Physics 776 Project: Extremal Black Holes in String Theory[1]

This note is an introduction to black holes in string theory and microstate counting of extremal black holes. In this note we follow the original calculation of Strominger and Vafa, where the state counting is done for a particular class of five dimensional black holes of Reissner-Nordstrom type.

April, 2005

[1] harsha@glue.umd.edu

1. Introduction

It is well known that black holes obey the laws of thermodynamics, and can be ascribed an entropy given by $S_{B.H} = \frac{A}{4G_N}$, where A refers to the horizon area of such a black hole and G_N is the gravitational Newton's constant. It is natural to wonder if these laws have a deeper statistical mechanical interpretation to them. If so, we would like to understand the entropy as referring to the microstates of the black hole that are accessible, fixing the macroscopic parameters like mass and charge. Specifically, $n = e^{S_{B.H}}$ states would be available to such a system. Since the formula for the Bekenstein-Hawking entropy is semi-classical (in the sense that it is non perturbative in G_N) we might expect that it can be understood once a non perturbative formulation of Quantum Gravity exists. String theory seems to be a UV finite theory of gravity (among other things), so it is natural to ask if the statistical description we seek emerges from String theory. In a seminal paper, Strominger and Vafa [1] showed that this is indeed true in certain classes of five dimensional black holes (the results have been extended to even near extremal black holes since then). The string theory technology is well understood for supersymmetric backgrounds, but so far non supersymmetric string backgrounds are rather poorly understood. For this reason, [1] focused on the extremal black holes of $\mathcal{N} = 2$ supergravity in $d = 5$, which are supersymmetric black holes. It is important to note that these black holes are of Ramond-Ramond type, which is incidentally responsible for much of the tractability of the state counting analysis (for NSNS black holes the identification of microstates involves understanding strongly coupled SCFT which has been accomplished in some cases, but this approach is beyond the scope of this review). The plan for the rest of the paper is as follows: section 2 is a (very) brief review of string theory. In section 3, we review toroidal compactifications of type II theories and argue the existence of Ramond-Ramond solitons in string theory whose space-time interpretation we also describe. In this process, we will describe a particular class of five dimensional black holes of the type considered by [1]. In section 4, we (following Polchinski) identify these solitons with a non perturbative object in string theory, called the Dirichlet brane (or D-brane for short) and discuss relevant aspects of D-brane dynamics. We also discuss the brane configuration relevant to describing the three charge $5d$ black hole of [1] at weak coupling. We argue that the number of BPS states is coupling independent so the black hole microstates can be reliably counted at weak coupling also. Then we proceed to perform the state counting in the D-brane picture and show that the Bekenstein-Hawking formula is precisely recovered.

2. Very basic string theory

It is impossible to give a satisfactory account of string theory in this review, so we will not attempt it, directing the reader to Polchinski's text, for an excellent introduction to the subject and for references to standard material. In this section therefore, we will merely give a cursory account of the subject and collect some information to make the rest of the paper coherent for the person untrained in string theory.

It is well known that QFT can be formulated (as long as we are only describing scattering) as a theory of maps of the world line into space-time, governed by the partition function $Z = \int dedXe^{-S}$, where $S = \int dt(\frac{\dot{X}^\mu \dot{X}_\mu}{e} + em^2 + iqA_\mu \dot{X}^\mu)$ is the point particle action and e is the Veilbein. It is easy to verify that gauge fixing this path integral and computing the n-point functions reproduces the Feynman diagram expansion of field theory. Analogously, the first quantized string theory is formulated by the Polyakov path integral, with an integral over the worldsheet (the worldsheet is topologically a cylinder for the closed string) of the Polyakov action $S = \int dzd\bar{z}\sqrt{h}h^{ab}\partial_a X^\mu \bar{\partial}_b X_\mu$ (h_{ab} is the worldsheet metric, playing the role of e in the case of the string). This mere jump in dimension by 1 has dramatic consequences for the interactions of strings. In particular strings can only interact via splitting and joining. Also, it turns out that the only fields that couple to a string are the ones that are present in the quantization of the worldsheet action. Moreover another nice property is that the theory is perturbatively finite and contains a graviton (among other fields). However, the action we have described above (the so called bosonic string) has the defect that its quantization reveals a tachyon in the spectrum, which indicates an IR divergence telling us the bosonic string vacuum is unstable. The tachyon seems to be a fairly generic problem, and one way to stabilize the theory (perturbatively) is to project out the tachyon. In order to do this consistently, one introduces worldsheet fermions ψ^μ, whose space-time interpretation depends on their moding on the cylinder. Along the S^1

