Features of the High Frequency QPO Power Spectrum in Accreting Black Holes

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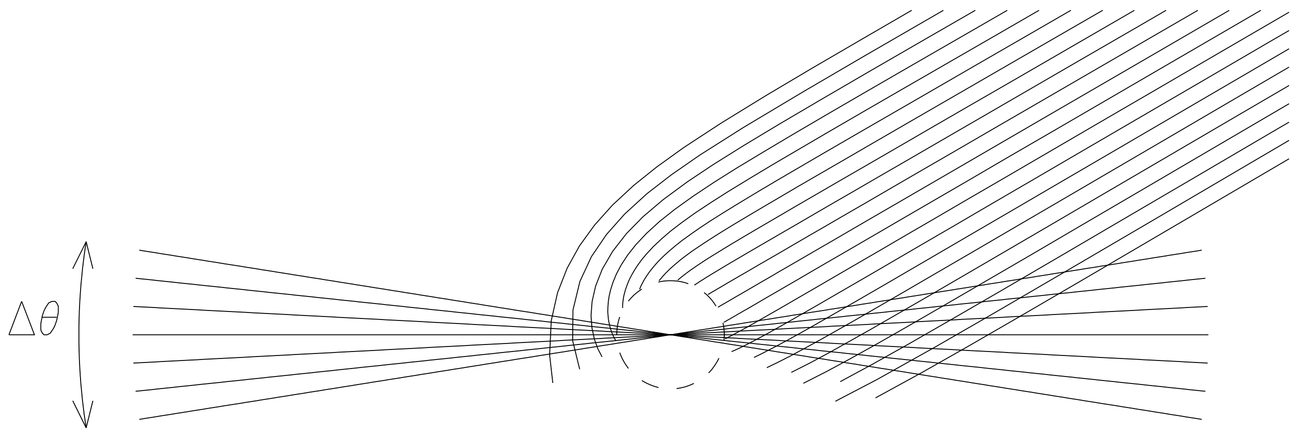
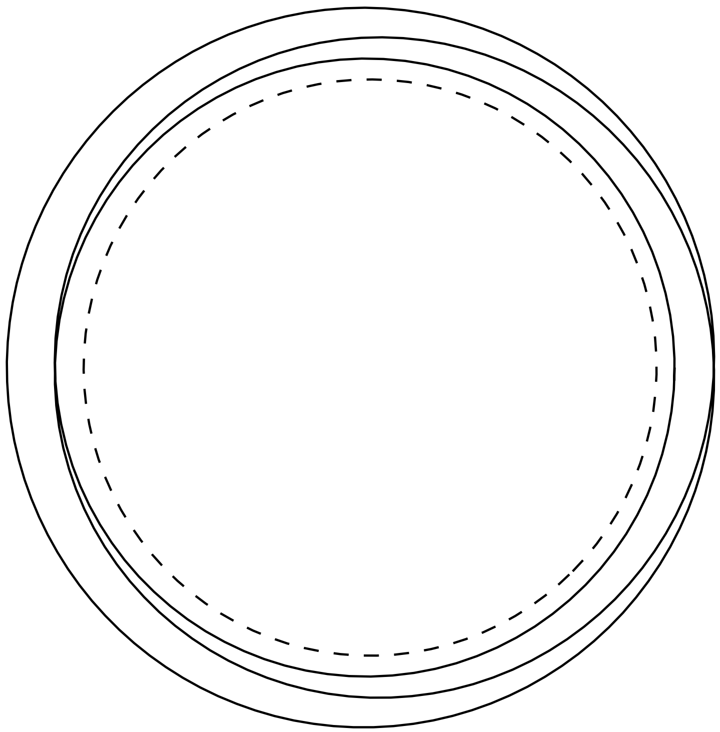
Hot Spot Model

Photons are ray-traced backwards in time from a distant observer through a finite-thickness computational grid.



Light curves are calculated by integrating emission from hot spots orbiting along geodesic orbits

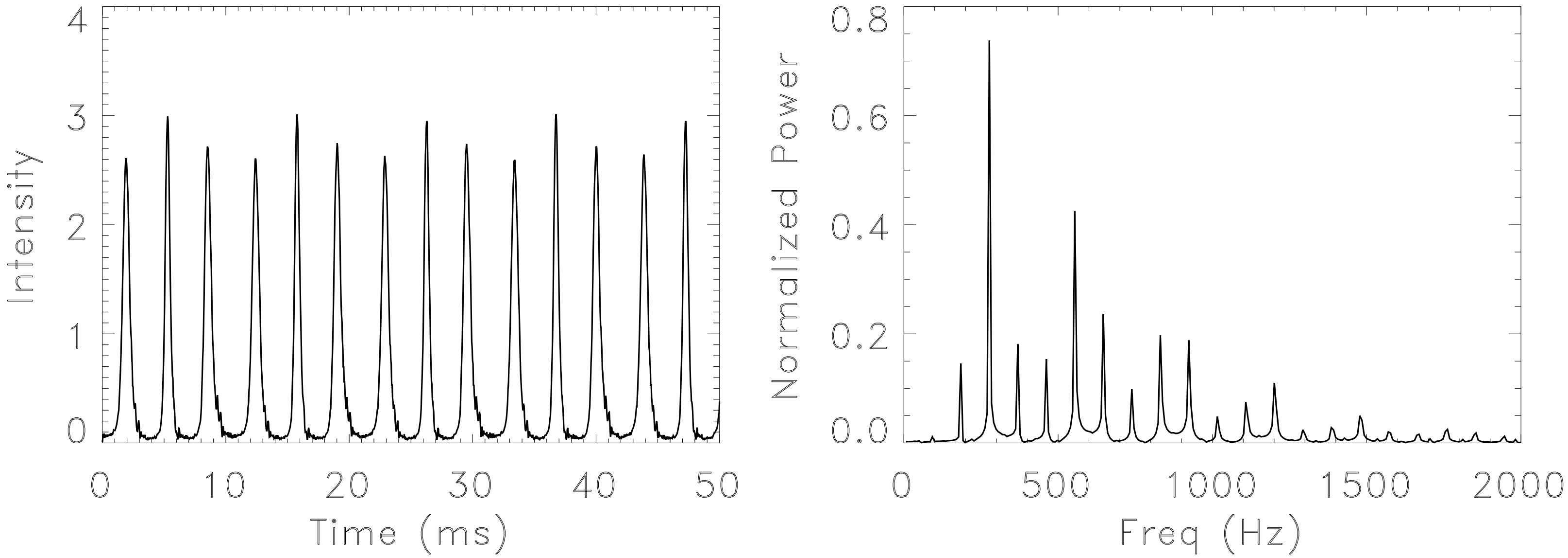
$$e \equiv \frac{r_{max} - r_{min}}{r_{max} + r_{min}} \approx 0.1 \quad R_{\mathrm{ISCO}} = 4.23M \quad \nu_\phi = 3\nu_r$$



