HW#11 —Phys374—Spring 2007
Due before class, Friday, April 27, 2007
www.physics.umd.edu/grt/taj/374b/

Prof. Ted Jacobson
Room 4115, (301)405-6020
jacobson@physics.umd.edu

1. Problems 15.6 g,h (Fourier transform of correlation and Parseval's theorem) (*Note*: The conventions (15.42), (15.43) are used here.) [10 pts.]

2. Find the Fourier transform of $f(t) = A\sin(\omega_0 t + \varphi)$. [10 pts.]

3. **Response function of an *RL* circuit** [20 pts.]

   Consider a circuit consisting of an inductance L and a resistance R connected in series. The input voltage $V_{in}(t)$ is applied to the series of L and R, whereas the output voltage $V_{out}(t)$ is taken from across the resistor R. The circuit equations are

$$V_{in}(t) = L\frac{dI(t)}{dt} + R\,I(t) \tag{1}$$
$$V_{out}(t) = R\,I(t) \tag{2}$$

   (a) Equations (1) and (2) are written in the time domain. Fourier-transform these equations to the frequency domain, i.e. obtain equations relating the Fourier transforms $\tilde{V}_{in,\,out}(\omega)$ and $\tilde{I}(\omega)$, which are related to $V_{in,\,out}(t)$ and $I(t)$ by

$$V_{in,\,out}(t) = \int d\omega\, e^{-i\omega t}\,\tilde{V}_{in,\,out}(\omega), \quad I(t) = \int d\omega\, e^{-i\omega t}\,\tilde{I}(\omega). \tag{3}$$

   (b) Eliminate $\tilde{I}(\omega)$ from the equations obtained in Part 3a and obtain a relation between the input and output voltages in the frequency domain:

$$\tilde{V}_{out}(\omega) = 2\pi\,\tilde{G}(\omega)\,\tilde{V}_{in}(\omega). \tag{4}$$

   The function $\tilde{G}(\omega)$ is the Fourier transform of the *response function*. Derive an explicit expression for $\tilde{G}(\omega)$.

   (c) Find an explicit expression for the response function $G(t)$ in the time domain. Show that it vanishes for $t < 0$ and explain why this makes sense physically.

   (d) Show that

$$V_{out}(t) = \int_{-\infty}^{t} dt'\,G(t-t')\,V_{in}(t'). \tag{5}$$

   Notice that if $V_{in}(t) = \delta(t)$, then $V_{out}(t) = G(t)$. That is, up to the numerical factor, the response function gives the response to a delta function impulse.

   Suggestions: Section 15.7 discusses the general theory of linear filters, of which this circuit is an example. Section 17.4 discusses a mechanical analog of this circuit, consisting of a particle moving through syrup under the influence of an external driving force. The mathematical description is identical, where the roles of L, R, $I(t)$, $V_{in}(t)$ and $V_{out}(t)$ are played by m, β, $v(t)$, $f(t)$, and $\beta v(t)$ respectively. You can use Problem 17.4 for guidance, but don't follow the steps directly. This circuit problem is organized differently, in order to help you see the general pattern.