direction, periodic moding of the fermions means the worldsheet fermions tranform in the spinor representation of the space-time Lorentz group, while anti-periodic moding means they transform in the vector representation. After introducing these extra fields one finds first of all that the theory makes sense only in $10d$, and that there is a discrete symmetry (called GSO projection), under which the tachyon is odd. It turns out that one needs to impose this GSO projection for consistency of the string theory. One way to understand this is to note that the quantization of the fermionic string in $10d$ reveals a tachyon (as usual) and a massless spin-$\frac{3}{2}$ particle. It is well known that a theory of an interacting massless spin-$\frac{3}{2}$ particle is inconsistent unless the space-time theory is supersymmetric. So it is clear that either the massless spin-$\frac{3}{2}$ object (called gravitino) has to be projected out, or the space-time theory has to be supersymmetric (in which case the tachyon is projected out). Based on our dislike of the tachyon, we choose the latter option, which yields two consistent interacting $10d$ string theories, called type IIA and B (the A and B refers to the chirality of the supercharges under the little group Spin(8) in $10d$). From these we can form the Type I theory by suitable projections of the type IIB theory (technically the procedure is called an orientifold). Apart from this there is one other perturbative string theory which consists of a left moving fermionic string and a right moving bosonic string (this is called the Heterotic string). Consistency of the string theory so constructed requires a space-time non abelian gauge field with gauge group $SO(32)$ or $E_8 \times E_8$. In all there were five perturbative string theories known in the 80s, and it is now recognised that these string theories are all related to each other by various dualities. In other words, by tuning various moduli, one can move between different string vacua.

String perturbation theory is captured by the genus expansion, and the loop counting parameter is the expectation value of the dilaton (more precisely the string coupling g_s is given by $g_s = e^{\phi}$). The worldsheet theory is given by the Polyakov action, which is a so called sigma model, schematically of the form $S = \frac{1}{2\pi\alpha'} \int dz d\bar{z} g^{\mu\nu} \partial X_\mu \bar{\partial} X_\nu$ (X^μ are coordinates on target space-time with metric $g_{\mu\nu}$) so that the sigma model coupling is $\frac{\sqrt{\alpha'}}{R}$, where R is the (local) radius of curvature of the target space-time. So for string perturbation theory to be under control, we need $g_s \ll 1$ and $\frac{\sqrt{\alpha'}}{R} \ll 1$. The requirement $\frac{\sqrt{\alpha'}}{R} \ll 1$ is the same thing as saying that the space-time geometry fluctuates at a much higher scale than the size of the string. As mentioned before, there are two possible modings of the worldsheet fermions. The periodic moding leads to the so called Ramond (R) sector, and the anti-periodic moding to the Neveu-Schwarz (NS) sector. R-sector fermions are space-time fermions and NS-sector fermions are space-time bosons. To form the states of a closed string we take a left moving sector and a right moving sector and tensor them (left and right moving refers to the light cone directions which is an invariant notion in $2d$). In this process space-time bosons come from NS-NS and R-R sectors, while fermions come from NS-R and R-NS sectors. From the NS-NS sector we get a metric, anti symmetric two form (called B-field) and the dilaton whereas from the R-R sector we get the form fields. GSO projection acts differently in the IIA and IIB cases, by virtue of which, in type IIA we get the 1,3,5,7,9-form fields and in type IIB we get the 0,2,4,6,8-form fields (electric-magnetic duality relates p-form and $(6-p)$-form fields). The elementary string is charged under NS-NS fields (to be precise the elementary string is electrically charged under the B-field while the NS5-brane is magnetically charged), but there are no perturbative string states carrying R-R charge. The low energy limit of the type II string theories is $\mathcal{N}=2$ supergravity in $10d$. The low energy limit of type IIA is non chiral supergravity while the low energy limit of type IIB is chiral supergravity. The low energy effective action takes the schematic form $S=\int d^{10}x\sqrt{-g}(e^{-2\phi}(R+4(\nabla\phi)^2-\frac{1}{4}H^2)-\frac{1}{4}F^2)$, where ϕ is the dilaton, H is the field strength of the B-field and F refers to the form fields.