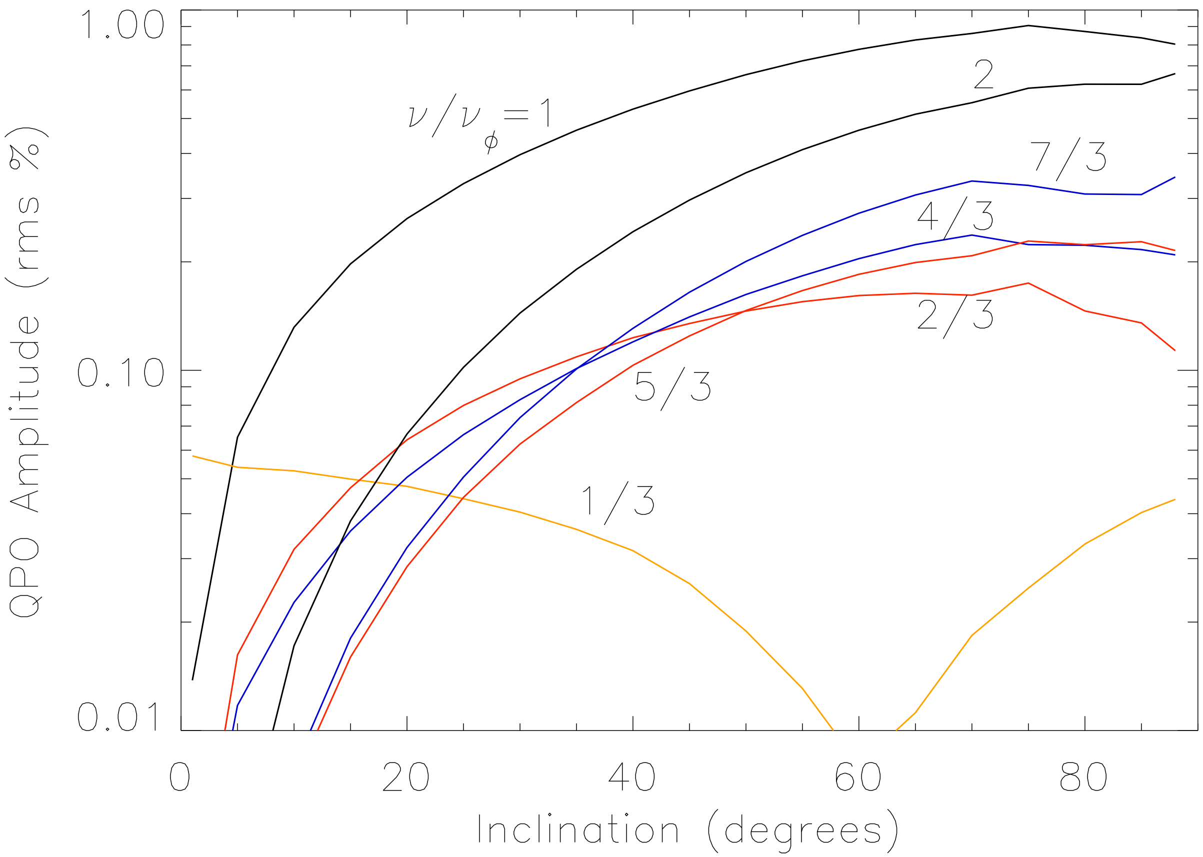
The power spectrum of the X-ray light curve shows the characteristic coordinate frequencies of the hot spot orbit

$$M = 10M_{\odot} \quad a/M = 0.5 \quad r/M = 4.887$$

$$\nu_{\phi} = 285 \text{ Hz} \qquad \nu_r = 95 \text{ Hz}$$



Higher inclination systems giver higher amplitude QPOs and stronger harmonic modes



Random Phase Broadening

We model the light curve as a superposition of periodic functions with random phases

If the Fourier amplitudes of the periodic light curve $f(t)$ are given by A_j, then the power spectrum $F^2(\nu)$ from a single hot spot with lifetime T is given by the convolution of the delta-functions $\delta(\nu - \nu_j)$ with the transform of a boxcar window function:

$$F^2(\nu) = \sum_j A_j^2 \frac{\sin^2[\pi(\nu - \nu_j)T]}{4T_f^2 \pi^2 (\nu - \nu_j)^2}, \tag{1}$$

where T_f is the total duration of the observation.

