ABSTRACT

Title of Dissertation:	STUDENT SENSE-MAKING IN QUANTUM MECHANICS: LESSONS TO TEACHERS FROM STUDIES OF GROUP-WORK AND REPRESENTATION USE
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This dissertation covers two distinct threads of research; both threads focus on understanding student-thinking in quantum mechanics and then draw implications for future research and instruction. The primary goal of this collection of work is, in any way possible, to improve instruction and find ways to better support students in their learning.

The first thread of research focuses on tension negotiation in collaborative group problem-solving. While group-work has become more commonplace in physics classes, this research provides instructors some means of seeing just how complicated group dynamics can be. In particular, I highlight one interactional pattern through which students resolve tension emerging in group interaction by closing

conversations or conversational topics. In doing so, students leave some conceptual line of reasoning unresolved. This work provides important insights into helping instructors understand and respond to group dynamics and conversational closings.

The second thread of work focuses on flexible representation use. This thread has two similar lines of research. The first focuses on how particular representations (wavefunction and external potential graphs) associated with the infinite-well and finite-well potentials can be used by students as tools to learn with. Adapting these models to new situations can lead to deeper understandings of both the model being adapted and the new situation. In some cases, the process of adaptation is not impeded by the student lacking a sophisticated understanding of the model being adapted.

The second line of research on representation use focuses on the reflexiveness of student inquiry with representations. In reflexive reasoning, the student's sense-making shapes, and is shaped by, the representations they draw and animate. This form of inquiry stands in contrast with traditional notions of proficiency in using representations which tend to highlight reproducing standard representational forms and then reading-out information from those forms. In this work, I highlight how this non-linear, reflexive sense-making is supported by the development of coherent, coupled systems of representations and attention to particular figural features, leading to the generation of new meaning.

STUDENT SENSE-MAKING IN QUANTUM MECHANICS: LESSONS TO TEACHERS FROM STUDIES OF GROUP-WORK AND REPRESENTATION USE

by

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Dedication

To: Rosemary Grant Ronayne

Acknowledgements

These will be brief.

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Table of Contents

List of Tables

List of Figures

Chapter 1: Introduction

This dissertation is a collection of work developed during my time working within the community of Physics Education Research. Work done in this community is normally spearheaded by physicists and aims to understand the teaching and learning of physics (McDermott & Redish, 1999). Broadly, the ultimate goal of work within this field is then improving physics instruction (McDermott, 2001). My work shares this orientation. On a large scale, there are three ways in which I contribute to this goal: 1) developing curricular materials for physics classrooms, 2) studying factors that influence students' reasoning and learning, and 3) understanding what lessons can be abstracted for instructors to then help support students. This dissertation focuses on the latter two of these sub-goals; studying student reasoning and drawing insights for instructors from these studies.

In this introductory chapter, I'll briefly touch on each of these.

Curriculum development in quantum mechanics sets the context of my research studying student thinking

My work in curriculum development helps set the context for my dissertation work. In particular, I have worked on two curriculum development projects for undergraduate quantum mechanics courses. The first project focused on engaging students in reasoning about ontology; ideas about the *properties* of quantum objects (i.e. particle or wave) and how those entities therefore interact with the world. The second project has focused on developing materials that support mathematical sense-making; different habits of mind in which students see coherence between mathematical and physical structure.

Examples of two tutorials I have taken the lead in designing can be found in the appendices of this manuscript. They both focus on laser-cooling of atoms. The instructional goal of these tutorials is to give students opportunities to think about more “real-world” situations than they might normally find in introductory quantum mechanics courses. Granted, by real-world I mean laboratory-generated. In any case, the tutorials allow students to apply their quantum formalism to understand how physicists go about studying many-body quantum behavior.

Curriculum development in quantum mechanics provides means of data collection for dissertation work

My work in curriculum development also provided a setting and pattern of data collection for the rest of my work. The first data that I collected and studied for these projects came from testing out tutorials in small collaborative groups of students. Trying to understand the efficacy of different tutorial prompts through these focus groups provided the data for my first body chapter. Data collection for the 2nd and 3rd body chapters was taken in a similar vein; problem-solving interviews with engineering and physics students. The data from these chapters were more ‘targeted’ in the sense that the problems given to students had a specific focus on representation-use. The data collected from these interviews maintained a focus on understanding how students generally respond to different opportunities for problem-solving.

Through this process, I eventually amassed a collection of videotapes of students working on tutorial-style problems. These problems are often fairly ‘short’ in nature, in that a given tutorial may contain on the order of a dozen questions. These problems are often conceptual in nature, tend to not involve sufficient calculation, and aim to provide a good basis for discussion among the students. In my experience, there can be a wide range in the amount of time it takes an individual student, or a group of students, to get through a single question. The amount of time may range from less than a minute to 30minutes. From consideration of the entire data collection, students typically spend an average of five minutes on each problem.

My research involved studying these relatively short moments of students’ reasoning and distilling implications for researchers and instructors. It may seem strange that a dissertation that focuses on learning and lessons for physics instructors would focus on student reasoning that spans only a few minutes in duration. However, in the next section, I briefly explain why I’m interested in such moments. More motivation for studying different aspects of sense-making can be found in the main chapters of this dissertation.

Studying student sense-making and learning: focusing on moments of reorganization

I see sense-making and learning as the coordination within a complex system of interaction (Newman, 2011; Hammer, Elby, Scherr, & Redish, 2005; Hutchins, 1995; Greeno, 1998). The collective behavior of the system arises from interaction between its constituent parts, those parts being people and their material surroundings. In this perspective, moments of reorganization or coordination become crucially important. This is because learning occurs when patterns of reorganization become internalized by individuals within the system (Hutchins, 1995). The question then becomes how students move from spending a few minutes making connections about a topic to deep, meaningful learning.

To answer this question, I will first discuss characteristics of sense-making that I see as valuable in reasoning about physical situations. Arguably, the main goal of physics is to develop coherent explanations and models to explain physical phenomena. And so sense-making that becomes particularly important here are forms of mechanistic reasoning. According to Russ, Scherr, Hammer, and Mikeska, mechanistic reasoning should involve any/all of the following: describing a target phenomenon, identifying set-up conditions, identifying entities, identifying activities, identifying properties of entities, and identifying the organization of entities. Connections among these features may discursively appear in the forms of chaining, analogies, or animated models (Russ, Scherr, Hammer, & Mikeska, 2008).

For example, consider a student reasoning about the probabilistic behavior of a bouncing ball. The student then makes an analogy, comparing the bouncing ball to a quantum particle. The analogy allows the student to reason about how the physical differences between the bouncing ball and the quantum particle gives rise to different probabilistic behaviors. Here, the student is sense-making through making connections about the entities involved (types, attributes, and activities) through an analogy to develop a deeper understanding of the classical “particle-in-a-box”[1].

This research is set-up to study short patterns of sense-making across multiple groups of students. This work is not geared towards studying long-term learning by students. Instead, when I talk about learning, particularly in chapters 3 and 4, I’m more specifically considering what Schwartz, Bransford, and Sears (2005) would call preparation for future learning. Preparation for future learning concerns ways in which students are preparing themselves to learn further about a topic. Such “seeds of learning” may be seen in growth in verbalizations about a topic, questions being asked, resources used or requested, or redirections in perceptual attention. Schwartz and Martin suggest that this type of student invention helps students notice distinctions or important features that then guide their future learning, (Schwartz and Martin, 2004).

My research, particularly chapters 3 and 4, typically looks for preparation for future learning in situations where students often do not have the requisite knowledge needed to simply replicate or directly apply what they already know. Instead, students must be adaptive and inventive in their sense-making. In these situations, the reasoning that students are doing may sometimes look non-canonical. However,

[1] See Chapter 3 for more detail.

previous work has shown that opportunities for invention early on can lead to better learning gains than providing students opportunities to participate in tell-and-practice methods, (Schwartz and Bransford, 1998; Schwartz and Martin, 2004). For example, Schwartz and Martin found that students who invented methods for statistical comparison were then better able to learn from a worked example than students who were initially given, and then practiced, the canonical method.

Two threads of research emerged in studying student reasoning

Chapter 2: Tension in collaborative group problem-solving

As mentioned earlier, a primary goal in watching focus group data is to understand how students respond to the tutorials written by my research group. The goal being that student responses to the tutorial prompts can help inform revisions of those prompts. In these viewings, an interesting episode stuck-out. In a bout of particularly tense sense-making, a group of students came to play on the wording of the tutorial to find an 'out' from their tense conversation. Moving between viewing the data collection and collective discussions with the research team, the pattern held. Students were finding creative ways to find 'outs' from tense episodes of sense-making. This work seeks to better understand this pattern of reasoning, which we call 'taking an escape hatch'. In particular, this work focuses on how tension can play a driving role in conceptual sense-making.