3. Compactification of Type IIA on T^4 and RR-solitons

Consider compactification of Type IIA string theory in $10d$ down to six dimensions on T^4. Such a compactification preserves all the supersymmetries of the uncompactified theory, so the resulting six dimensional theory (for massless modes) has 32 supercharges and is thus maximally supersymmetric. The massless matter arise from dimensional reduction of the $10d$ fields. In particular we get 25 scalars and 16 gauge fields the scalars parameterizing a moduli space $\mathcal{M}=\frac{SO(5,5;\mathbf{R})}{SO(5;\mathbf{R})\times SO(5;\mathbf{R})}$ while the gauge fields form a **16** of $SO(5,5;\mathbf{R})$. Of the 16 gauge fields, 8 come from the NSNS sector and 8 from RR sector. So the six dimensional $\mathcal{N}=4$ supergravity has a global symmetry $SO(5,5;\mathbf{R})$. It is believed that a $SO(5,5;\mathbf{Z})$ sub group is unbroken in the full string theory and this group is called the

U-duality group (by now there is considerable evidence supporting this conjecture). As we saw, the U-duality requires the gauge fields to transform in a **16** of $SO(10)$ so U-duality interchanges RR and NSNS gauge fields, so it also interchanges RR solitons with NSNS solitons. It is well known that there are BPS states[1] carrying NSNS charge in the six dimensional theory, so U-duality requires the existence of BPS states carrying RR charge. Even though we have formulated this in the context of six dimensions, the story holds in every dimension less than 10, indicating that there are indeed states that carry RR charge in string theory. For example, let us consider a five dimensional compactification on T^5. We get 42 scalars and 27 gauge fields along with the graviton by KK reduction (of course with the fermionic super partners in $\mathcal{N} = 4$ multiplets). The scalars parameterize a moduli space that is locally $\mathcal{M} = E_{6,6}/Sp(4)$ while the gauge fields transform as the **27** of E_6. Of the 27 gauge fields 11 arise from the NSNS sector while 16 arise from the RR sector. U-duality clearly mixes the RR and NSNS gauge fields as we have mentioned before. BPS states in $\mathcal{N} = 4$ supergravity in $d = 5$ must also form representations of the U-duality group $E_{6,6}(\mathbf{Z})$. Some of these BPS states are easy to identify: for example, strings wound n times carrying momentum p on any S^1 of the torus T^5 is such a BPS state. Clearly this BPS state carries charge only under the NSNS fields. We would like to identify the BPS states carrying RR charge, which are required by U-duality[2]. In order to do this, one has

[1] The simple supersymmetry algebra in $4d$ for instance is of the form $\{Q_a, Q_{\dot{b}}\} = P_{a\dot{b}}$ (where Q are the supercharges and P refers to the four momentum). With twice as much SUSY the algebra can admit central charges, the modified algebra being of the form $\{Q^I{}_a, Q^J{}_{\dot{b}}\} = \delta^{IJ} P_{a\dot{b}}$ and $\{Q^I{}_a, Q^J{}_b\} = \epsilon^{IJ}\epsilon_{ab} Z$ where Z is the central charge and I, J run from 1 to 2. This leads to the BPS bound $M \geq |Z|$, for the mass of the massive multiplet with central charges and the ones that saturate this bound are known as BPS states. They form short (8 dimensional) representations in the $\mathcal{N} = 2$ theory we took as an example.