For a large number of hot spots with an exponential distribution of lifetimes (with characteristic lifetime T_l), if there are an average of $N_{\rm spot}$ hot spots in existence at any time, the number of hot spots formed with a lifetime between T and $T + dT$ is given by

$$dN(T) = N_{\rm spot} \frac{T_f}{T_l^2} e^{-T/T_l} dT. \tag{2}$$

Since the hot spots all have random phases, their contributions to the power spectrum add in quadrature giving a total power spectrum of

$$\begin{aligned} \tilde{I}^2(\nu) &= \int_0^\infty F^2(\nu, T) dN(T) \\ &= N_{\rm spot} \sum_j \left(\frac{A_j}{2\pi T_f} \right)^2 \int_0^\infty \frac{\sin^2[\pi(\nu - \nu_j)T]}{\pi^2 (\nu - \nu_j)^2} \frac{T_f}{T_l^2} e^{-T/T_l} dT \\ &= 2N_{\rm spot} \sum_j A_j^2 \frac{T_l}{T_f} \frac{1}{1 + 4\pi^2 T_l^2 (\nu - \nu_j)^2}. \end{aligned} \tag{3}$$

This result can actually be generalized for any set of self-similar sampling functions $w(t,T) = w(t/T)$, with corresponding power spectra $W^2(\nu,T)$ approximated around $\nu = 0$ as a Lorentzian:

$$W^2(\nu,T) \approx \frac{T^2}{T_f^2} \frac{1}{1+\beta^2 T^2 \nu^2}, \tag{4}$$

with β a dimensionless constant over the set of $w(t,T)$. The characteristic width of $W^2(\nu,T)$ is thus defined as $1/(\beta T)$. Integrating over the exponential lifetime distribution $dN(T)$, the net power $\tilde{I}^2(\nu)$ around $\nu = \nu_j$ is given by

$$\tilde{I}^2(\nu) \approx \tilde{I}^2(\nu_j) \frac{1}{1+12\beta^2 T_l^2 (\nu-\nu_j)^2}. \tag{5}$$

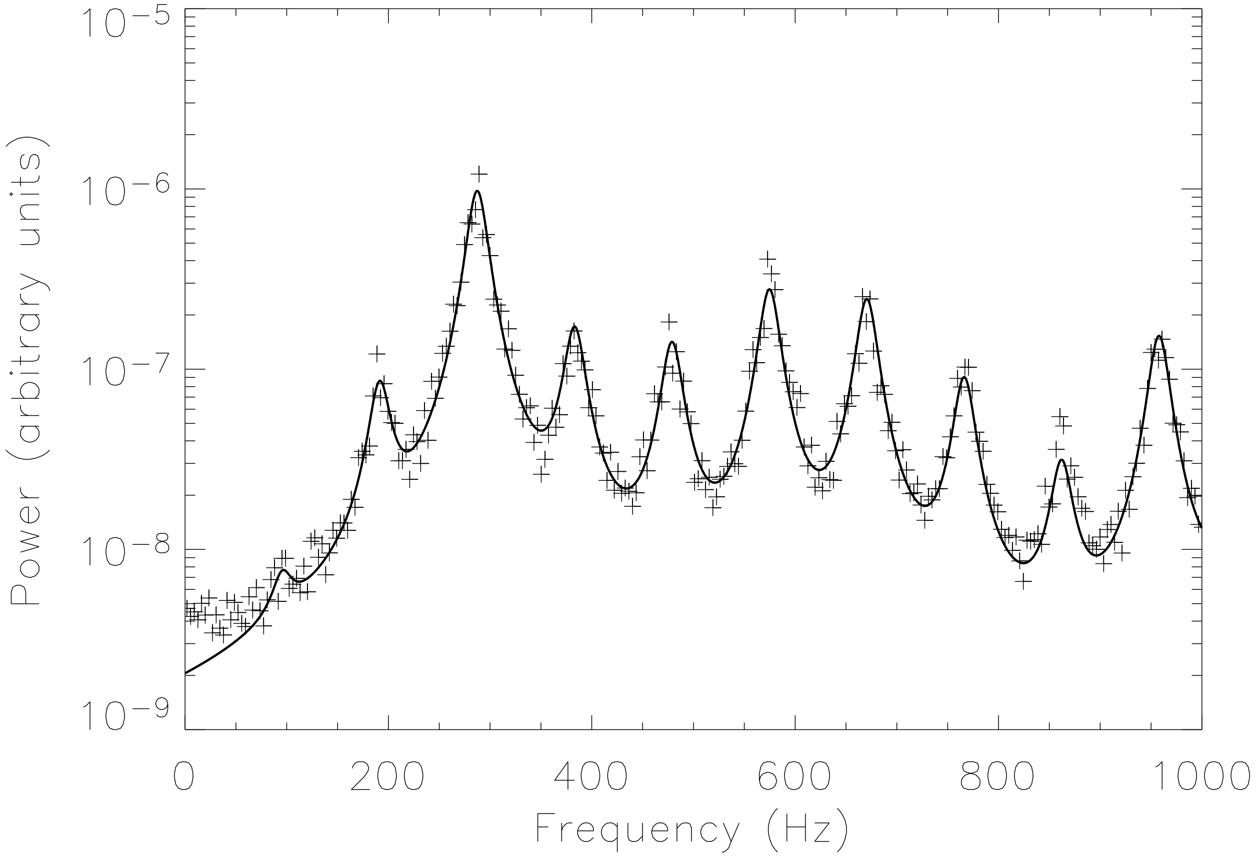
We see now that the general effect of an exponential distribution of hot spots with random phases is to decrease the peak width, and thus increase the coherency, by a factor of $\sqrt{12}$.