Chapters 3 and 4: Representation-use in individual student interviews

These chapters focus on student reasoning with representations. Unlike the first study on tension in group problem-solving, the data from these chapters come from individual student interviews. There were two interview protocols used to collect the date for these chapters. These protocols can be found in the appendices at the end of this dissertation.

In proctoring and reviewing these interviews, I saw that the interview space and prompts allowed particular types of 'representational play.' Students drew representations and pictures in their reasoning and would proceed to break them apart, manipulate them, piece them back together, etc. These acts were highly non-canonical and very creative. It seemed further that these actions were generative towards the student's endeavor of developing deeper understandings of the situations I posed to them. I found these actions to stand in stark contrast to more traditional notions of representation-use, where a representation simply reflects a student's thinking or makes it easier to perform simple manipulations or read-outs.

These two chapters are both geared towards understanding this type of 'flexible representation use,' which will be defined more thoroughly in Chapter 4. Though similar, I have chosen to split the work into two separate chapters in order to help focus instructor attention on different aspects of student sense-making. From these chapters I draw complementary instructional implications.

In the concluding chapter of this dissertation, I reflect more on the decision to separate the two lines of work on representation use. I also touch more on the instructional ‘lessons learned’ from this work. Many of these takeaways have to do with helping develop instructional practices of noticing. I.e. As an instructor, what are the things I should attend too and how? Additional implications about task design are also discussed.

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Chapter 2: Taking an Escape Hatch: Managing Tension in Group Discourse

Abstract

Problem solving in groups can be rich with tension for students. This tension may arise from conflicting approaches (conceptual and/or epistemological), and/or from conflict emerging in the social relations among group members. Drawing on video records of undergraduate students working collaboratively on physics worksheets in groups of 4-5, we use three cases to illustrate the multifaceted ways in which conflict arises—combining conceptual, epistemological, emotional, and social dynamics—and a specific way of managing the tension that can emerge from the multifaceted conflict, that we call "taking an escape hatch." An escape hatch is a set of discourse moves through which participants close the conversational topic, thereby relieving tension, but before a conceptual resolution is achieved. We describe how epistemological twists and turns can be recruited as a means of managing the strong emotions experienced by the students, showing the coupling of emotion and epistemology in students' conceptual sense-making during group-work. In doing so, we help to provide the groundwork necessary for instructors to *notice*, understand, and respond to one way in which conceptual-epistemological—social-emotional aspects of interaction are coupled in the emergence of tension, rather than narrowly targeting instructional moves based on only conceptual or epistemological considerations. Instead, instructors should often respond to— and help students

become aware of—the emotional component of peer interactions and its entanglement with the "cold cognitive" conceptual and epistemological components.

Introduction

Collaborative, active learning in small group settings using research-based materials can have many benefits (Alexopoulou & Driver, 1996; Barron, 2000; Heller, Keith, & Scott, 1992). Group learning allows students to share knowledge as they build on and critique each other's ideas and reasoning strategies. This creates the opportunity for students to participate in better problem-solving approaches and solutions than when working individually (Heller, 1992). However, collaborative problem solving can create challenges due to the necessarily social negotiation of ideas, approaches, and communication styles (De Dreu & Weingart, 2003; Johnson and Johnson, 1979). Collaborative learning can give rise to conflict for a variety of reasons. The ideas introduced in the group and their connection to the end goal may be unclear. What is taken to be understood by the group can fluctuate quickly. Because common ground is so variable, it demands constant attention by participants (Barron, 2000). Conflicts may arise from dominant personalities (Heller, 1992), unequal opportunities to participate (Sullivan & Wilson, 2013), failure to obey turn-taking norms, or students' insistence on their own strategies (Barron, 2000). Previous research has shown students using epistemic distancing (proposing ideas without taking ownership of them; (Conlin, 2012) or slipping into less collaborative modes (Barron, 2003) as ways of mitigating the tension that arises in the face of these very different types of conflict. A better understanding of the nature of group conflicts, including the interactional processes underlying their generation, sustenance, and resolutions, can help both designers and facilitators of small-group learning activities. This is particularly true for conflict that gives rise to emotional tension.

In this paper, we study different analytical dimensions of interaction (conceptual, epistemological, social, and emotional) and their interaction. In particular, we highlight how the emotional/affective analytical dimension of interaction is entangled with the other dimensions. In doing so, we (i) contribute to the small body of work focusing on the entanglement of conceptual, epistemological, emotional, and social dynamics in small group work, and (ii) characterize a type of student interaction during tense group negotiations, which we call "escape hatches." By "escape hatches" we mean collaboratively achieved closings of tense discussions leaving unresolved the core conceptual issue(s) that formed the context of the local conflict. To do so, we analyze three episodes of students engaged in collaborative physics problem-solving, showing how tension arises in the emergence of multifaceted conflict. In doing so, we document a variety of conversational moves that can initiate students' taking an escape hatch, thereby relieving tension.

We show, in one case, that escape hatches can emerge as epistemological stances or humor. This complicates the facilitator's job, as the nominal meaning and discursive function of students' utterances can differ radically in hard-to-notice ways. We start by reviewing literature on conflict and tension in teamwork, in both professional and in educational settings. Then we outline the data collection and analytical flow of our work. Next, we present our analysis of three episodes of small group work showing how the escape hatch the students' take function in the groups' discourse. We conclude with implications for research and instruction.

Literature Review

Collaborative problem-solving groups have long been seen as helping people learn complex skills (Collins, Brown, Newman, 1989; (Brown & Palincsar, 2013; Heller et al., 1992). Still, when people work together, disagreements often arise. The literature on argumentation and conflict in collaborative work (Bricker & Bell, 2008; Mortimer and Machado, 2008; Kutnick, 1990; Lawson, 1995; Berland & Reiser, 2011; Aikenhead, 1985) often focuses on the conceptual and epistemological aspects of group work. Few studies simultaneously attend to the emotional and relational aspects of group work, and even fewer simultaneously attend to those aspects and the conceptual and epistemological aspects. In this brief walk through the literature, we focus on the latter studies to document that (i) some studies suggest that the conceptual, epistemological, and social aspects of conflict during group-work are coupled and (ii) students have a variety of tools to manage the tension that can arise with conflict in an interaction. For (i), we draw on management studies of conflict in professional settings before turning to education research, which typically addresses the two points simultaneously. We close this section by arguing that fine timescale investigations of how these conflicts arise and are resolved are still needed, motivating this paper.

Tension in group-work: A view from management studies

Within organizational and management studies, researchers have classified conflict as affective/interpersonal or as cognitive, with cognitive conflict arising from different conceptualizations of the task and from disagreements about resource and process management (K. A. Jehn & Mannix, 2001) Shah & Jehn, 1993; (Amason & Sapienza, 1997; De Dreu & Weingart, 2003; K. a Jehn, 1997). These studies document complicated patterns of coupling between conflict and team performance, influenced by the interactions between different conflict types, task types, and team dynamics. Overall, teams experiencing less conflict (affective and cognitive) tend to perform better (De Dreu & Weingart, 2003; K. A. Jehn & Mannix, 2001). Yet, Jehn and Mannix (2001) found that more successful groups tended to experience rising conflict over time, while some lower-performing groups experienced a dip in task conflict halfway through. In contrast, De Dreu and Weingart (2003) found that conflicts are less disruptive for simpler and shorter-term tasks than for complex, long-term projects. And, task conflict has a lower impact on performance when task and relationship conflicts are weakly correlated. They argue that “teams benefit from task conflict when they cultivate an environment that is open and tolerant of diverse viewpoints and work with cooperative norms preventing those disagreements from being misinterpreted as personal attacks (Amason, 1996; De Dreu & West, 2001; K. Jehn, 1995; Lovelace, Shapiro, & Wiengart, 2001; Simons & Peterson, 2000).

These findings in organizational/management studies have implications for research in science learning. For one, even in science problem-solving tasks, we should expect that cognitive conflicts (resulting from conceptual and/or epistemological differences) might entangle with affective or relational conflict. This entanglement is still underexplored in science education, where most studies have focused on cognitive or interpersonal/affective conflict. Another implication is methodological. The finding that a group's performance depends not simply on the amount but on the types and timing of the conflicts suggests that, in studying small-group learning and problem-solving in science, we will obtain incomplete or even misleading results if we look only at coarse-grained relations between "level of conflict" and "performance." We need fine timescale examination of conflict arising during group work, including their genesis and resolution. This is precisely the charge this paper takes on.

