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Aharonov-Bohm effect

Imagine an infinite straight solenoid along the z axis, with radius R and magnetic field $\mathbf{B} = B\hat{\mathbf{z}}$ inside and zero outside.

1. Show that the integral $\oint \mathbf{A} \cdot \mathbf{dl}$ of the vector potential $\mathbf{A}$ around any path that loops around the solenoid must be equal to $\Phi = B\pi R^2$, the magnetic flux threading the solenoid.

2. Find a vector potential for this magnetic field, both inside and outside the solenoid. For the part inside use problem 5.24(a) from hw5. For the part outside use the previous part of this problem to make a guess. Then check that the curl of your vector potential is indeed equal to $\mathbf{B} = B\hat{\mathbf{z}}$ inside, and $\mathbf{B} = 0$ outside.

3. The wave-function of a charge q moving along a path γ in a magnetic field picks up a phase $\exp\Big(i(q/\hbar)\int_\gamma \mathbf{A} \cdot \mathbf{dl}\Big)$. A gauge transformation changes this phase, but that doesn't affect the probability density. However phase *differences* matter. Show that if the charge follows a superposition corresponding to two paths γ_1 and γ_2, both connecting point a to point b, then the *relative* phase associated with the two paths picks up the factor $\exp\Big(i(q/\hbar)\oint \mathbf{A} \cdot \mathbf{dl}\Big)$, where the integral loops around the path consisting of γ_1 followed by the reverse of γ_2.

4. The above results imply that the charged quantum particle can "feel" the presence of a magnetic field, even if the particle travels only in the region where the magnetic field is zero! This is called the *Aharonov-Bohm effect* (see Wikipedia for references). Show that the effect of the magnetic field on the particle depends on the magnetic field only through the total flux, modulo integer multiples of the magnetic flux quantum $\Phi_0 = h/q$, where $h = 2\pi\hbar$ is Planck's constant. In particular, if the flux is an integer multiple of h/q there is no effect at all.