For the case of boxcar sampling functions, the power spectrum has Lorenztian peaks of constant width

$$\boxed{\Delta\nu_{\rm phase} = \frac{1}{2\pi T_l}}$$

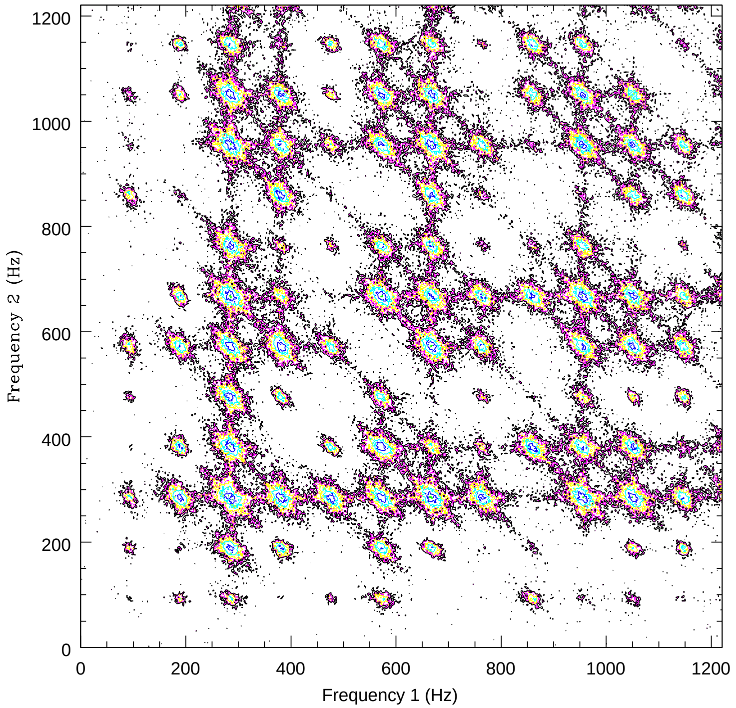
We can verify the results of the analytic model by simulating the light curves from many hot spots with random phases

$\Delta\nu = 11$ Hz $\quad T_l = 14.5$ msec ≈ 4 orbits



The bicoherence of such a light curve shows symmetric broadening around the coordinate frequencies [1]

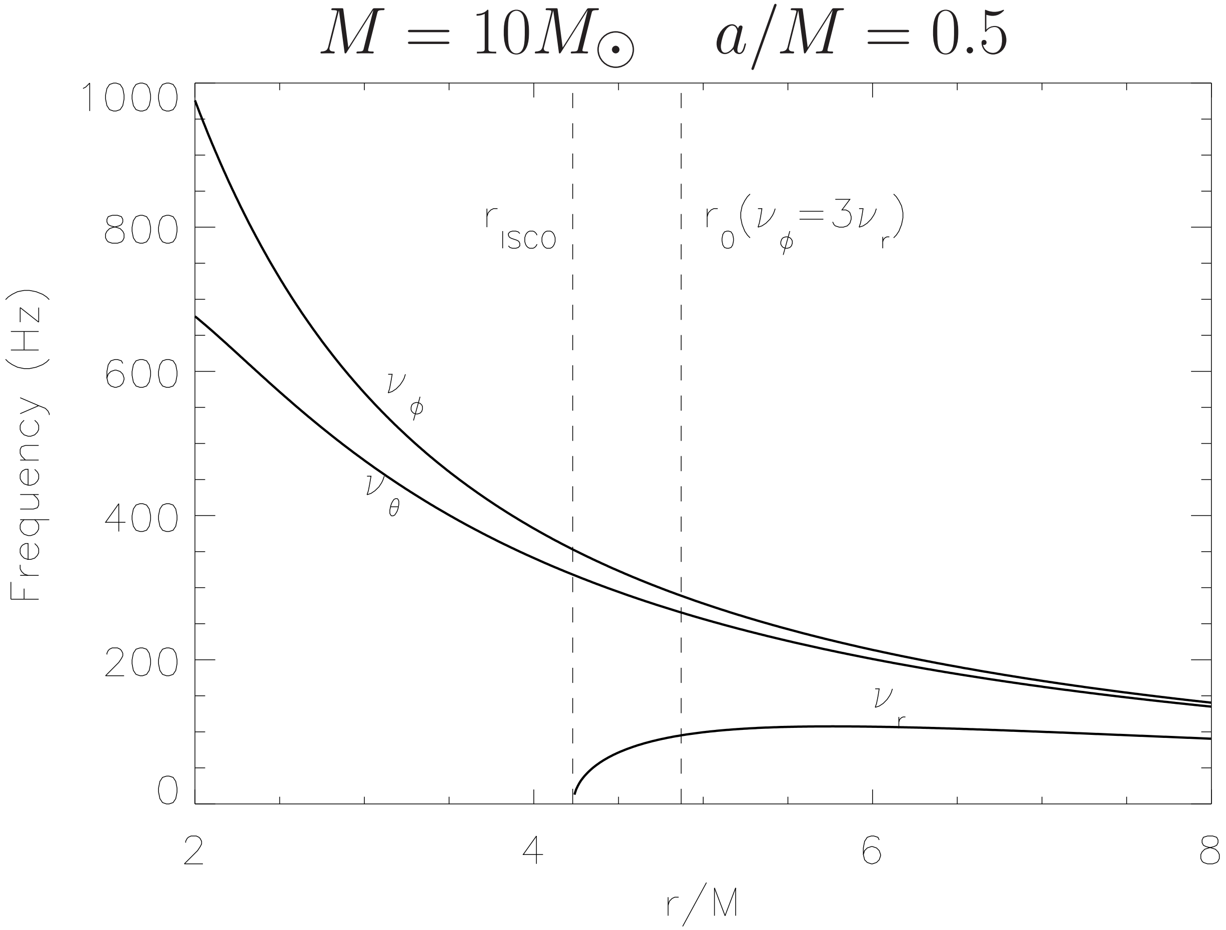
$$b^2(k,l) = \frac{\left|\Sigma F_i(k)F_i(l)F_i^*(k+l)\right|^2}{\Sigma\left|F_i(k)F_i(l)\right|^2 \Sigma\left|F_i(k+l)\right|^2}.$$