Resources for managing tension during group work in learning environments

In this section we present illustrative episodes from the few studies in science and mathematics education literature that explore possible entanglement between the cognitive and affective/relational aspects of group-work, to provide a feel for and to situate our argument within previous work.

Lampert et al. (1996) discuss fifth graders' actions in the face of disagreement while working on a math problem concerning a car traveling at constant speed. A group of four students in the back of the room, "talking loudly and gesturing toward one another" (p. 748), stands out to Lampert (the teacher/researcher). Within this group, a disagreement occurs when Sam misreads Connie's answer, mistaking "min" to denote miles. While the group discusses whether the answer should be in minutes or miles, Sam "seems to be trying to reduce [Connie's] credibility with the others in the group, especially when he accuses Connie of 'guessing' rather than 'figuring it out.'" Thus, the conflict in this situation is simultaneously characterized by a conceptual layer (minutes or miles), an epistemological layer (guessing versus figuring it out), and an interactional positioning layer (establishing relative status). The group settles on "minutes" and moves on to another disagreement over the correct numerical solution. Again, Connie and Sam are at odds, with Connie supplying the correct answer in response to Sam's incorrect solution. Sam and Connie go back and forth trying to persuade the other group members, Enoyat and Catherine. Sam and Connie then implicitly agree to disagree with Sam noting "I'm just putting 1 hour 20," and Connie noting that "I'll put 1 hour 40." This move confuses the other group members, who conceptualize the mathematical activity as including coming to consensus. Enoyat is unsure of how to accomplish this task. However, he proposes that he average the two responses in an "attempt to resolve the discomfort he feels in choosing between Connie and Sam" (pp. 751).

According to Lampert, Sam's action initially supported the belief that the mathematical discussion should include coming to consensus, but later "his mathematical intention also gets confounded with a social one as he seems satisfied with everyone 'writing what you think the answer would be.'" (pp. 754) Specifically, in this conflict, the group begins with a conceptual negotiation (over units, then numbers), which gives rise to both conceptual and interpersonal conflict. However, the group manages these conflicts at the boundaries of the social and epistemological layers; they renegotiate the rules about what counts as a valid answer, with consensus no longer a criterion. For Enoyat, the situation involved an emotional layer as well, in that his resolution aimed to resolving the tension that was associated with the epistemological and social conflict being created by Sam and Connie. So, as Lampert emphasizes, the "joint activity [of generating tension and resolution] is not just an expression of what they bring to this conversation by way of beliefs about how to disagree—they are shaping those beliefs dynamically as they interact" (Lampert, 1996, p. 754). Hence, "reasoning and social negotiation become intermingled. In a mélange of social and mathematical moves, the students struggle to figure out how to both maintain their relationships and do what the teacher has asked." (pp. 751). In summary, the epistemological, social, emotional components of conflict were coupled and negotiated in the moment.

Taking another tack, a few researchers have looked at humor, playful talk, and skillful positioning of ideas as ways to navigate conflict in group-work. Conlin (2012) shows how students use humor and irony to manage the affective risk of threatening face (Goffman, 1955) when making repairs to each other's conceptual reasoning. Students also manage the threat to face through "epistemic distancing", a shift of footing (Goffman, 1955) wherein the student positions herself as the messenger of someone else's claim rather than the claim's author. If the claim is rejected or repaired, the loss of face is therefore shifted away from the messenger. Methodologically, Conlin found that epistemic distancing can be evident not just in the substance of a student's utterance but also through other "paralinguistic channels, such as shifts in register and prosody, facial expressions and gestures" (Conlin, 2012; Goodwin, 2007).

Similarly, Sullivan and Wilson (2015) document young students' use of playful talk (humor, puns, teasing, music making, and wordplay) (Lytra, 2009; Sullivan & Wilson, 2013) as a means navigating conflict in small group work. In their case study of 6th grade science students building a robot, conflict arises with respect to status within group and associated access to the work of the project (who gets to build the robot, whose ideas are taken up), perceived gender identities, and other flashpoints. The students used playful talk to manipulate opportunities to participate within the group by positioning themselves or others as more or less capable. For example, one of the group members was often positioned as less competent by her peers. When the group was allocating building tasks, she playfully offered to build the entire device, positioning herself as a competent builder within her group and staking out a slot on the building team. So again, in this study, cognitive conflicts (e.g., over what ideas get taken up) are entangled with social conflicts (e.g., over who gets to participate in what ways).

Barron (2000, 2003) argues that conflicts during group work can arise because students bring different orientations towards what it means to collaborate. Some students behave in ways that support equitable participation and joint attention to ideas and artifacts. They align their task-orientation through referencing and building on one another's ideas and approaches. Others want to dominate the discussion, insisting on control and authority. Note that these two different orientations have both an epistemological component (is knowledge collaboratively constructed or authoritatively transmitted?) and a social component (more equitable vs. less equitable participation patterns). When group members consistently approach collaboration in these different ways, conflicts arise and performance can suffer. Different orientations towards collaboration are made visible through "struggles of control, failures to understand one another, repeated attempts at explanation, rejections of that explanation (even when invited), self-focused talk, admissions of confusion," (Barron, 2003, p. 366), etc. Barron argued that students try to manage and negotiate their forms of participation, and expectations thereof, during group-work. And whether initial differences in participation converge or further diverge depends both on social and cognitive factors.

How people organize their participation in conversations and the generation and resolution of conflict is also an area of study within sociolinguistics. Goodwin (2007), for example, describes the organization of embodied participation frameworks in an episode in which a father is attempting to help his daughter with her homework. Embodied participation frameworks concern the embodied alignment and organization for talk and action within an interaction. Like the students in Barron's (2000) paper, the father and daughter also brought different expectations of collaboration to their interaction, as made visible through their talk (substance of utterances as well as tone, pitch, etc.), body posture, gestures, and gaze. While the father wants to help the daughter figure out the homework, the daughter wants him to just tell her the answer. This conflict leads to a breakdown and collaboration could only resume once their participation frameworks were better aligned.

Situating our argument in this landscape

This brief walk through prior work suggests that (i) for researchers, understanding conflict arising in group work requires the simultaneous attention to social-interactional aspects and the cognitive aspects of the interaction, and (ii) for students, a resolution of tension that enables the collaborative work to proceed smoothly often requires alignment along some of the cognitive and/or social dimensions. This manuscript both builds and expands upon this prior work, by

(i) illustrating that in the genesis and sustenance of group tension, the cognitive, affective, and social components are not only simultaneously present but dynamically coupled, mutually affecting each other; and
(ii) introducing the notion of "taking an escape hatch" as one way in which groups relieve tension.

In taking an escape hatch, the group relieves the affective and social conflict but without resolving the cognitive disagreements that helped produce those conflicts. Lampert et al.'s documentation of "agreeing to disagree" is an example: By renegotiating what counts as an acceptable answer (deciding that consensus isn't necessary), the group closes the tense conceptual discussion about how much time the car takes, thereby relieving the social conflict generated by Sam and Connie. The episodes we present below suggest that “agreeing to disagree” is just one of many ways of taking an escape hatch, and that “taking an escape hatch” may be common in students’ collaborative small-group work in science.

This paper also contributes to the need for more empirical analyses that bridge cognitivist and interactionist analysis (diSessa, Sherin, Levin, 2015). In addition, little research provides fine timescale analyses of discourse in undergraduate-level collaborative learning, especially in upper-division disciplinary contexts such as quantum mechanics.

Methods

Data Context

As part of our design process in creating curriculum materials for upper division quantum mechanics courses, we video recorded groups of 3-5 students engaging with the materials developed. The curricular materials were in the form of worksheets which posed sequences of conceptual questions, to be answered collaboratively by the group. Physics and engineering students (mostly juniors and seniors), were recruited for these groups through an email to the first semester of the physics-major quantum mechanics class or through a department-wide email.

Focus group sessions were held in a room in the physics building. Each session was attended by 3-5 students and the interviewer. These sessions started with the interviewer explaining to the students that the researchers were interested in how the students responded to the tutorials and how the students tend to think and talk about quantum mechanics, more generally. The groups of students typically proceeded through the tutorial with very minimal, unprompted input from the interviewer.

In some cases, the same group of students attended multiple sessions. Because of this, the students were able to get to know the researchers (myself included), the other students (if they did not already know them), and the norms associated with the focus group space. Each episode presented below will include some detail about the students in groups, including the nature of their participation in focus groups. In the discussion section, I will make some conjectures about how differences in relationships with these students may have applicability for the findings of this paper.

