HW#4 —Phys374—Spring 2006
Due before class, Wednesday, Mar. 1, 2006
www.physics.umd.edu/grt/taj/374a/

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The goal of this homework set is to develop familiarity and intuition with various notions of vector calculus as applied to physics. Most of the problems are essentially already solved in the textbook or the Maxwell's equations supplement at the course web page. Your job is to digest this material and write out the solution, clearly explaining the reasoning. Try to be very succinct, i.e. precise and concise (short and sweet—but complete). I suggest you get started right away, and let me know if you get stuck on anything—I don't want you to get bogged down.

1. Problems 5.2a,b. (*Pressure gradient*) (For 5.2a, photocopy the map. For 5.2b, you'll have to work out or look up the distance across Ireland in the relevant direction.) [6+6=12 pts.]

2. Problems 5.2e,f,g. (*Pressure force*) Snieder basically does these for you, so all that's left to do is just put in a few words explaining why each equation is true. [3+3+3=9 pts.]

3. Problems 8.4a,b,c,d,e,f & 11.1a,b (*Quantum mechanical probability current density*) [8×3=24 pts.]

4. Problems 9.6a,b,c (*Wingtip vortices*) [3+3+3=9 pts.]

5. In the supplement on Maxwell's equations, equation (14) expresses conservation of electromagnetic field energy plus charged particle energy. Derive this equation from Maxwell's equations. (*Hint*: The supplement gives some guidance.) [12 pts.]

6. We showed in class that the electric field in vacuum satisfies the 3d wave equation with a wave speed equal to $c = (\epsilon_0\mu_0)^{-1/2}$.

   (a) Show using Maxwell's equations that the magnetic field satisfies the same wave equation. [6 pts.]

   (b) For fields that are plane symmetric, which depend only on z and t, each field component is therefore a superposition of a function of $z - ct$ and a function of $z + ct$ (see the string wave problem on hw3). Assuming a purely unidirectional wave, $\mathbf{E} = \mathbf{E}(z - ct)$ and $\mathbf{B} = \mathbf{B}(z - ct)$, show that Maxwell's equations imply that, apart from constant fields, $\mathbf{E}$ and $\mathbf{B}$ are perpendicular to each other and to the z-direction, that $|\mathbf{E}| = c|\mathbf{B}|$, and that $\mathbf{E} \times \mathbf{B}$ is in the $+z$ direction. (*Hint*: If you need some help see the discussion in the supplement on Maxwell's equations.) [18 pts.]