HW#7 —Phys374—Spring 2007
Due before class, Friday, March 30, 2007
www.physics.umd.edu/grt/taj/374b/

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1. *Gravity waves on water* supplement, Exercise **b**. This involves showing that the individual bits of water move in ellipses whose size and ellipticity depend upon the depth. [10+2+1+2=15 pts.]

2. Problems 11.5a,b,c,d,e,f,g,h,i (*Explosion of a nuclear bomb*)
Note: When Snieder says at the beginning of this section that the neutron current is given by (11.29), he means that it is given *by analogy with* this equation. *Hints*: (b) If a function of only t is equal to a function of only r, then the two functions must both be constant. The value of this common constant is what is called in the book the "separation constant". (e) The neutron density is finite at $r = 0$, and it goes to zero at $r = R$, since once the neutrons reach the surface they quickly fly off. (i) Justify your answer. [1+3+1+2+2+2+1+2+1=15 pts.]

3. Problem 12.2f (*Estimated and actual derivative*) [5pts.]

4. *Quantized superfluid vortices*

   A superfluid is a fluid whose microscopic thermal motions are strictly absent because of quantum mechanical behavior at low temperature. The velocity field of a superfluid composed of particles of mass m has the form $\mathbf{v} = (\hbar/m)\nabla\varphi$, where $\hbar$ is Planck's constant. The scalar function $\varphi(\mathbf{r})$ is actually the phase of a complex function $\psi(\mathbf{r}) = A(\mathbf{r})e^{i\varphi(\mathbf{r})}$ that describes the quantum state of the superfluid. [2+2+3+3=10 pts.]

   (a) Show that the vorticity vanishes in a superfluid.

   (b) Suppose there is a circulating flow in a superfluid, so that on some loop the circulation $\oint \mathbf{v} \cdot \mathbf{dl}$ is nonzero. Show that this implies that somewhere on a surface spanning the loop the vorticity must be nonzero.

   (c) You might conclude from the previous two parts that there cannot be a circulating flow in a superfluid, however this is not so. The loop in the previous part may encircle a vortex, inside of which the fluid is not in the superfluid state. On the other hand, it is still true that everywhere on the loop, which lies entirely in the superfluid state, the velocity is proportional to the gradient of φ.

      i. Show that if φ were a continuous function, the circulation would necessarily be zero on any loop, no matter what is inside the loop.

      ii. Since φ is by definition a *phase*, it need not be continuous. It is sufficient that $e^{i\varphi}$ be continuous. This allows φ to have jumps that are an integer multiple of 2π. Show that this means that the circulation around and enclosed vortex can take on only a discrete set of values, labeled by an integer. What are these values?