Background and Analytical Flow

In studying the data collection, we were broadly interested in students' reactions to the worksheets, moments of struggle (conceptual or otherwise), moments of negotiation and coordination among students, and the role of ontologies (Brookes & Etkina, 2007) and metacognition in students' reasoning. We worked inductively and deductively (Erickson, 2006), viewing the data on a larger scale and then selectively investigating areas of interest more closely (Derry et al., 2010; Jordan & Henderson, 1995).

Specifically, the first author began by watching the data with an inductive orientation, looking for patterns in the data, as guided by emergent interest and commitment to attending to fine time scale variations in students' talk and interaction. When viewing a group of students working through our Particle in a Box worksheet, the first author noticed an interactional pattern that occurred twice during the session. During these episodes the discussion became quite tense, as evidenced through volume of speech, patterns of cutting other speakers off, and body language. At this stage of analysis, the first author was attending to these indications more intuitively rather than following any strict methodology or pursuing a specific research question. In the midst of these two tense moments, which both occurred during the throes of group problem-solving, students escaped the tension by making and taking up a bid to close the conversation or topic (Schegloff & Sacks, 1973), but without coming to a conceptual resolution.

The first author then brought the video episodes and transcripts to video analysis sessions (Jordan & Henderson, 1995) attended by all the authors and sometimes by collaborators from another university as well. The transcripts at this stage did not yet include intonation, stresses, or gestures; for those, we relied on the video. As a group, we formed alternative interpretations of the data and tested those via repeated viewings in which we would expand on the layers of multi-modal analysis (Stivers & Sidnell, 1998) to see which interpretations were best supported by coherence across multiple channels of talk and action. We labeled the interactional pattern "taking an escape hatch," and worked collaboratively to characterize the mechanics of this type of interaction. Further nuance to the phenomenon developed through reflexively moving between the data and operationalization of the phenomenon. Ultimately, this process lead us modeling the interaction of "taking an escape hatch" as having three main, coupled characteristics.

1) The move functions to relieve tension within the group.
2) The move closes discussion of the current conceptual and/or epistemological topic of the conversation.
3) Taking an escape hatch circumvents finding a conceptual resolution for that topic.

Within these constraints, a variety of conversational moves can function as bids for taking an escape hatch, as we document in this paper.

Methodological Orientation and Tools

We are examining the emergent intertwining of the conceptual, social and epistemological dimensions of group interactions around physics problem-solving. In order to make empirical claims to whether or not a move constitutes an escape hatch, we utilize talk-in-interaction as a primary data source (Derry et al., 2010; Goodwin, 2007; Jordan & Henderson, 1995), analyzing multimodal semiotic channels (speech, gesture, material ecologies) to develop a coherent story of a group's collaborative interaction, which can then be binned into more conceptual, affective, and epistemological layers. Because we want to understand the process and mechanisms comprising and supporting the taking of an escape hatch in group interaction, microgenetic analytical methods provide an empirical framing for doing so (Parnafes & diSessa, 2013; Siegler & Crowley, 1991).

Knowledge-in-use analysis to attribute conceptual substance. In unpacking the conceptual substance of students' talk, we attend to fine shades of meaning (e.g., the same word taking on different meanings at different moments) and to the changes in conceptual meaning that happen at short timescales. We don't assume coherence of "conceptions" across or within students unless warranted by features of their talk and action. Work that exemplifies analysis of conceptual knowledge-in-use comes from Beth Warren, Ann Rosebery, and colleagues (Warren, Ogonowski, Pottier, 2005; Rosebery & Puttick, 1998). Like Rosebery and Warren, we loosely draw on knowledge analysis (DiSessa,1993; Hammer, 2000) without aiming to model the knowledge being enacted in terms of cognitive elements or making claims about the ontology of knowledge-in-use.

Epistemological statements and strategies to attribute epistemological substance. To understand the role of epistemology in students' interactions, we attend to students' explicitly stated stances towards knowing in the moment, as well as their tone, hedge words, disclaimers, organization of available material resources, and coordination of their activity in order to produce knowledge. Students' negotiations around what counts as a satisfactory answer and how to approach a problem provides strong empirical characterization of how knowing and learning are being enacted in the moment. For example, students approaching a problem and only discussing formulaic or highly mathematized ideas are functionally approaching the problem as having a mathematical solution path. This methodology is consistent with Goodwin's (2007) analysis of epistemic stances. Thus our analysis takes a "social practices" rather than a "beliefs" perspective towards epistemology (Kelly, McDonald, & Wickman, 2012).

Interaction analysis to attribute smooth vs. tense interaction. Tools from interaction and conversation analysis provide a means for understanding the micro-scale organization of talk-in-interaction. We describe interaction in through the following dimensions/structures:

- Turn-taking; individual turns at conversation
- Repair; attempts to alleviate conversational trouble or breakdowns in mutual understanding
- Turn construction; conversational turns are structurally comprised of turn-construction units, which may be single words, clauses, questions, etc.
- Adjacency pairs (Sidnell, 2010; Sacks and Schegloff, 1973); distributed conversational sequence of two utterances, where the first-pair part mutually constrains second-pair part
- Preference; some second-pair parts are organizationally "preferred" in the sense that some second-pair parts make more significant progress towards the joint enterprise underway

- Progression; on a larger scale than preference, there is a sense that the conversation should move towards accomplishing the mutually determined purpose

We also rely on paralinguistic features of speech, such as tone, volume, and pauses. Taken together, these tools help us understand what group members are (more or less) jointly trying to accomplish, how they are going about accomplishing these actions, what resources (knowledge, skills, experiences, etc.) the group utilizes in doing so, and what emotions are evident through physical presentation. This analysis shows when joint action unfolds smoothly, or when there is some conflict within or across any of the aspects of action described above. A paradigmatic example we draw upon is Goodwin's (2007) analysis of a father helping her daughter with math homework; the father initially approached the interaction as helping his daughter figure out the answers, while the daughter initially wanted her father to simply provide the answers. Goodwin used the substance of utterances as well as tone, pitch, body posture, gestures, and gaze to document tension arising from different orientations toward the interaction. Evidence of this tension comes from measures of embodied opposition such as vowel lengthening, volume, gesture, posture, pauses and polarity markers at the outset of conversational turns (Goodwin & Goodwin, 2002).

Attribution and categorization of conflict. We then sought to model the conflict present in each interaction, which we characterized as social, conceptual, or epistemological. Categorizing conceptual, social, or epistemological conflict involved describing any extended opposition/decoherence in: how people are relationally involved in the interaction (social), how knowledge is being enacted or constructed (epistemological), and the content of the interaction (conceptual). For example, a lack of conceptual progression, with students positioning different conceptual ideas or approaches against each other, would be evidence only of conceptual conflict. We might also expect these types of conflicts to be correlated in their emergence. For example, a conversation in which there are a high degree of cut-offs and interruptions, particularly of one person is an example of social conflict, where there is a large degree of opposition, disjointness with respect to how people are relationally related. When this interactional pattern comes to affect how knowledge is being enacted or constructed, the conflict has clearly taken on both social and epistemological dimensions.

Adjacency pairs to attribute conversational closings. We now turn to the mechanics of the discursive moves that typically constitute an escape hatch. (From here forward, we use "escape hatch" as shorthand for "taking an escape hatch.") Because an escape hatch is a way to close a conversation in response to tension, we utilize Schegloff and Sacks' formulation of *adjacency pairs* as a prevalent means of identifying conversational closings. Adjacency pairs, also called "possible pre-closings," are particular examples of a two-part sequence of conversational turn-taking in which the first utterance, a bid to close the discussion, constrains the second utterance. (Schegloff & Sacks, 1973). For example, a student may make a bid to close with "alright?" which when met with "alright" is an agreement to close, but when met with “wait, what happens tomorrow?” is rejection of the bid. Thus, closings are interactionally distributed.

Analysis

In the following sections, we present analysis of three episodes of students working in extra-curricular focus groups on worksheets of quantum mechanics problems.

The first two episodes come from a focus group that took place in late 2014. Five students participated in the group; they were all male, and junior or senior physics majors. They knew each other, to varying degrees from the quantum class they were currently enrolled in, and other common courses. Their pseudonyms are Al, Bob, Chad, Dan and Ed. Approximately four months later, Al and Ed returned for a second focus group session. This time, they were joined by Karen and Larry, also upper-level physics majors. The third episode in this paper comes from this second focus group session. We named the episodes “Because math,” “Can we define” and “Reframing” based on the content of student interaction during the episodes.

Episodes 1 and 2: “Because math” and “Can we define”

The first two episodes of escape hatch that we present, occurred in the clinical focus group session using the worksheet on the Particle in a Box (PIAB)[2]. The PIAB worksheet has students consider the properties of the quantum particle bound within a

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[2] https://www.physport.org/curricula/QuantumEntities/

square potential well, a standing wave on a string (as an analogy to the energy eigenstates), and a classical particle in a box. Episodes 1 and 2 occurred about 5minutes and 15minutes into the hour-long focus group, respectively.

