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1. Show that if $\mathbf{v}$ has vanishing divergence, then $\mathbf{v}$ is equal to the curl of $\mathbf{w}$ defined by the components

$$w_x(x,y,z) = \int_0^z dz'\, v_y(x,y,z') \tag{1}$$
$$w_y(x,y,z) = -\int_0^z dz'\, v_x(x,y,z') + \int_0^x dx'\, v_z(x',y,0) \tag{2}$$
$$w_z(x,y,z) = 0. \tag{3}$$

This proves by explicit construction that every divergenceless vector field can be expressed as the curl of another vector field. [5 pts.]

2. (i) Show that $\mathbf{F} = yz\,\hat{\mathbf{x}} + zx\,\hat{\mathbf{y}} + xy\,\hat{\mathbf{z}}$ is both divergence free and curl free, so that it can be written both as the gradient of a scalar and as the curl of a vector. (ii) Find scalar and vector potentials for $\mathbf{F}$. (iii) Explain why the scalar potential must satisfy Laplace's equation, and verify that your scalar potential does so. [5+5+5=15 pts.]

3. Prove that solutions to Laplace's equation $\nabla^2 f = 0$ in a compact volume $\mathcal{V}$ are uniquely determined the values of f on the closed boundary surface $\partial\mathcal{V}$. [5 pts.]
(*Hint*: Suppose two solutions f_1 and f_2 agree on $\partial\mathcal{V}$. Consider the function $g = f_2 - f_1$, and remember what you know about maxima and minima of harmonic functions.)

4. *Gravity waves on water* supplement, Exercise **a**. [3+2=5 pts.]