[2] The U-duality group is E_6 while the T-duality group of the torus is $SO(5, 5)$. Decomposing the **27** of E_6 under $SO(10)$ we get $\mathbf{27} = 10 \oplus 16 \oplus 1$, so that T-duality mixes the RR fields among themselves, while it mixes 10 of the NSNS fields among themselves, the final NSNS field (the unreduced B-field) is T-duality invariant. The BPS states from the NS sector are fundamental strings carrying momentum and winding, NS5-branes wrapping T^5 and bound states of strings and NS5-branes, the latter being $\frac{1}{4}$-BPS.

to start with the action:

$$S = \int d^5x \sqrt{-g}(R - G_{IJ}\partial_\mu X^I \partial^\mu X^J - G_{IJ}F^I_{\mu\nu}F^{J\mu\nu} - \frac{1}{6}C \wedge F \wedge F) \qquad (3.1)$$

where X^I are the scalar moduli in the vector multiplets, and G_{IJ} is the $E_{6,6}$ invariant metric on the moduli space, the Chern-Simons term being peculiar to five dimensions and required by supersymmetry (only the bosonic part of the matter action is shown in (3.1)). To find BPS states we need to determine all solutions that preserve part supersymmetry. One such solution which is simple to determine is in the case where all moduli are frozen, and only two gauge fields are excited. Calling those gauge fields F and H to refer to the RR and NSNS sectors respectively, we find the following solution (its validity as a string solution requires $1 \ll Q_F \ll Q_H$):

$$ds^2 = -(1-(\frac{r_0}{r})^2)^2 dt^2 + \frac{dr^2}{(1-(\frac{r_0}{r})^2)^2} + r^2 d\Omega_3{}^2 \quad F = \sqrt{2}(\frac{8Q_H {Q_F}^2}{\pi^2}) * \epsilon_3 \quad H = (\frac{8Q_H {Q_F}^2}{\pi^2}) * \epsilon_3 \qquad (3.2)$$

where Q_H and Q_F refer to the charges associated with H and F, while $r_0 = (\frac{8Q_H {Q_F}^2}{\pi^2})^{\frac{1}{6}}$. The metric (3.2) is the $5d$ extreme Reissner-Nordstrom black hole, with horizon size r_0. Putting back factors of G_N, we see that $r_0 \simeq g_s\sqrt{\alpha'}$, so the size of this object is much smaller than string length when $g_s \ll 1$. This is a BPS state coming from the mixed RR and NSNS sector, and is also required by U-duality, but we see that this object has a good interpretation as a space-time black hole only when $r_0 \gg \sqrt{\alpha'}$. However, the corresponding BPS state must exist even in the limit $g_s \ll 1$, so the question is, what happens to this extreme black hole as the coupling is tuned to small enough values. It was the remarkable insight of Polchinski that these objects have a perturbative string description as worldsheets with boundary. In other words, in the limit $g_s \to 0$, the correct description of the space-time RR solitons is not in terms of a supergravity background, but in terms of hyper planes in flat space, where open strings can end. So we seem to have understood the role played by RR-solitons, but now we get more for the time we invested. Since we saw that there were RR solitons whose space-time interpretation at strong coupling was in terms of a black hole, and we also argued that at weak coupling they have a perturbative string description, it is clear that we stand a chance of counting the number of BPS states carrying fixed charge, in the D-brane picture. However, since these states are required to exist by U-duality for all g_s, it is also clear that the number of such states cannot change with coupling, so we must indeed be counting the microstates of the extremal black hole which must replace this D-brane description at strong coupling. We seem to have found a way to verify the Bekenstein-Hawking entropy by a microscopic calculation. In the next section, we will make this approach more precise and develop enough technology of D-branes to allow us to perform this computation, but for now we will point out one more fascinating aspect of these black holes. Going back to the metric (3.2), we can compute the Bekenstein-Hawking entropy as:

