HW#1 —Phys374—Spring 2007

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1. **Scaling of cruising velocity of flying objects with respect to mass**: Problems 2.5d,e from from textbook. (You need not (but may) follow the steps and instructions in parts a,b,c,d. The book's method seems a bit inelegant to me. Note that eqns (2.30) and (2.31) should have $\sim$ signs rather than $=$ signs.)

2. **Flow rate through a pipe**: Problems 2.6a,b,c from textbook. (*Hint for the last part*: the flow velocity is always zero at the walls...)

3. **Speed of surface waves on water**

   The *phase velocity* of a wave is the speed of wavefronts of constant phase, while the group velocity is the speed of a wavepacket. Both have dimensions of velocity, of course, so dimensional analysis will not distinguish them.

   The speed of water waves might depend upon various quantities: (i) properties of the water such as density ρ or surface tension (energy per unit area of stretching of a surface) σ, (ii) properties of the environment such as gravitational acceleration g or depth of the body of water h, and (iii) the wavelength of the wave λ. Depending on the wavelength of the wave different quantities are important in determining the wave speed. The speed is determined by the restoring force(s) that tend to return the surface to equilibrium, and the inertia of the water.

   (a) First consider wavelengths long enough that surface tension can be neglected but short enough that the depth of the body of water can be neglected (*deep water waves*). (a) Use dimensional analysis to find how the wave speed depends on the remaining quantities ρ, g, λ. (b) Estimate (modulo the unknown dimensionless coefficient) the speed of a one meter wave on a lake.

   (b) Next consider wavelengths much longer than the depth (*shallow water waves*). Now the speed may depend upon the depth h as well, so there is a dimensionless ratio h/λ that the speed could depend on. However, one might reason that when this ratio is very small, the dependence on λ should drop out, since the restoring force is a local process and the concept of wavelength involves very distant parts of the wave. How then would the wave speed depend upon the various quantities ρ, g, h? Estimate (modulo the unknown dimensionless coefficient) the speed of a (long wavelength) tsunami on the Pacific ocean of depth 4 km.

   (c) Finally consider wavelengths short enough that the surface tension is the dominant restoring force and gravity can be neglected (*capillary waves*). How does the speed of capillary waves depend upon the remaining quantities ρ, λ, σ?

   (d) The phase velocity v_{ph} of a wave of angular frequency ω and wave vector k is ω/k, while the group velocity v_g of a wavepacket consisting of waves with wave vector near a given k is $d\omega/dk$. What is the exact ratio of group velocity to phase velocity for the three cases considered above? (Recall that $k = 2\pi/\lambda$.) (*Hint*: Consider the question for a dispersion relation of the general form $\omega = ck^n$ for some constant c...)

(e) In the above cases, only the capillary wave speed depends on the density of the water. Explain physically why this is the case.

4. **Ideal gas law**: Near the beginning of the nineteenth century Avogadro inferred from experiments the remarkable fact that the number of molecules N in a gas contained in a volume V at a pressure P and temperature T is the same for all gases. He deduced that the molecules must occupy only a very small fraction of the total volume. This led to the kinetic theory of gases, in which the pressure arises from collisions between the molecules and the walls.

   Use dimensional analysis to determine the form of the function $N(V, P, T)$. Although it was not known at the time of Avogadro, make use of the fact that there is a universal constant k, *Boltzmann's constant*, such that kT has dimensions of energy. (kT is proportional to the average kinetic energy of a single molecule.) Note that your result is just the ideal gas law!