HW#10 —Phys374—Spring 2006 Prof. Ted Jacobson
Due before class, Wednesday, April 24, 2006 Room 4115, (301)405-6020
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1. Evaluate the integral $\int_0^\infty dx/(x^2+1)^3$ by relating it to a contour integral. (Note that you'll need the residue at the relevant pole of $(z^2+1)^{-3}$. This has poles of order 3 at $z = \pm i$, but also has a nonzero residue there. Be careful! The previous problems I gave did not involve higher order poles like this. See the updated complex numbers supplement for a brief explanation of how to find the residue in a generic case like this.)

2. In class we found that the Fourier series on the interval $x \in (-L, L)$ for a square step equal to 1 in the interval $x \in (-a, a)$ and zero outside is

$$(2a/L) + \sum_n (2/n\pi)\sin(n\pi a/L)\cos(n\pi x/L).$$

These Fourier components do not die off very fast as n grows, a consequence of the sharp corners on the square step. Let's replace the step by a smooth bump $B_a(x) = (2/\sqrt{\pi})\exp(-x^2/a^2)$ which is equal to $2/\sqrt{\pi}$ at $x = 0$, falls away to zero very quickly for $x > a$, and has the same integral over the range $(-\infty, \infty)$ as the square step.

   (a) Find the Fourier series for $B_a(x)$ assuming that a is sufficiently small compared to L that you can extend the x integration from $(-L, L)$ to (∞, ∞). (It's OK to look up the integral, you need not work it out yourself.)
   (b) Compare the Fourier coefficients for the square step and this smooth bump in terms of the relative importance of different values of n.
   (c) What happens to the Fourier transform of $B_a(x)$ as a gets smaller or larger?

3. Lets go back to section 11.5, *Explosion of a nuclear bomb*, and use the Fourier series to understand the time development of the neutron density function $N(r,t)$. In that section, you assumed a factored form $N(r,t) = F(r)H(t)$, and found the equations satisfied by $F(r)$ and $H(t)$. Then you wrote $F(r) = f(r)/r$ and found that $f(r)$ must be a sine function. After applying the boundary conditions $f(0) = 0 = f(R)$ the solution took the form $N_n(r,t) = A_n \exp(\mu_n t)\sin(k_n r)/r$, where n is a positive integer, A_n is an arbitrary constant, and μ_n and k_n are determined by the diffusion constant κ, the production rate λ, the radius of the sphere R and the integer n. A general solution is a linear combination of such solutions, $N = \sum_n N_n$, with different values of the constants A_n.

   Consider the case when the neutron density at time $t = 0$ is equal to $\bar{N}$ inside a sphere of radius a and zero outside.

   (a) Find the values of the coefficients A_n in this case. Given these, $N(r,t)$ is known for all time.
   (b) Assume the bomb is not a dud, i.e. at least one of the μ_n is positive. (This requires that R be large enough.) The component $N_n(r,t)$ that grows the fastest corresponds to the largest positive μ_n.
      i. Find this component and write it out explicitly as a function of r and t.
      ii. Sketch its graph as a function of r for fixed t.
      iii. Describe what happens to the graph as t grows.

continued on the next page...

4. Problem 17.4 a-j, *Response of a particle to a driving force in syrup*. *Hint*: For part a use the fact that two functions are equal if and only if their Fourier transforms are equal. *Notes*: (i) After (17.37) Snieder refers to (17.33) when (I think) he means to refer to (17.34). (ii) For part i, put into words the argument that you use to “convince yourself”. (iii) As usual with Snieder’s problems, there’s not much to do here since he tells you almost everything. The point of the problem is for you to read through and understand this very nice example, and to write down all the logic in your own hand.