In "Because math," the students are considering the question:

> **Why isn't the ground state *n* = 0? That is, why isn't it possible for the particle to have zero energy?**

In this episode, the students engaged in a tense discussion of the mathematics of eigenstates and eigenvalues before dissipating that tension via terminating that line of reasoning. In the second episode, students are discussing the question;

> **Can we define a 'speed' for the wave?**

This question references a classical standing wave on a string. Here, the students engage in a long period of tense reasoning before defusing tension by taking an escape hatch afforded by the wording of the question.

Episode 1: Because math

After Chad begins reading the question out loud, Al suggests a conceptual solution, but it isn't taken up. Al then suggests a more mathematical path forward. (Transcript conventions used are presented in the Appendix I.)

Segment 1/6: Lack of input influences framing

Chad: Why isn't the ground state n=0?

Al: Uncertainty principle? (3.0) I guess mathematically, I don't know why. But. ((*taps pen on paper twice*))

There is a three second pause after Al suggests the “uncertainty principle”. Chad looks up from his paper and frowns as he looks to Al but he, and the rest of group, remain silent. Al amends his suggestion with a hedge: “I guess mathematically, I don’t know why.” indicating that the group’s silence and Chad’s re-focusing his attention are taken up by Al as the group *not* taking up his suggestion. This amendment has epistemological connotations, by suggesting that “know[ing] why” may involve thinking “mathematically.” In the subsequent conversation, the group takes up the bid to pursue a mathematical explanation, exploring entry points while drawing heavily on mathematical language.

Segment 2/6: Grappling for an entry point in a mathematical space

Bob: Well. And. We, we, we. Is this a definition we don't. [Oh wait. That's why it.

Al: //NO, but if you.

Dan: We just--

Al: ['Member if you…

Dan: //We talked about the difference between that and the [harmonic oscillator.

Al: //harmonic oscillator.

Dan: 'Cus they, harmonic oscillator starts at n=0.

Al: Right, but when you have like the state n=0 for a harmonic oscillator, you still like, in your equation for the energy levels, you still use n=1. You just call it [n=0.

Chad: //Well, no. [It's n=0 but it's n plus one half.

Dan: //Well you're energy would be [one half hbar omega.

Al: //Okay. Yeah, yeah, yeah. You're right but for like these ((points pen into paper)), like the square well or whatever ((two-handed flat gesture shows shape of a box)), it's umm… there's an n multiplied by it so like, if you had n=0, the energy would be zero ((open right hand facing down, sweeps horizontally from center of body out to fully extended arm))

Dan: Right.

25 Ed: Which would be no wavefunction ((mimics Al’s horizontal
26 gesture, hand sweeps horizontally in front of body)). It would just
27 be a flat [bar.

Here, although the students are on the same page about taking a mathematical approach (as indicated by repeated mentions of the energy equation for a harmonic oscillator vs. a square well), some social and conceptual conflict starts to emerge as students debate whether the energy of the system can be zero. First, the various “starts” by Bob, Dan, and Al (lines 4, 6, 7, 8, 9, 13, 16) correspond to different potential entry points, none of which take hold unchallenged. Indeed, the starts of many of the utterances (lines 6, 9, 13, 16, 17, 18) serve to counter the previous utterance either through direct challenge or through proposing a different path. It is unclear in some instances if the disagreements are based on failures to actively listen or on intentional disagreement. Either way, though, this initial volley of embodied oppositional stances constitutes a tense exchange among the group members.

To support this conclusion, we now walk through the discourse line by line. In lines 8 and 10, Dan suggests that looking to the harmonic oscillator might provide a clue because in that case, the ground state starts at n=0. Al immediately follows with an argument that even for the harmonic oscillator, the lowest energy state uses a value of n=1, but is just referred to as the “n=0” state. The “right, but...” on which Al begins signifies that what follows is likely to challenge Dan’s utterance (Goodwin & Goodwin, 2002). In response, Dan and Chad talk over each other to correct Al’s reasoning. Even before their simultaneous turns of talk are over, Al, leans back, gestures, and rapidly says, “okay. yeah yeah yeah yeah, you’re right, but…” Al’s response to Chad and Dan, (lines 18-24), is punctuated with continuous gesturing using both hands. During his utterance, Al taps his paper repeatedly with his pen and pushes his paper towards the center of the group, thereby offering and loudly animating an object (the paper) around which the group can converge their attention. Al finishes his utterance using large gestures that involve almost fully extended arms. Al’s expanding embodied counters to his group members’ challenges, his subsequent pushing for his group member’s attention to his paper, and his final use of extended gestures demonstrate a growing tension within this short interaction (Goodwin, 2007; Goodwin & Goodwin, 2002). His utterance acknowledges that he made a mistake, concedes that Dan and Chad are right about the mathematics, but follows that with bringing the conceptual and mathematical substance back to the square well problem.

So, the tension building up here emerges through conceptual and social conflict. Their search for an explanation is bound up in challenging one another and saving face. We can imagine less charged interactions in which a group tries to unpack how the square well relates to the harmonic oscillator, and why a substitution of n=0 makes sense for one but not for the square potential, in a way that does not continuously put the speaker’s face at risk. But this space, as currently constructed, is one in which physics knowledge also serves as a tool for establishing superiority. In the rapid exchanges that challenge previous ones, repairs, and acknowledgments of who is right and who is wrong, we see coupled conceptual and social dynamics as contributing to tension building up within the group.

Segment 3/6: Status negotiation at the expense of a peer

28	Chad:	//Well if the energy was zero, wouldn't it be like there was no
29		particle in the box, anyway? So it's not the same problem?
30	Bob:	Yeah. That's right, we're talking about [particle in a box here.
31	Al:	//Well…
32	Bob:	So that means we're talking about um, I imagine in this case, an
33		infinite square well. I think that's what it usually refers to?
34	Chad:	Yeah.
35	Al:	Yeah.
36	Bob:	So, [if they're saying..

36 Bob: So, [if they're saying..

37 Chad: //Walls that you can't pass.

38 Al: Yeah.

39 Bob: A particle has zero energy..

40 Chad: It's not a particle.

41 Bob: A particle always has energy. Always has some sort of energy.

42 Chad: You can't have a particle with no energy. That's like saying I have a
43 whole bushel of no apples ((both hands show shape of a
44 container))

45 Al: NO::. What I think they're saying is like the difference... Like you
46 could imagine in a classical sense ((hand forms a loose fist and
47 falls loudly onto table)), like a ball in a well, [it could just be
48 sitting there.

49 Bob: //Well classically, it can just be sitting there.

We see continued building of tension. The conflict in the group takes on conceptual, epistemological, and social components. Bob and Chad begin to collaboratively suggest that a particle must always have some sort of energy, and hence zero energy would suggest the non-existence of the particle. Chad makes an analogy to a "bushel of no apples." Al responds to Chad's statement with "NO::" with the strength of his disagreement indicated by loudness and vowel elongation (line 45). His gestures add to the explicitness of his disagreement: when he mentions "a ball in a well," (lines 47-48) he shapes his right hand into a loose fist which he raises up, and then allows to fall loudly on the table. So, his talk and actions embody opposition to

Chad’s suggestion, with the forcefulness of the opposition bringing a social and emotional component to what on paper looks like a conceptual disagreement about the possibility of a zero-energy particle.

Al also introduces an epistemological disagreement to the discourse. He adds to the authority of his counter to Chad when he takes a position of privilege as someone who is able to interpret what the tutorial is asking for (in line 45, “What I think they're saying is...”) (Sullivan & Wilson, 2015). He says the tutorial is asking for “the difference,” which from the lines 45-48 and subsequent talk, we take to mean the difference between a classical and a quantum particle, with respect to whether a particle can have zero energy. Unlike Bob and Chad, for whom the answer is an assertion about whether a particle can have zero energy (without specifying what kind of particle), Al wants different answers for a classical vs. a quantum particle. And while Bob in line 49 takes up and adds onto Al’s assertion about classical particles, acknowledging that a classical particle “can just be sitting there” presumably with no energy, he does not take up Al’s suggestion to separately consider classical vs. quantum particles, as we’ll see in the next section. In any case, Al’s positioning of himself as uniquely able to interpret the tutorial’s intent and his forceful conceptual disagreement with Chad generates tension in the group, as evidenced by Bob smirking at Ed during line 45.

Segment 4/6: Limited collaboration turns the conversational focus

In this segment, the tension continues. Al cuts off Bob with a reassertion of what the tutorial writers are looking for, but then Bob and Chad do not respond to

Al's bid. They instead pursue another line of reasoning, with no participation from Al.