$$S_{B.H} = \frac{A}{4G_N} = 2\pi\sqrt{\frac{1}{2}Q_H {Q_F}^2} \tag{3.3}$$

The remarkable feature of (3.3) is that it has no dependence on any of the moduli (including the dilaton). We obscured this fact by choosing to solve the BPS equations for fixed moduli, but it is not too hard to solve the more general case of varying moduli and convince oneself that the entropy formula is moduli independent. This is certainly necessary for the black hole entropy to have a statistical interpretation, since the entropy is required to be an adiabatic invariant. However, what is more remarkable is that the entropy depends only on the topological invariants of the compactifying manifold (it depends on quantities like the intersection numbers of suitable cycles of the compactifying manifold). Coming back to the entropy formula (3.3) we would like to determine the number of states $n = e^{S_{B.H}}$ that carry the charges Q_H and Q_F. This is the same as determining the degeneracy of the $\frac{1}{4}$-BPS states with these charges. For example, in a simple field theory like $\mathcal{N} = 2$ Super Yang-Mills in $4d$ (see footnote 1), BPS states form 8 dimensional representations, whereas we are looking for a $e^{2\pi\sqrt{Q_H {Q_F}^2}}$ dimensional representation. In the next section, we will see which field theory is responsible for such degeneracy of BPS states.

4. Dirichlet Branes

Before we discuss the brane configuration that is related to the black hole (3.2), let us consider a simpler situation. The following metric and dilaton solve the low energy supergravity equations in $10d$ (we have dropped numerical constants that are irrelevant for this discussion):

$$ds^2 = Z(r)^{-\frac{1}{2}}\eta_{\mu\nu}dx^\mu dx^\nu + Z(r)^{\frac{1}{2}}dy^m dy_m \quad Z = 1 + (\frac{\rho}{r})^{7-p} \quad \rho^{7-p} = \alpha'^{\frac{7-p}{2}} g_s Q \quad e^{2\phi} = Z^{\frac{3-p}{2}} \tag{4.1}$$

This is the metric of a black p-brane, which has a $ISO(1,p) \times SO(9-p)$ global symmetry. For all these p-branes, fixing the RR charge Q and taking $g_s \to 0$ tells us that the size becomes much smaller than string scale, so the supergravity background is not reliable. However, (4.1) is a $\frac{1}{2}$-BPS state, so it preserves 16 supercharges. Such BPS states are known not to decay, so even though the supergravity background is unreliable for small g_s, there must be an alternative description. Polchinski argued [2]that this alternate description (called a Dp-brane), valid in the $g_s \to 0$ limit was in terms of $p+1$ dimensional hyper planes embedded in $10d$ space-time. More precisely, Polchinski showed that if instead of considering worldsheets without boundary(appropriate for closed strings) one considered worldsheets with boundary (mapping the closed string worldsheet into $10d$ space-time and the worldsheet boundary being mapped into a $p+1$ dimensional flat plane)then these objects carried RR charge of precisely the same magnitude as in (4.1). Moreover one could show that low energy scattering of string and D-branes off Dp-branes reproduced the geometry of (4.1) in the large impact parameter regime. The can be taken as evidence for the conjecture of Polchinski that D-branes were the right description of the black p-branes valid in the $g_s \to 0$ limit.

