HW#1 —Phys374—Spring 2006
Due before class, Monday, Feb. 6, 2006
www.physics.umd.edu/grt/taj/374a/

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1. **Scaling of cruising velocity of flying objects with respect to mass**: Problems 2.5d,e from from textbook. (You need not (but may) follow the steps and instructions in parts a,b,c,d. The book's method seems a bit inelegant to me. Note that eqns (2.30) and (2.31) should have $\sim$ signs rather than $=$ signs.) [6 pts.]

2. **Flow rate through a pipe**: Problems 2.6a,b,c from textbook. [6 pts.]

3. **Speed of surface waves on water**

   The *phase velocity* of a wave is the speed of wavefronts of constant phase, while the group velocity is the speed of a wavepacket. Both have dimensions of velocity, of course, so dimensional analysis will not distinguish them.

   The speed of water waves might depend upon various quantities: (i) properties of the water such as density ρ or surface tension (energy per unit area of stretching of a surface) σ, (ii) properties of the environment such as gravitational acceleration g or depth of the body of water h, and (iii) the wavelength of the wave λ. Depending on the wavelength of the wave different quantities are important in determining the wave speed. The speed is determined by the restoring force(s) that tend to return the surface to equilibrium, and the inertia of the water.

   (a) First consider wavelengths long enough that surface tension can be neglected but short enough that the depth of the body of water can be neglected (*deep water waves*). (a) Use dimensional analysis to find how the wave speed depends on the remaining quantities ρ, g, λ. (b) Estimate (modulo the unknown dimensionless coefficient) the speed of a one meter wave on a lake. [3 pts.]

   (b) Next consider wavelengths much longer than the depth (*shallow water waves*). Now the speed may depend upon the depth h as well, so there is a dimensionless ratio h/λ that the speed could depend on. However, one might reason that when this ratio is very small, the dependence on λ should drop out, since the restoring force is a local process and the concept of wavelength involves very distant parts of the wave. How then would the wave speed depend upon the various quantities ρ, g, h? Estimate (modulo the unknown dimensionless coefficient) the speed of a (long wavelength) tsunami on the Pacific ocean of depth 4 km. [3 pts.]

   (c) Finally consider wavelengths short enough that the surface tension is the dominant restoring force and gravity can be neglected (*capillary waves*). How does the speed of capillary waves depend upon the remaining quantities ρ, λ, σ? [3 pts.]

   (d) The phase velocity v_{ph} of a wave of angular frequency ω and wave vector k is ω/k, while the group velocity v_g of a wavepacket consisting of waves with wave vector near a given k is $d\omega/dk$. What is the exact ratio of group velocity to phase velocity for the three cases considered above? (Recall that $k = 2\pi/\lambda$.) [3 pts.]

(e) In which of the above cases does the wave speed depend upon the density of the water? Explain physically why this is the case. [3 pts.]

4. **Period of a harmonic oscillator**

In this problem you find the period of a harmonic oscillator by a complicated method that will be useful when considering more general problems. Consider a particle of mass m moving in a potential $V(x)$.

(a) For the case of a harmonic oscillator $V(x) = \frac{1}{2}kx^2$, use dimensional analysis to find how the period depends upon m, k, and the amplitude of oscillation x_{max}. [3 pts.]

(b) Show that the period of an oscillation with total energy E and amplitude x_m in an arbitrary symmetric potential $(V(-x) = V(x))$ is given by

$$T = \sqrt{8m}\int_0^{x_{\text{max}}} \frac{dx}{\sqrt{E - V}}.$$

(*Hint*: The time dt it takes the particle to travel a distance dx is $dx/(dx/dt)$.) [3 pts.]

(c) Evaluate the integral for the case of a harmonic oscillator $V(x) = \frac{1}{2}kx^2$ and show that you recover the usual result. Before carrying out the integral adopt units adapted to the problem at hand, in which $m = 1$ and $k = 1$ and $x_{\text{max}} = 1$. (How do you know such units exist and are unique?) This will simplify the intermediate steps. After obtaining a result, express your answer in arbitrary units by using dimensional analysis to insert the unique combination of m, k, and x_m with the dimension of time. [3 pts.]