Exam#1 —Phys374—Spring 2006
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www.physics.umd.edu/grt/taj/374a/

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1. We previously used dimensional analysis to determine the dependence of drag force on a body moving through air. In so doing we took account of the density of the air but neglected its viscosity. For this problem consider the effect of viscosity: suppose a sphere of radius R is moving with speed v through a fluid with viscosity η.

   (a) Using dimensional analysis determine how the viscous drag force can depend on R, v, and η. (*Hint*: Recall that the viscous force in the presence of a velocity gradient perpendicular to a surface is $\eta(dv/dx)$ per unit surface area.) [8 *pts.*]

   (b) Using "geometrical analysis", would you expect any factors of π in the exact result for the force? If so, how many, and exactly where? [2 *pts.*]

2. Consider the cubic equation $ay^3 + y + 2 = 0$, with $a > 0$.

   (a) Display the location of the real roots graphically, by sketching the graphs of $y+2$ and $-ay^3$ and seeing where they intersect. Show in your sketch three cases: a is small, equal to, and large compared to 1. [2 *pts.*]

   (b) Determine the leading order a dependence of the roots in the limits (i) $a \ll 1$ and (ii) $a \gg 1$. [8 *pts.*]

3. A particle of mass m moves in a region with potential energy $V(x) = \lambda(x^2 - a^2)^2$, where $\lambda > 0$.

   (a) Use the Taylor expansion to show that near $x = a$ the potential energy has the form of a harmonic oscillator potential centered at $x = a$. Find the effective "spring constant" k of this oscillator. [8 *pts.*]

   (b) Determine the condition on $|x - a|$ required for the potential at x to be well approximated by a harmonic oscillator. [2 *pts.*]

4. Consider the function $\varphi(x, y, z) = z - 3xy$.

   (a) Find $\nabla\varphi$. [2 *pts.*]

   (b) What is the rate of change of φ at the point $(0, 1, 2)$ in the direction of the vector $\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$? [3 *pts.*]

   (c) What is the rate of change of φ in the direction of most rapid increase at the point $(0, 1, 2)$? [3 *pts.*]

   (d) What is the average of φ over a sphere of radius 2 centered on the point $(0, 1, 2)$? [2 *pts.*] Justify your answer.

5. We showed from Maxwell's equations that in a region of space containing no charges the electric field satisfies the wave equation $\partial_t^2 \mathbf{E} = c^2 \nabla^2 \mathbf{E}$.

   (a) Show that in a region of space containing no charges the x-component of a time-independent electric field satisfies $\nabla^2(E_x^2) = (\nabla E_x) \cdot (\nabla E_x)$. [5 *pts.*]

   (b) The same goes for y and z components. Explain how this can be used to show that there can be no maximum of the magnitude of the electric field in such a region.[1] [5 *pts.*]

[1] As explained in the text, this is why lightning never starts in mid-air away from charges.