[1] Maccarone, T. J., & Schnittman, J. D. (2004), in preparation

Coordinate Frequency Broadening

Kerr orbits have three distinct coordinate frequencies



A finite distribution in orbital radii gives a characteristic width to the coordinate frequencies

Like many other resonances in nature, we assume a Lorentzian distribution in orbits around the commensurate radius r_0 (where $\nu_\phi = 3\nu_r$):

$$P(r)dr = \frac{dr/(\pi \Delta r)}{1 + \left(\frac{r-r_0}{\Delta r}\right)^2}. \tag{6}$$

For a relatively small resonance width Δr, we can linearize the coordinate frequencies $\nu_j(r)$ around $r = r_0$ with a simple Taylor expansion:

$$\nu_j(r) \approx \nu_{j0} + (r - r_0) \left.\frac{d\nu_j}{dr}\right|_{r_0}, \tag{7}$$

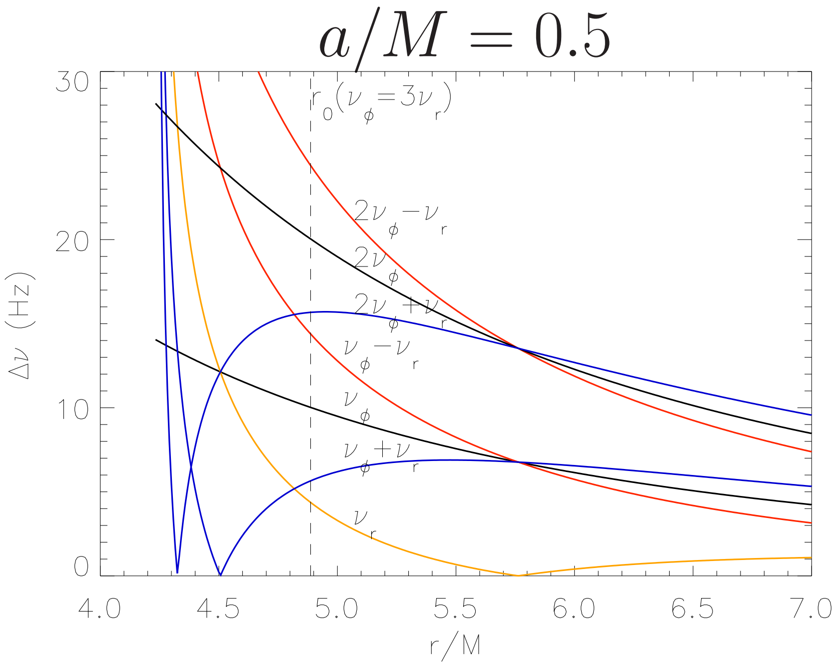
in which case the probability distribution in frequency space is also a Lorentzian:

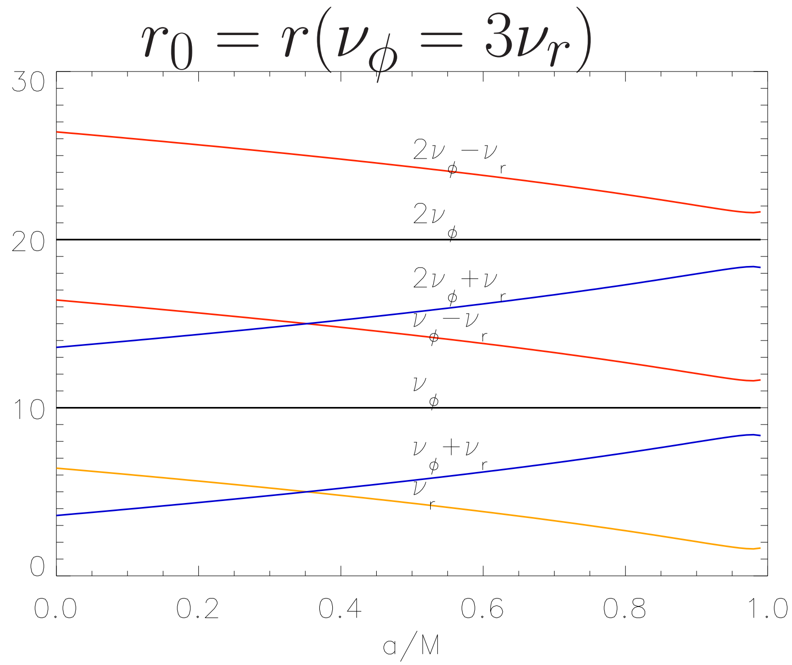
$$P(\nu_j)d\nu_j = \frac{d\nu_j/(\pi \Delta \nu_j)}{1 + \left(\frac{\nu_j-\nu_{j0}}{\Delta \nu_j}\right)^2}. \tag{8}$$