50 Al: It could have no kinetic energy whatsoever ((*takes loose fist from*
51 *table, moves back and forth in front of body*)).
52 Bob: Although I do remember reading [the other…
53 Al: //What they're saying I think ((*points repeatedly into paper*)), is like
54 why in like a quantum realm, why it can't ((*moves loose fist back*
55 *and forth*)).
56 Bob: I do remember like reading like yesterday in another physics book
57 I have, uhh that there's a minimum speed that a particle can have.
58 And for a macroscopic object like a ball it's like 10^-36. Which is
59 basically zero, but ummm... ((*laughs)*) Meters per second. Umm--
60 Chad: Then it would also be at zero Kelvin, wouldn't it? If it had no speed.
61 Bob: Well it's just translational speed ((*horizontal sweeping motion*)),
62 not its molecular ((*gestures small back and forth motions*))...
63 Chad: Oh ok.
64 Bob: For a particle though, it was much larger. I think for an electron it
65 was like... what was the number? I don't remember, but it was a lot
66 larger. But anyway, umm for the particle in a box... Isn't there
67 some sort of theorem in uhhh, in linear algebra that says that, zero
68 can't be an eigenvalue? Or is it can't be an eigenvector?

In line 53, Al continues explicating his interpretation of the tutorial question doing so, he cuts off Bob in line 52. Bob's response in line 56 addresses neither the interruption nor the substance of Al’s bid. Instead, he starts engaging in a new epistemic activity, trying to remember some information from an authoritative source (“physics book”) about the minimum speeds that objects of different sizes can have. Chad takes up Bob’s line of reasoning by requesting (line 60) and then affirming (line 63) Bob’s clarification of what “minimum speed” means. In summary, this segment of discourse is non-collaborative between Al and Bob, both in a social sense (Al cuts off Bob, Bob ignores Al’s ideas and Al is shut out of conversation) and in an epistemological sense (Al and Bob are engaged in two different, non-interacting epistemic activities). This non-collaborativeness, we claim, sustains the earlier tension.

Segment 5/6: Epistemological statements close mathematical topic

In this segment, the epistemological conflict is at least temporarily resolved as the entire group, including Al, takes up Bob’s bid at the end of segment 4 to explore what linear algebra, which is the mathematical formalism used in quantum mechanics, has to say about the possibility of a zero energy particle. However, the tension continues until, at the end of the segment, Al makes a bid for taking an escape hatch.

Al: It's zero isn't an eigenvector.

Chad: 'Cus it's arbitrary.

Al: Right, cus then everything would be...

Bob: Oh it's eigenvector? Ok never mind, so that doesn't work.

Al: Zero can't--

Chad: Can't be an eigenvalue, 'cus eigenvectors –

Al: 'Cus then any vector could be an eigenvector.

Bob: Oh, so it can't be an eigenvalue?

Al: No::, no. Zero can be an eigenvalue. ((*drops pen taps table*))
Right?

Dan: No,[zero can't be an eigenvalue. ['Cus--

Chad: //No, it can't.

Al: //Then zero can be an eigenvector.

Dan: Yes, [I think.

Bob: //So then if zero can't be an eigenvalue ((*Dan laughs*)) and if the
way [you

Al: //It's whatever one that makes it like trivial

Dan: Trivial? I think it's the eigenvalue.

Chad: Yeah.

Dan: 'Cus that would be like H [psi equals zero psi . Right?

Chad: //(?) y equals zero.

Al: So it would work for any [wavefunction.

Dan: //Effectively.

Bob: So, in other words, if I take H on [psi and get zero as an eigenvalue.

Chad: //If the Hamiltonian was zero.

Al: So we can say linear algebra. ((*Ed smiles, rest laugh*))

Dan: Because math. ((*Ed and Chad smile, rest laugh, Bob leans into the table*))

Starting immediately in line 69, the group takes up Bob's suggestion to discuss whether an eigenvector/eigenvalue can be zero, giving tacit approval to this direction. Al first responds, saying that "it's zero isn't an eigenvector." His tone lacks inflection and his words are well-enounced, indicating some confidence in his response. In line 74 Chad interrupts Al to finish his statement. Al initially responds with "eigenvector" but Chad answers with "eigenvalue." Chad promptly opposes Al's idea, cutting him off before he can voice it. Al responds in kind, taking the floor from Chad to apparently summarize Chad's point for him, "'cus then any vector could be an eigenvector." We see Al as espousing Chad's position for him, rather than Al's own potentially changing position, because Chad is arguing for eigen*values* not being able to be zero (line 74) while Al is in favor of eigen*vectors* being unable to take on zero value (line 69). Accordingly, Bob asks for clarification from the group after Al finishes Chad's utterance with, "oh, so it can't be an eigenvalue?" Bob's request for clarification is met with forceful opposition from Al. During his utterance in line 77, Al shakes his head, drops his pen and hits the table with his pointer finger repeatedly while reiterating his point (finger pointing into the table). The drawn-out "no" by Al and his embodied response as a whole, highlights his opposition to Bob's suggestion that it is an eigenvalue that cannot be zero. Al's initial statement in line 69, and his reframing of this statement in line 77, indicate that his belief that zero cannot be an eigenvector is somewhat stable through this piece. This further suggests that what Al is doing in line 75 when he finishes Chad's utterance for him, is taking away opportunities to participate from Chad. Al voices an opinion for Chad that contradicts his own views, only to strongly push back against this opinion in his next turn.

After Al suggests that zero can be an eigenvalue in line 77, both Chad and Dan disagree with him. Opposition in the group is apparent in these lines, as three subsequent speaker turns begin with "no," (Goodwin & Goodwin, 2002). The group's opposition might be enough to make Al reconsider his point of view, and he reiterates the rest of the group's position with "then zero can be an eigenvector." It now almost appears as if Chad, Dan and Al all have settled on eigenvalues being unable to be zero. Bob then takes the next steps in the group's reasoning, attempting to make inferences based on the group's apparent position, "so then if zero can't be an eigenvalue and if the way you--" However, Bob is cut off by Al who responds with "it's whatever one that makes it like trivial." This utterance allows Al to superficially acknowledge what condition the group's solution must satisfy, without actually identifying which possibility—no zero eigenvectors or no zero eigenvalues—satisfies that condition. Al is also indicating, or making a bid, that the issue has not been resolved within the group.

In line 92, Bob tries to synthesize the group's position. However, Al's response to Bob is not one that incites further discussion but makes a bid to close down the conversational topic altogether—a bid for an escape hatch, as we'll argue below. Al proposes a close with "so we can say linear algebra," to which Dan agrees with "because math"—an adjacency pair type (Schegloff & Sacks, 1973). Al's first pair part is a joke about the mathematical conclusion reached by the group, to which Dan responds in kind, with the joke, "because math."

The rest of the group implicitly agrees to close through their laughter and their openness to Al's redirection of conversational topic, discussed in the next section below. The "because math" joke reiterates the group's epistemic stance that mathematics was the preferred place to look for warrants for their arguments while also acknowledging, through humor, that their mathematical "resolution" is perhaps not fully satisfying. For the argument of this paper, however, the key point here is that the joke relieves tension in the group. The group members smile, laugh and lean into the table.

This episode is not an escape hatch simply because the group interactionally achieved relief of tension. *It's an escape hatch because (1) the conflict that emerged was multifaceted (conceptual, social, emotional, and epistemological) in nature, (2) closing move(s) contribute to relief of the tension as an interactional achievement, but (3) without resolving the conceptual issue and/or epistemological issues that* *helped produce the tension in the first place—in this case, reconciling between competing ways of understanding why a quantum particle in a box cannot have zero energy.*

Segment 6/6: Coming to an uncertain conclusion

With the escape hatch having been taken, the discourse gets a fresh start. Al restates an approach he had introduced back in segment 1, but this time the group takes up the approach in a collaborative way.

98	Al:	No, what, what I was arguing at the very beginning is that umm, I
99		thought, at least qualitatively, it boiled down to like the uncertainty
100		principle. Like there's always, you can't say it has zero energy
101	Bob:	O::h yeah! I think that's what it is. I think--
102	Al:	Well YEAH. I really think so too, but like…
103	Dan:	How do you explain that?
104	Bob:	That's--
105	Al:	[I don't know, like beyond that.
106	Bob:	//If it has no energy, then it now has a definite position. And [if..
107	Al:	//Something along the lines of that.

Dan: [And a definite momentum?

Bob: //And. Yeah! A definite position and a definite momentum, which is impossible. ['Cus you know its momentum is zero, and you know its position is right there, which is not possible.