Just as quantization of worldsheets without boundary revealed a rich spectrum of massless and massive particles, we similarly get matter fields from quantizing the open strings that end on the Dp-brane. It turns out that the theory on the world volume of a Dp-brane is the dimensional reduction to $p+1$ dimensions of $\mathcal{N} = 1$ Super Yang-Mills in $10d$. Parallel D-branes preserve supersymmetries that are common to the ones preserved by individual D-branes. Such considerations lead to the fact that a Dp-Dp' system preserves $\frac{1}{4}$-supersymmetry iff $p - p' = 0 \mathrm{mod} 4$. Another important observation about Dp-branes is that a stack of n Dp-branes has a world volume theory which is a non abelian $U(n)$ gauge theory with 16 supercharges (this fixes the low energy lagrangian uniquely).

Placing a stack of n D1-branes parallel to m D5-branes, with their common world volume direction wrapped on a S^1, while the remaining directions of the D5-brane are wrapped on T^4, gives a point like object (a $\frac{1}{4}$-BPS state) in five dimensions. Giving quantized momentum p on the common S^1 gives rise to a 3 charge object in five dimensions, which is nothing but the black hole we discussed in the previous section with the identification $\frac{1}{2}Q_F^2 = mn$ and $Q_H = p$. We would like to determine the degeneracy of BPS states corresponding to this configuration.

The theory on the world volume of the D1-D5 system (for the moment ignoring the momentum on S^1) preserves eight unbroken supersymmetries. The theory has a gauge group $U(n) \times U(m)$, with respect to which various fields are charged. Specifically, the massless fields on the D1-D5 world volumes can be grouped into vector and hyper-multiplets. From 1-1 strings we get a vector multiplet in the adjoint of $U(n)$ (this is a $U(m)$ singlet). From 5-5 strings, we get a vector multiplet in the adjoint of $U(m)$ (this being a $U(n)$ singlet). From 1-5 and 5-1 strings, we get m hyper-multiplets in the n of $U(n)$. The theory on the D5-brane decouples as it is IR free, so we can focus on the D1-brane theory, which is a $\mathcal{N} = (4,4)$ supersymmetric field theory. On the Higgs branch of this field theory, there are $4mn$ massless scalars and fermions each[3]. So low energy excitations of the D1-D5 system are captured by this gas of $4mn$ scalars and fermions. If we put this gas on a circle and provide right-moving momentum p (this is the relevant configuration to describe the black hole of [1], the left movers must be in their ground state to have a BPS configuration) we can determine the number of ways of partitioning the integer p into this gas of particles. This is a purely combinatorial problem whose solution is given by the partition function:

$$Z(q) = \sum d(k) q^k = [\frac{1+q^k}{1-q^k}]^{4mn} \tag{4.2}$$

where we are interested in the coefficient $d(k)$ for $k = p$. For large p, this coefficient

[3] The theory on the D1-brane world volume is a 1+1 dimensional supersymmetric field theory, which has interacting vector and hyper multiplets, whose lagrangian is fixed by $\mathcal{N} = (4,4)$ SUSY. The theory has a moduli space of supersymmetric vacua, and classically the moduli space has two branches, the Higgs branch, where hypers acquire VEVs, and the Coulomb branch, where scalars in vector multiplets acquire VEVs. The Higgs branch is what is relevant for describing the bound state of D1-D5 branes, while the Coulomb branch describes seperation of D1-branes from the D1-D5 system and is not what we are interested in, if we are trying to describe the black hole.