For planar orbits, the QPO power will be focused around the harmonics and beat modes with $\nu = n\nu_\phi \pm \nu_r$. Each peak will be a Lorenzian of width given by

$$\boxed{\Delta\nu_{\text{freq}} = \Delta r \left(n\frac{d\nu_\phi}{dr} \pm \frac{d\nu_r}{dr}\right)_{r_0}}$$

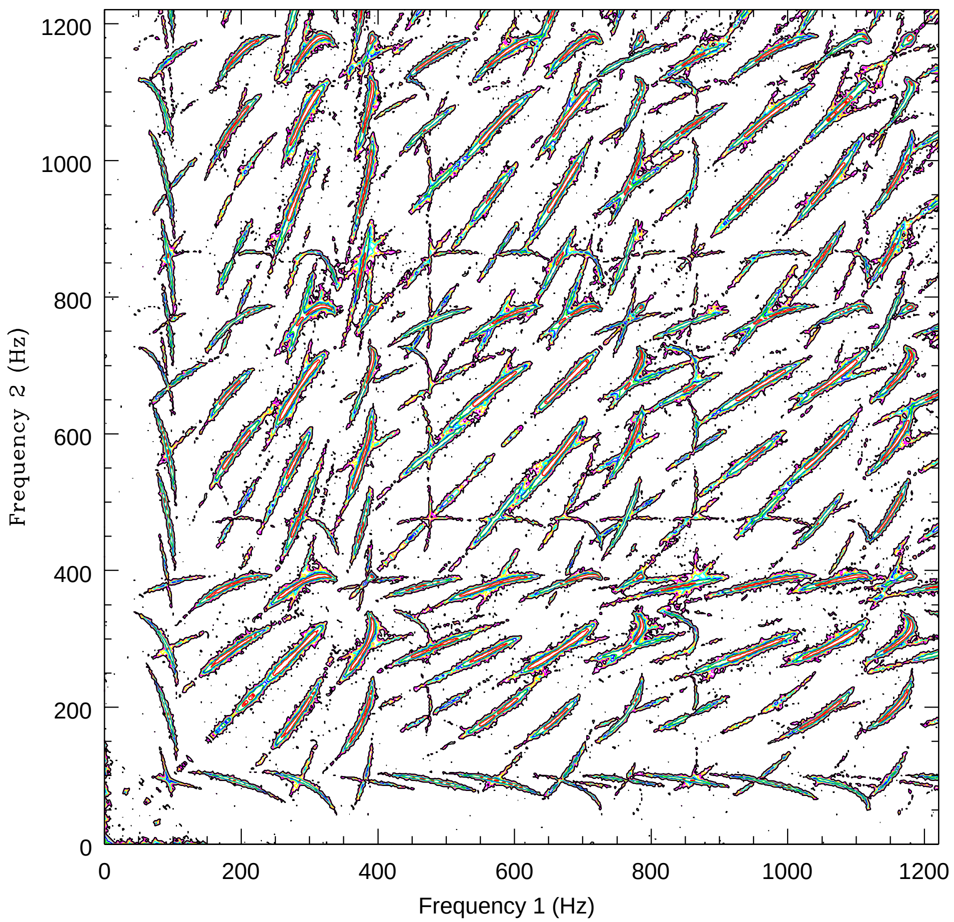
By measuring the relative widths of multiple QPO peaks, we should be able to map out the GR metric near the ISCO[2]





[2] $\Delta\nu$ are normalized to give $\Delta\nu_\phi(r_0) = 10$ Hz

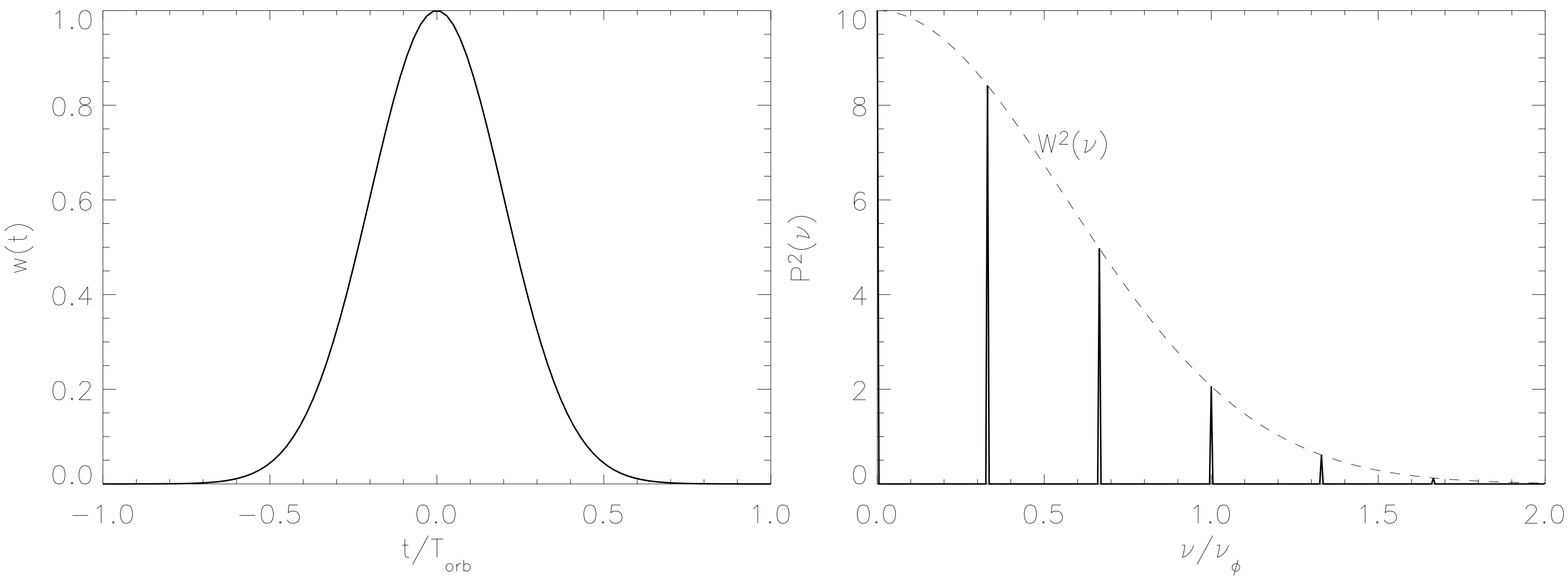
The bicoherence could be particularly useful for mapping out the space-time around the resonant orbits