Al: //If you knew, if you knew—

Chad: [Well if you have--

Chad: //Well do you know where its position in the box is, if it has no momentum? Un-- Unless you know the initial state, you don't know. Which, [I don't know how that plays into it.

Bob: //But you know it's somewhere in a discrete position and you know it's, once you find it, it's gonna be right there.

Chad: Well yeah, but it's still a probability distribution ((pencil traces probability distribution in air)) of where it's going to be.

Bob: Well that's exactly the point. So it's, it's gonna to have a probability distribution and I think that gives rise to some sort of energy.

Chad: Well yeah, that would give it energy, if it had a distribution, 'cus then you could do the Hamiltonian and it won't be a zero eigenvector... eigenvalue.

Bob: Well yeah that makes sense. I have no idea how to answer this question. ((laughs))

Chad: I think it's just indistinguishable from no particle, so you can't [have a particle with no energy.

Dan: //O:h yeah.

Bob: //I'm going with that. I'm going with that 'cus I don't know. ((laughs, all lean into their papers and begin writing))

Chad: Then it just becomes trivial.

Al's emphasis on "qualitative" in line 98 suggests that he might be thinking of his reasoning based on the uncertainty principle as distinct from the mathematical reasoning they have been pursuing for the last few minutes. The group shows their support for the need for a conceptual response by allowing Al to complete this relatively long statement without interruption (lines 98-100), after which they begin to collectively explore what the uncertainty principle may offer. Bob is the first to offer support to Al's suggestion with his exclamation (line 101). Al then concedes that he doesn't know how to explain his solution further (line 105). Bob and Chad subsequently continue the line of reasoning, negotiating what would happen to the particle's energy, momentum and position. Although Bob and Chad make repairs to each other's contributions, they still build on each other's reasoning (and on Al's and Dan's)—a collaborative discussion that contrasts with the lack of collaboration in segment 4. Although the students don't reach a full resolution using qualitative reasoning centered on the uncertainty principle, they make progress and end up connecting that qualitative reasoning with the mathematical ideas they had been discussing earlier. So, as we discuss later in more detail, the escape hatch in this case provided tension relief that enabled the group to restart their discussion in a way that helped them make progress addressing the question at hand.

Episode 2: "Can we define"

Our second episode comes from later in the same session. Here, the "escape hatch" is a locally closing segment of conversation in which the students re-interpret the worksheet question in a way that allows the group to move on to the next question without resolving the preceding conceptual disagreement.

The episode begins with Chad reading the question out loud. Al then proposes considering points on the wave (transcript lines are relabeled, starting from 1.)

Segment 1/4: Which speed and which frame?

Chad: ‘Can you define a speed for the wave?’

Al: Well you could talk about how fast, like it moves, like the points on it move up vertically up and down ((moves thumb up and down)). Then if it's...

Bob: Well that's a good question. Which [speed is it talking about?

Dan: //Translational [versus...

Chad: [Transverse… ((sweeping, horizontal gesture))

Al: //Well it says it's [standing wave.

Bob: //A standing wave, right, right, ok so... To define the speed of...Why do they say speed of the wave?

Chad: I would just say it's--

Bob: Translational speed wouldn't be the speed of the wave, it'd [be the speed of the particles.

Al: //Yeah when they say speed of the wave they usually mean like how fast it's propagating. ((horizontal gesture))

Dan: So there's no translational speed.

Al (2-4) begins talking about the physical situation, noting that the motion of individual points on the wave could be interpreted in terms of speed. Bob follows up with a clarifying question (line 5). Bob's question seems to be a bid to first make meta-level sense of the task, i.e., what is the tutorial asking us to do? The ensuing discussion raises two possibilities for “defining a speed for the wave”: defining the horizontal speed with which the wave propagates (Dan, line 6; Chad, line 7), or the vertical speed with which particles/point(s) on the wave move up and down (Al, lines 2-4).

Segment 2/4: Oppositional Stances to Defining "Speed"

17	Chad:	Are we [assuming that…
18	Bob:	//A standing wave is equivalent of two waves moving in opposite
19		directions, superimposed on each other.
20	Chad:	That's true. So. If we just picked out the one segment of it that's
21		going to the right ((thumb on right hand gestures to right)), that
22		would be the speed of it. 'Cus it would be the same in the [other
23		direction ((thumb on left hand gestures to his left)).
24	Bob:	//Accor-- Yeah, they're both going the same speed. Yeah.
25	Chad:	It'd have to be the same speed. ((thumbs pointing out to his sides))
26	Al:	Uhhh, I don't know about that because let's say if it was, if it was
27		actually a traveling wave, then it could be one wave going this way
28		((right thumb moves right)) and then one going a little slower this
29		way ((left thumb slowly moves left)).
30	Bob:	[But the whole point is it's a standing wave.
31	Dan:	//But they're saying it's a standing wave so it's going this way
32		((thumb on right hand moves to his right)) and this way ((thumb on
33		left hand moves to his left)) and they're the same speed.

Al: No. I get that ((tapping on table with both hands))

Chad: It must be the same speed or else it wouldn't be a standing wave ((quickly gestures thumbs out to respective sides)), it'd be beating patterns.

Chad and Bob (lines 18-25) co-construct a conceptual approach to the problem of decomposing the standing wave into its rightward and leftward traveling-wave components, and taking the speed of one of the components. Al contests their claim, using his thumbs to demonstrate a similar situation, one in which their decomposition idea does not make sense. Al may be suggesting that, because using decomposition to define speed applies only to the particular case of a standing wave, it might not be a valid way to define wave speed more generally. Bob, Dan, and Chad emerge as aligned with one another, and dis-aligned with Al, in repeatedly making the same point to Al that the case under consideration is a standing wave. Bob and Dan add additional epistemological layers to their disagreement through their choice of words of, “the whole point is it’s a standing wave,” and “they’re saying it’s a standing wave,” which invoke external authority.

So here, the initial conceptual disagreement on defining speed gets coupled with conceptual, epistemological and social components, in that one group member’s questioning of how generalizable a line of reasoning should be is not interactionally taken up by the other group members. Instead, the other group members provide repeated, distributed, and epistemologically weighted protests against one member’s bid for an epistemological reconsideration of what counts as a valid definition.

Segment 3/4: Al’s questioning is shut down

Al: I know what you're saying, but like, ((tapping on table with both hands)) if you, if you did. Like… Remember when we did the umm, like the free particle, how there's like the group velocity ((thumb on right hand moves to his right)), and the...

Bob: And the phase velocity.

Al: Phase velocity. It's like... ((holds right thumb up pointed to his right)) when we like solve the Schrodinger Equation, you, we had the, we pictured one going this way ((thumb on right hand moves to his right)) and one this way ((thumb on left hand moves to his left)).

Chad: Yeah.

Al: What I'm saying is like ((holds both thumbs up)), if you're doing what you're saying, that like oh, a standing wave is just they're going in opposite directions ((thumbs move outwards)) at the, or the same speed, then like, and you just pick one ((right thumb moves right)) and say that's the speed, then how do you determine which one ((both thumbs move out))?

Chad: [Well speed doesn't have a direction.

Bob: //Wouldn't you just say speed...

Al: No:. No, no, no. What I'm saying is like, if it's a traveling, like if you ((right thumb moves right). Well… [I guess it's a standing wave so--

Chad: //If you were to release the edges of the standing wave ((hands start pinched in front of body, move out to sides with fully extended arms)) and put it to a string, there would just be two separate waves that go off at the same speed ((hands move out to sides with fully extended arms)).

Chad: [((fully extended arms)

Bob: //If you had an infinitely long string ((pinches fingers together)), but we have these boundary conditions ((hands flat in front of body, facing each other)), that literally [cause those

Chad: //((hands flat in front of body, facing each other))

Bob: [two traveling waves traveling in opposite directions to be

Chad: //((pinches fingers together))

Bob: [reflecting off the

Chad: //((arms form shape of an ‘S’ to show reflection))

Bob: [boundaries.

Chad: //They have to be imposed on each other.

Bob: Yeah. And that’s what’s causing it to…

Chad: Yeah, 'cus the nodes don't move on a standing wave. So. Total speed...

In this excerpt, the epistemological+social conflict grows, in Al reiterating his concerns over the rest of the group’s reasoning and the other members continuing their distributed, embodied counters to Al's concerns in a way that shuts Al out of the conversation after line 59.

