HW#4 —Phys374—Spring 2007
Due before class, Friday, Feb. 23, 2007
www.physics.umd.edu/grt/taj/374b/

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1. Problems 5.2a,b. (*Pressure gradient*) (For 5.2a, photocopy the map.) For 5.2b, you'll have to work out or look up the distance across Ireland in the relevant direction.) [5+5=10 pts.]

2. Problems 5.2e,f,g. (*Pressure force*) Snieder basically does these for you, so all that's left to do is just put in a few precise and concise words explaining why each equation is true. (Note $\hat{\mathbf{x}}$ should not be present in (5.13) & (5.14) since teh left hand side is just a *component* of the force vector. [4+3+3=10 pts.]

3. Consider the function $\varphi(x, y, z) = z - 3xy$.

   (a) Find $\nabla\varphi$. [3 pts.]

   (b) What is the rate of change of φ at the point $(0, 1, 2)$ in the direction of the vector $\hat{\mathbf{i}} + \mathbf{2}\hat{\mathbf{j}} + \mathbf{3}\hat{\mathbf{k}}$? (*Be careful*, this is not a unit vector.) [4 pts.]

   (c) What is the rate of change of φ in the direction of most rapid increase at the point $(0, 1, 2)$? [3 pts.]

4. Derive the identity
$$\nabla \cdot (f\mathbf{v}) = \nabla f \cdot \mathbf{v} + f\nabla \cdot \mathbf{v}, \tag{1}$$
where f is a scalar field and $\mathbf{v}$ is a vector field. [5 pts.]

5. Evaluate the expression
$$\nabla \cdot \mathbf{r}, \tag{2}$$
where $\mathbf{r}$ is the position vector from the origin to the point $\mathbf{r}$, using (i) Cartesian coordinates and (ii) spherical coordinates (cf. (6.32)). [3+2=5 pts.]

6. (a) Find the most general function $f(r)$ of only the radial coordinate $r = |\mathbf{r}|$ such that $\nabla \cdot (f\mathbf{r}) = 0$ except possibly at the origin $\mathbf{r} = 0$. (b) Find the most general function $g(r, \theta, \varphi)$ such that $\nabla \cdot (g\mathbf{r}) = 0$, again except possibly at the origin $\mathbf{r} = 0$. [4+1=5 pts.]

7. Problems 10.1a,b,c,d. (*Curvature of a function*)
[2+1+4+3=10 pts.]

8. Problem 10.1 g (*Stability of equilibrium*)
[5 pts.]