can be evaluated by saddle point methods to yield:

$$d(p) = e^{2\pi\sqrt{mnp}} \tag{4.3}$$

Using the fact that $Q_H = p$ and $\frac{1}{2}{Q_F}^2 = mn$, we see that (4.3) matches $e^{S_{B.H}}$ with $S_{B.H}$ as given in (3.3)(the above derivation was first performed in [3]). One point to be noted is that the derivation of (4.2) required $p \gg mn$, which is the same as $Q_H \gg {Q_F}^2$ whereas the derivation of (3.3) only used $Q_H \gg Q_F$. This is easily rectifiable, if we note that (4.2) amounts to distributing a momentum of p on mn strings each singly wound on the S^1, whereas for $p \simeq mn$, it becomes thermodynamically favorable to have one long string wound mn times on the S^1. This rectifies the problem for all $Q_H \gg Q_F$ and we recover (4.3) again, from which we see that for all $Q_H \gg Q_F$, the microscopic entropy precisely matches the macroscopic prediction based on the Bekenstein-Hawking formula. A more sophisticated computation of the black hole entropy which is valid throughout the range $Q_H \gg Q_F$ proceeds as follows [1]: the Higgs branch of the D1-D5 system has a moduli space of dimension $4mn$. In the IR, the gauge coupling of the $U(n)$ gauge theory blows up, so the entire $U(n)$ vector multiplet decouples in the IR. This leaves behind $4mn$ scalars and fermions which parameterize a sigma model, which is a hyper-Kæhler sigma model[4] (on the Higgs branch any $\mathcal{N} = (4,4)$ supersymmetric sigma model has as target space a hyper-Kæhler manifold, in our case the specific target space is $\frac{(T^4)^{mn}}{S_{mn}}$, which has dimension $4mn$). This leads to a $1+1$ dimensional $\mathcal{N} = (4,4)$ SCFT in the IR, with a central charge $c = 6mn$ (a hyper-Kæhler sigma model is super conformally invariant for all radius, so scaling to the large radius limit, we get a flat target space with dimension $d = 4mn$, whose central charge is simple $\frac{3}{2}d$, where we used the fact that in $2d$ a scalar counts as 1 degree of freedom while a fermion counts as $\frac{1}{2}$-degree of freedom). By Cardy's formula[4], one can determine the asymptotic density of states of the right moving states in the SCFT

[4] A complex manifold is a real $2n$ dimensional manifold with a type(1,1) form J, which squares to -1 and which can be put in the form $J^a_b = i\delta^a_b$, in suitable complex coordinates patch by patch on the manifold. A complex manifold is Kæhler if there exists a hermitian metric g such that the 2-form $J^{[a}_c g^{c|b]}$ is covariantly constant. Such a Kæhler manifold is called hyper-Kæhler if it in addition contains a nowhere vanishing symplectic 2-form. The simplest non trivial example of a hyper-Kæhler manifold is $\mathrm{T}^*\mathbf{P}^n$, the holomorphic cotangent bundle of complex projective space.

for large p (where p is the momentum on S^1). This in turn yields a degeneracy exactly as in (4.3). Cardy's formula determines the asymptotic form of the partition function of a unitary SCFT on a torus schematically of the form $Z(q) \simeq \sum q^{L_0 - \frac{c}{24}}$ (L_0 refers to the scaling weight) as a function of the modular parameter q, to be $Z(q) \rightarrow e^{\frac{\pi^2 c}{6(1-q)}}$, for $q \rightarrow 1$ from which we can determine the asymptotic degeneracy by a saddle point evaluation to be given precisely by (4.3).

References

[1] A. Strominger and C. Vafa, "Microscopic Origin of the Bekenstein-Hawking Entropy," Phys. Lett. B **379**, 99 (1996) [arXiv:hep-th/9601029].

[2] J. Polchinski, "Dirichlet-Branes and Ramond-Ramond Charges," Phys. Rev. Lett. **75**, 4724 (1995) [arXiv:hep-th/9510017].

[3] C. G. . Callan and J. M. Maldacena, "D-brane Approach to Black Hole Quantum Mechanics," Nucl. Phys. B **472**, 591 (1996) [arXiv:hep-th/9602043].

[4] J. L. Cardy, "Operator Content Of Two-Dimensional Conformally Invariant Theories," Nucl. Phys. B **270**, 186 (1986).