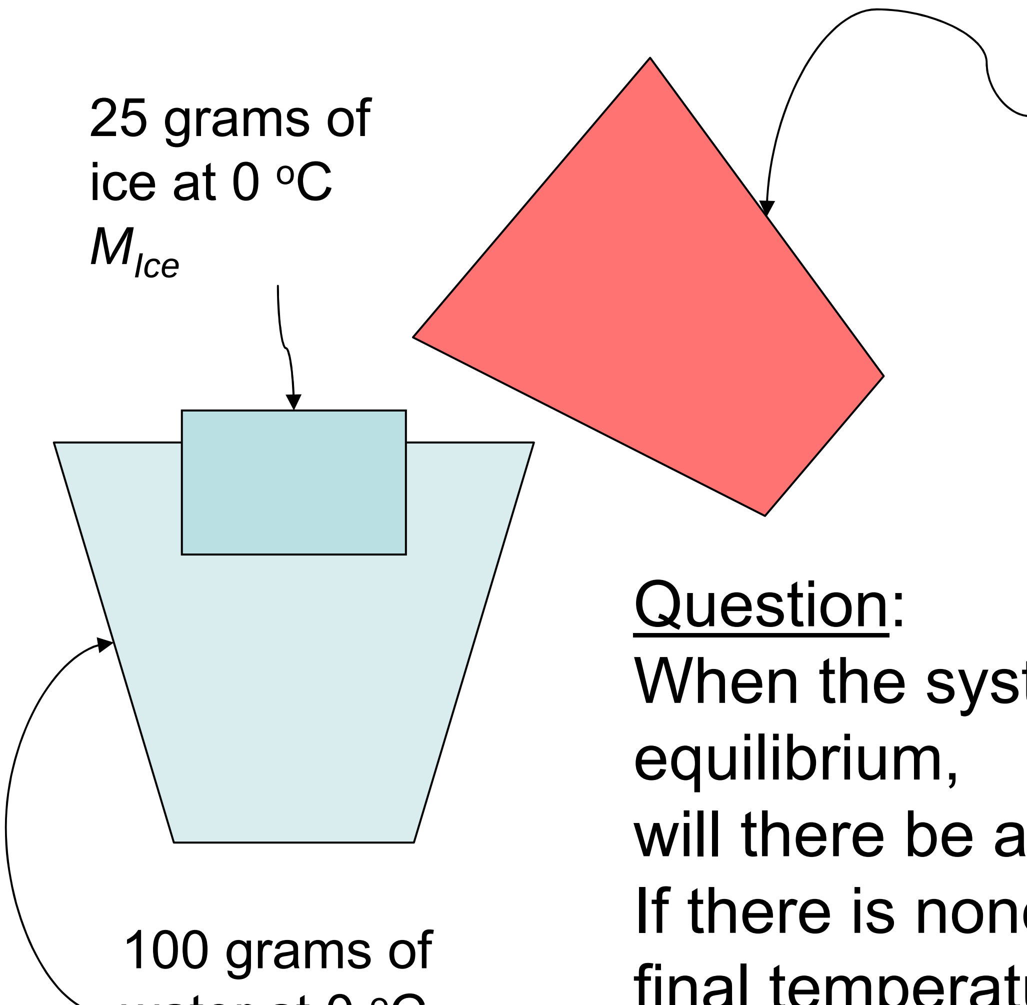
→ Note: I do not claim that this framework represents *actual* student thinking, rather I present it as a *tool* for thinking about mathematical thinking in physics.

Previous Research

- Facets
  Minstrell, J. (1992) Facets of students' knowledge and relevant instruction, in Duit, R, Goldberg, F., and Niedderer, H. (Eds.) Proceedings of the International Workshop: Research in Physics Learning—Theoretical Issues and Empirical Studies. The Institute for Science Education at the University of Kiel (IPN), Kiel, Germany.
- Symbolic forms

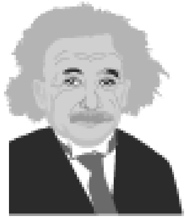
  Sherin, B. (2001) How Students Understand Physics Equations. *Cognition and Instruction*; **19**, p479-541.
- Phenomenological Primitives
  DiSessa, A. A. (1993) Toward an epistemology of physics. *Cognition and Instruction*, 10 (2-3), 105-225.

Example: Homework Problem



<u>Question</u>:
When the system comes to thermal equilibrium,
will there be any ice left?
If there is none, what will the final temperature of the water be?

Example:
An Ideal Answer



Step 1: Determine how much hot water it takes to melt all the ice.

The heat gained to melt the ice

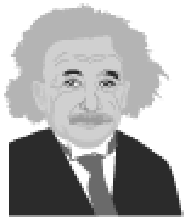
$$M_{\text{ice}} L_F = M c \Delta T$$

The heat lost by the hot water

$$= 20 \text{ grams}$$

Mass of hot water needed to melt all the ice.

Example:
An Ideal Answer



Step 2: Find the final temperature of the mixture.