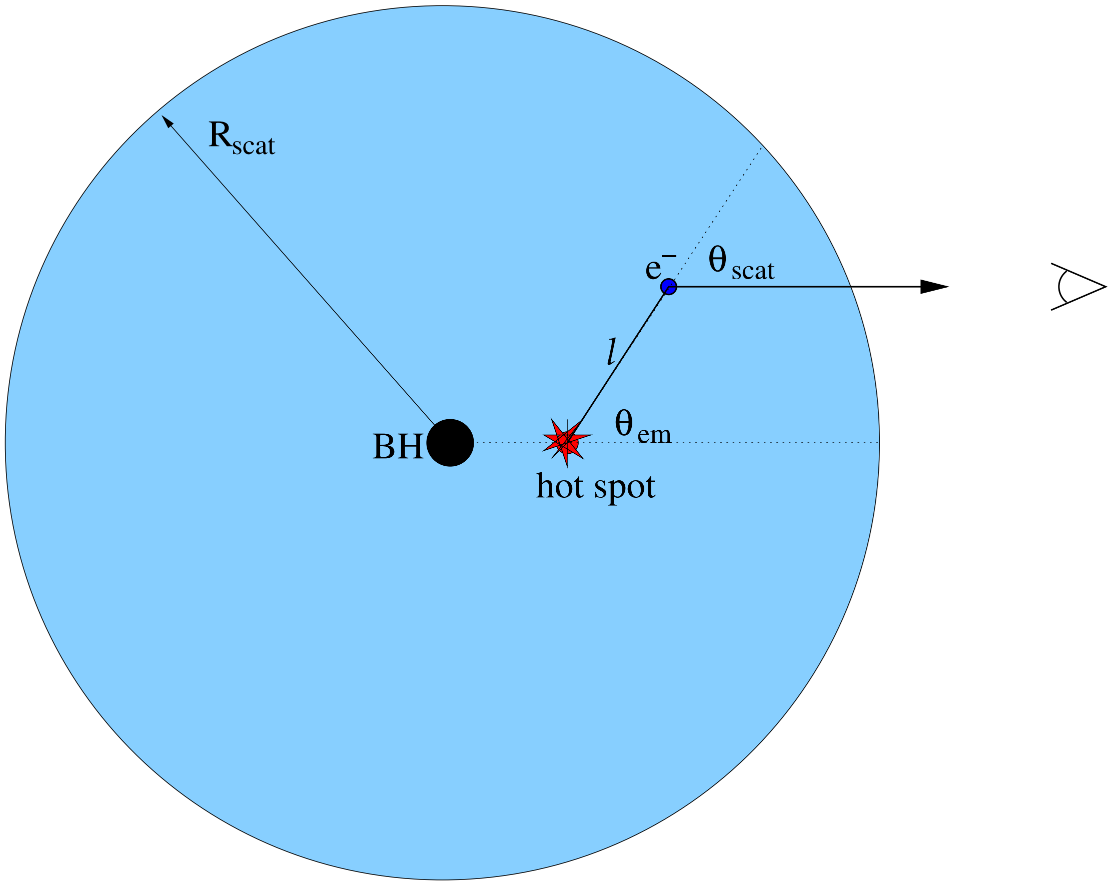
Harmonic Damping

Stretching the hot spot into an arc damps out higher harmonics while increasing the low frequency power

An arc length $\Delta\phi$ can be modeled by convolving the light curve with a window $w(t, T)$ of length $T \sim \Delta\phi/(2\pi\nu_\phi)$. For $\Delta\phi \approx 180°$, most power above $\nu = \nu_\phi$ is strongly suppressed.



We include a simple scattering model to study the effect of a low-density corona on the power spectrum



For a corona with mean free path $\lambda_{\rm es}$, the probability of scattering after a distance l is

$$P(l)dl = \frac{dl}{\lambda_{\rm es}} e^{-l/\lambda_{\rm es}}. \tag{9}$$

Our geometry requires $\theta_{\rm em} = \theta_{\rm scat}$, giving the differential cross section as

$$\frac{d\sigma}{d\theta} = \frac{3}{8}\sigma_T \sin\theta(1+\cos^2\theta), \tag{10}$$

where σ_T is the classical Thomson cross section. Thus a single scattering event contributes an additional path length of

$$D(l,\theta) = l(1-\cos\theta) \tag{11}$$

compared to a photon that does not scatter between the source and the observer.

