HW#5 —Phys374—Spring 2006
Due before class, Wednesday, Mar. 8, 2006
www.physics.umd.edu/grt/taj/374a/

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1. Derive the following identities:

$$\nabla \cdot (f\mathbf{v}) = \nabla f \cdot \mathbf{v} + f\nabla \cdot \mathbf{v}, \qquad \nabla \times (f\mathbf{v}) = \nabla f \times \mathbf{v} + f\nabla \times \mathbf{v} \tag{1}$$

   where f is a scalar field and $\mathbf{v}$ is a vector field.

2. Evaluate the following expressions:

$$\nabla \cdot \mathbf{r}, \qquad \nabla \times (f(r)\mathbf{r}) \tag{2}$$

   where $\mathbf{r}$ is the position vector from the origin to the point $\mathbf{r}$, and $r = |\mathbf{r}|$.

3. In the previous problem you should have found that all spherically symmetric radial vector fields are curl-free, but such vector fields are not generally divergence-free. (a) Find the most general function $f(r)$ of only the radial coordinate r such that $\nabla \cdot (f\mathbf{r}) = 0$ except possibly at the origin $\mathbf{r} = 0$. (b) Find the most general function $g(r, \theta, \varphi)$ such that $\nabla \cdot (g\mathbf{r}) = 0$, again except possibly at the origin $\mathbf{r} = 0$.

4. A harmonic function f is one whose Laplacian vanishes, $\nabla^2 f = 0$. A remarkable fact about such a function is that the average of its value over any two concentric spheres is equal. Since the average over an infinitesimal sphere at a point is just the value at that point, this is equivalent to saying that the average over any sphere is equal to the value at its center. Prove this remarkable fact in the following steps

   (a) Use Gauss' theorem to show that the flux of the gradient of a harmonic function f through any closed surface vanishes, $\oint \nabla f \cdot \mathbf{dS} = 0$.

   (b) The average of f over a closed surface is defined as $(\oint f dA)/(\oint dA)$. Use the previous part to show that for a fixed center, the average of f over a sphere of radius r is independent of r.

5. Problems 11.5a,b,c,d,e,f,g,h,i (*Explosion of a nuclear bomb*)
   Note: When Sneider says at the beginning of this section that the neutron current is given by (11.29), he means that it is given *by analogy with* this equation.