Total mass of water at 0 °C
100 + 25 + 20

Mass of hot water left over to heat the mixture
50 – 20 = 30

The heat gained by the mixture at 0 °C

$$m_1 c(T-0) = m_2 c(100-T)$$

The heat lost by the remaining hot water

$$T = 18\ ^{o}\text{C}$$

Change in temperature of the cold water.

The final temperature of the entire mixture.

Change in temperature of the hot water.

Example: Student's Response

<u>Student's Equation</u>

$$mL_F = mc\Delta T$$

"It's M L F…the heat gained by the ice.

And then I took M C delta T… which is the energy that is lost by the hot water.

And then I found…the final temperature,

knowing that…the initial temperature is 100 degrees."

<u>Student's Story</u>

1. "I separated that—I put just the ice by itself."
2. "And, add the hot water poured into it to melt it first."
3. "And, then find out the temperature that it was after…thermal equilibrium,"
4. "and then pour that water into the other water."

The Framework: Intro

Different *resources* serve different functions

- **Knowledge elements**
  bits of information

Both resources can exist at different levels of abstraction

- **Reasoning strategies**
  cognitive mappings giving meaning to knowledge elements

The Framework: Specific

Student's Equation

$$mL_F = mc\Delta T$$

"It's M L F…the heat gained by the ice.

And then I took M C delta T…

which is the energy that is lost by the hot water.

And then I found…the final temperature,

knowing that…the initial temperature is 100 degrees."

Student's Story

1. "I separated that—I put just the ice by itself."
2. "And, add the hot water poured into it to melt it first."
3. "And, then find out the temperature that it was after…thermal equilibrium,"
4. "and then pour that water into the other water."

The Framework: Specific

Specific Knowledge Element

The heat gained by the ice [equals]…the energy that is lost by the hot water.

Student's Story

1. "I separated that—I put just the ice by itself."
2. "And, add the hot water poured into it to melt it first."
3. "And, then find out the temperature that it was after…thermal equilibrium,"
4. "and then pour that water into the other water."

The Framework: Specific

Specific Knowledge Element

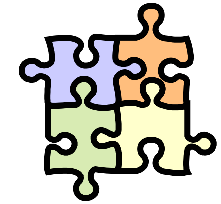
Facets

Specific Reasoning Strategy

The heat gained by the ice [equals]…the energy that is lost by the hot water.

I…add the hot water…to melt it first. And, then find out the temperature that it was after…thermal equilibrium.

The Framework: Specific



Facets (Minstrell)

Knowledge elements and reasoning strategies that are applied in *particular* problem situations.

Note: A relationship between *specified* entities that is used in *particular* problem situations.

e.g. *"more force means more motion"*

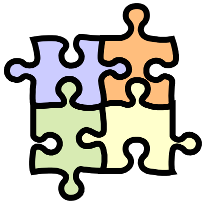
The Framework: Abstract knowledge elements

"It's M L F…the heat gained by the ice." ← mL_F

Symbolic Form:
Parts-of-a-whole

1. Conceptual component: A whole entity can be comprised of different parts.
2. Symbolic component: The symbolic representation of which falls into the pattern □*□.

The Framework:
Abstract knowledge elements



<u>**Symbolic Forms (Sherin)**</u>

- **Symbol Template**: elements of knowledge that give structure to a mathematical expression; e.g. ☐=☐ or ☐*☐*☐…
- **Conceptual Schema**: elements of knowledge that offer a conceptualization of the symbol template.

<u>Note</u>: A relationship, between *unspecified* entities, which is used in *multiple* problem situations.

e.g. *“more is more”*

The Framework: Abstract Reasoning Strategies

Conservation

Entities mapped into each side of the equation is associated with a different moment in the process.

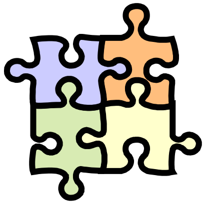
the heat gained by the ice...
to melt the ice *first*.

energy lost by the water...
after thermal equilibrium.

$$mL_F = mc\Delta T$$

Interpretive Mapping

The Framework: Abstract Reasoning Strategies



Interpretive mappings

Cognitive mappings from abstract knowledge elements in general situations to specific knowledge elements in particular situations.

Note: A cognitive mechanisms connecting the abstract to the specific.

The Framework: Summary

Facets

the heat gained by the ice…
to melt the ice *first*.

energy lost by the water…
after thermal equilibrium.

Interpretive Mapping
Conservation

$$mL_F = mc\Delta T$$

And then I found…
the final temperature

$$\Delta T = T_f - T_i$$

Symbolic Form
Parts-of-a-whole

Symbolic Form
Balancing

Analysis of Errors

Conclusions

- Builds coherence between different models of knowledge that appear in the literature.

- Views students’ mathematical errors in physics as inappropriate mappings between abstract and specific knowledge.

<u>Future Work</u>

- Development of instructional practices.

- Build an understanding of how coherent knowledge structures are created.