Integrating this distribution over all forward-scattered photons ($\theta < \pi/2$), we find the average additional path length to be $\langle D \rangle = 7l/16$. Thus scattering once in the corona adds a time delay Δt to each photon, giving an effective convolution in time with window function

$$w_{\rm scat}(\Delta t) = \frac{1}{T_{\rm scat}} e^{-\Delta t/T_{\rm scat}}, \tag{12}$$

where the average scattering time delay $T_{\rm scat}$ is given by

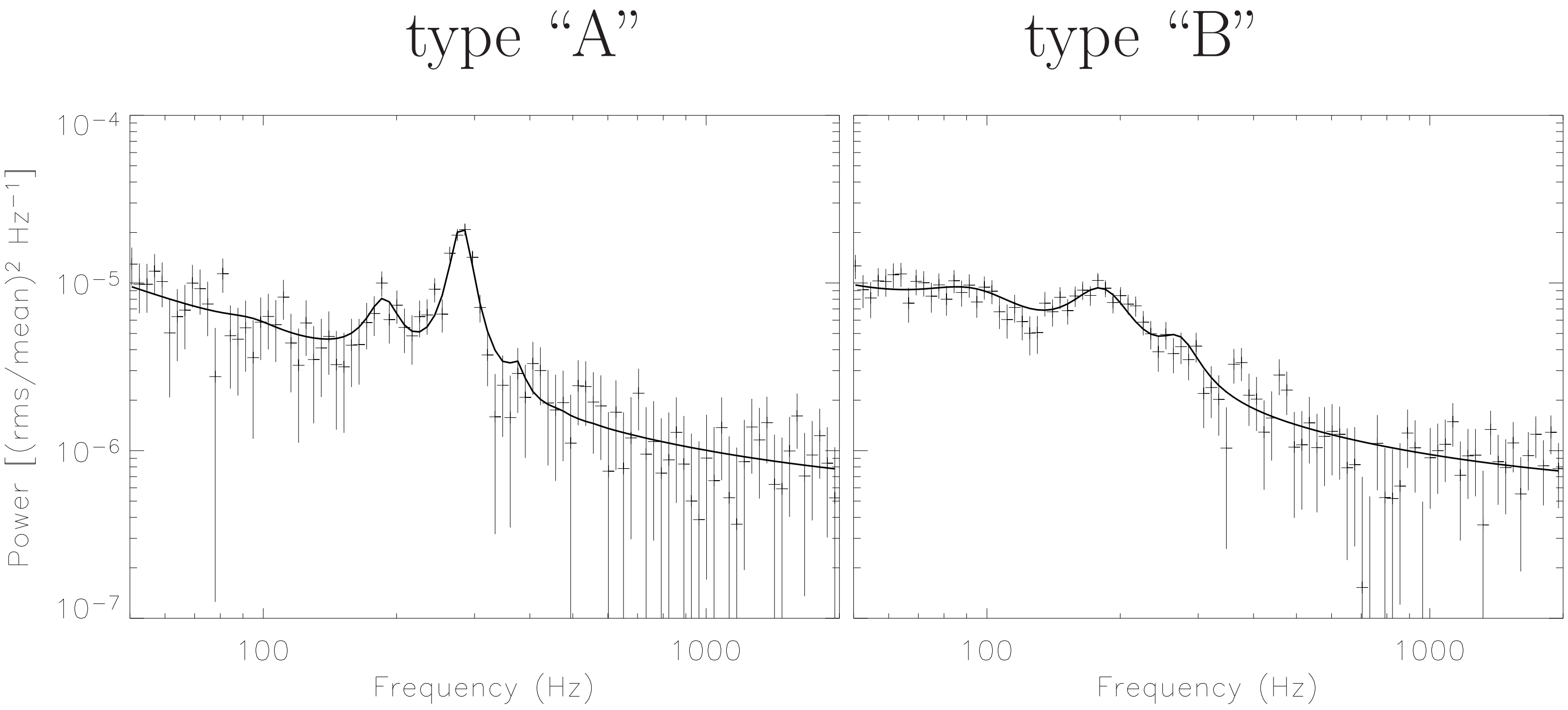
$$T_{\rm scat} = \frac{7}{16}\frac{\lambda_{\rm es}}{c}. \tag{13}$$

The corresponding power spectrum is thus damped by a Lorentzian

$$W^2_{\rm scat}(\nu) = \frac{1}{1+(2\pi T_{\rm scat}\nu)^2}. \tag{14}$$

Comparison with Data

The complete hot spot model can be applied successfully to the QPO observations of XTE J1550–564[3]



[3]Remillard, R. A., et al. 2002, ApJ, 580, 1030

The two QPO types can be characterized by different hot spot lifetimes and arc lengths

Hot Spot model parameters for XTE J1550–564. (1σ) confidences are shown in parentheses.

Parameter	**Type A**	**Type B**
orbital frequency ν_ϕ (Hz)	280.1(2.4)	270.5(12)
lifetime T_l (ms)	10(2.0)	5(1.5)
(orbits)	2.8(0.55)	1.4(0.4)
resonance width Δr (M)	0.02(0.05)	0.025(0.12)
scattering length $\lambda_{\rm es}$ (M)	5(10)	10(20)
arc length $\Delta\phi$ ($^\circ$)	155(30)	285(20)
flux ratio $\frac{I_{\rm hotspot}}{I_B+I_{\rm hotspot}}$	0.085(0.025)	0.38(0.05)

Mode	**A** rms (%)	**A** FWHM (Hz)	**B** rms (%)	**B** FWHM (Hz)
ν_r	0.57(0.15)	33.1(6.2)	2.03(0.21)	63.6(16.0)
$\nu_\phi - \nu_r$	1.62(0.26)	35.7(5.9)	2.57(0.14)	67.6(15.5)
ν_ϕ	3.35(0.17)	34.6(5.5)	1.48(0.24)	65.9(15.3)
$\nu_\phi + \nu_r$	0.75(0.19)	33.4(5.8)	0.06(0.02)	64.1(15.8)

A next-generation X-ray timing mission could distinguish between QPO broadening mechanisms by measuring the bicoherence