Al begins the excerpt by expanding on his bid to have the group consider the epistemic validity of their reasoning. During Al's utterance, both Chad and Bob smirk at Al and each other, both exchanging glances with Dan. Bob and Chad talk over each other to respond to Al's comment, elaborating on why the components of a standing wave have the same speed but not addressing Al's point that in other physical situation the components could have different speeds. Chad responds that "speed doesn't have a direction," which while technically correct directly contradicts his gesture animating the wave components in Segment 2, lines 20-23. Al conveys a sense of frustration when he responds with, "No:. No, no, no," and goes on to reiterate, "what I'm saying," a phrase Al has used in back-to-back turns. His repeated use of this phrase indicates that he may feel his point is not getting across, but he concedes before finishing his explanation. Pronoun use in his utterance, clearly marking his utterance as his own ("what I am saying") and marking ideas he is challenging as belonging to the group (through the use of "you"), also indicate his sense of separation between himself and the group.

Chad's utterance (lines 60-64), immediately following Al's initial protest and then resignation, seems directed at Al: Chad glances away from Al only once during lines 60-64, and fully removes his gaze from Al only when Bob begins his utterance in line 66. However, Al is unengaged during this time, looking and leaning into his paper. His physical positioning and actions differ starkly from the rest of the group, who are upright and attending to Chad. Bob and Chad go on to co-construct a story about the speed and reflection of the components of a standing wave (lines 60-78). In lines 69-75, Chad provides an embodied complement to Bob’s reasoning, gesturing with Bob’s utterance. The high level of collaboration between Bob and Chad here makes their misalignment with Al even starker.

Thus in this segment, we see a further construction of alignment between Bob, Chad, and Dan, and their growing dis-alignment with Al, with interactional markers that the alignment/dis-alignment with respect to conceptual and epistemological approach to the question also has affective and social dimensions.

Segment 4/4: Escaping continued tension: undefinable speed

79	Ed:	So how would you say we can define the speed?
80	Chad:	Uhh, phase velocity in one direction?
81	Bob:	I don't know.
82	Al:	I, I just don't think it's good to define the speed of the wave in like,
83		the speed of one of the things that's superimposed. [Superposition.
84	Bob:	//Okay so, how are we defining it ((looks to Al))? How should we
85		define it?
86	Chad:	Well, what would be a speed then ((looking at Al))?
87	Al:	I don't think there is one [(gestures towards tutorial)).

88 Chad: //((looks at his tutorial))

89 Al: For it's a standing wave, I don't think it has [like--

90 Chad: //Oh! It's CAN we define!

91 Al: Yeah. ((Bob laughs))

92 Chad: No. No, we cannot. ((Chad, Al, and Dan laugh))

93 Bob: No:.

In this segment, the coupled conceptual, epistemological, and social divergence between Al and the rest of the group continues (contributing to tension) until playing on the wording of the tutorial provides a means of closing the discussion—the escape hatch.

This segment functions as a conversational closing, initiated with Ed's bid in line 79. His utterance starts with the hedge word "so", indicating that he might be wanting a shift from the earlier discussion (Bolden, 2009); and in contrast to the earlier discussion, Ed refers back to the wording of the tutorial question, "can you define the speed?" Through reference to what opened the conversation, Ed demarcates this conversational point as an opportunity to close (Schegloff and Sacks, 1973).

Ed's bid for the group to find an answer is met with the group revealing uncertainty in their tentative conceptual solutions and a continuance of the tension between Al and the rest of the group. Chad's opening and uncertain "uhh" and his upward inflection at the end of his utterance suggest that Chad is unsure about whether "phase velocity in one direction" is a correct resolution. Bob displays similar uncertain feelings of the group's conceptual footing as he smiles, shakes his head and quietly says "I don't know." Al then restates his earlier stance towards Chad's conceptual solution. In response, Bob and Chad collectively position Al as now responsible for finding the solution (lines 84-86).

As Al completes his utterance in lines 87 and 89, gesturing towards his paper while noting that he doesn't think there is one, Chad looks to his own paper and exclaims, "oh! It's CAN we define!" This utterance marks a stark shift in the group's affect; the earlier tension gives way to mirth. Al voices his affirmation and the rest of the group laughs and leans into the table. Chad laughs as he leans over his paper, "no. no, we cannot." Similarly, Bob voices a drawn-out, lilted "no:" sharing in the group's humor. Throughout the discussion, the students were implicitly framing their task as finding an appropriate speed of the wave, proposing and opposing many different definitions along the way. However, Chad's comment (line 89) allows the group to re-frame the discussion as addressing whether it is possible to define a speed, a reframing supported by the "Can we define" wording of the tutorial.

For this conversational closing to be an escape hatch, an underlying conceptual disagreement must remain unresolved. Al may have reached some sort of conceptual resolution, his principled argument against the decomposing the standing wave into components in order to define speed. For Bob and Chad, by contrast, it's not evident that a conceptual resolution has been reached. Chad keeps suggesting the decomposition (phase velocity) line of reasoning until the re-framing at the very end. Additionally, Chad seems to be responding more towards the wording of the tutorial, rather than some notion that there is not a well-defined speed, as Al might be suggesting. Re-framing the "can we define" wording acted as a pivot for the group, helping the conversation to close and to relieve the tension created earlier.

Episode 3: Agreeing to disagree

The students in the group are Al and Ed (from episodes 1-2 above) now joined by Karen and Larry. When we held the focus group meeting, Al, Ed, and Larry were enrolled in the second semester of a quantum mechanics course for physics majors. Karen had taken these courses the year before. In this Episode, the students are addressing the question:

> **Consider the hydrogen atom. One student described the relationship between energy and position by saying ‘The hydrogen atom is in a higher energy state when the electron is farther from the nucleus. Do you agree or disagree? Why?**

In what follows, the students’ are unable to reconcile their perspectives on whether higher energy means farther from the nucleus.

Segment 1/4: Two conceptual ideas posited against each other

Larry: 'Consider the hydrogen'... Oh no. ((Karen and Ed laugh)) 'One student described the relationship between'...

Ed: Uhh, yes. So it would be.

Larry: Not necessarily, though.

Al: It could be like in a higher, like a …further away but in a lower ((traces circle)) --

Larry: Ground state is a sphere ((gestures sphere)). Second state is like the balloon thing ((fingers trace figure-eight)), but that's not necessarily farther away--

Karen: Oh and then we come to those pictures that are kinda like two balloons ((draws)), right? [Like that kind of--

Ed: //Like a ring in the middle.

Karen: Yeah.

Ed: Ahh, spherical harmonics.

Larry: So no. I don't think it's--

Al: You--, maybe you could say in general, but [not necessarily.

Karen: //Not necessarily.

Al: You could be in a, I guess, like you could have a higher principle quantum number and be further from the nucleus but have different angular momentum. Right?

Larry: I guess I never thought about the actual distance. If, if, let's say we have two electrons in the two different states [and one is in its ground state.

Al: //Well it's hydrogen.

Larry: Yeah. Or let's, okay... So electron in ground state is spherical, and the next higher state is I think the balloon thing ((gestures figure-eight))...

Al: Sure. Yeah.

Larry: Does that mean... you're right. Probabilistically it's gonna be out here ((gestures to top of figure-eight)), rather than here ((gestures to middle of figure-eight)). That's why the balloon's bigger out here ((gestures to the top/bottom of the figure-eight))--

Al: Right, what I'm saying is like there's more... It's not as simple as you're just getting further away… Um. Do you guys remember in like high school chemistry doing that, like the electron shells and everything?

Conversational patterns emerge here that are largely sustained. These include cutting each other off and talking over one another (lines 6-7, 9-10, 6-17, 23-24, 32-33), with Larry and Al the primary interlocutors. Al’s and Larry’s gazes tend to stay on each other, and not fall on Ed or Karen, even when Karen explicitly adds on (lines 10-11) to Larry’s idea. This further pushes Ed and Karen to the periphery of the conversation.

Considering the content of Al and Larry's discussion, we see each speaker as attempting to reason out the question as individuals rather than building on each other's' ideas: the individual speakers engage in self-repairs (lines 5-6, 16, 18-20, 21-23, 25-26, 29-32, 33-36), but rarely engage each other's' lines of reasoning, e.g., Larry responds to Al's question about angular momentum by talking about his own thinking about distance, (Barron, 2003). Hence, through the (mild) social conflict of weakly collaborative discourse, conceptual conflict also emerges, with Larry and Al's conceptual approaches (orbitals versus electron shells) posited against each other.

Segment 2/4: Sense-making with limited participation

37	Karen:	Oh yeah. Like--
38	Al:	Like [how you wouldn't necessarily go all the out in one shell, you
39		would jump to one before. Because even though it was further out,
40		it was like....
41	Larry:	//Yeah.
42	Larry:	That's if you're considering two shells. 'Cus there [are—
43	Al:	//Yeah. Right. Which, what I'm saying is the shells are what we're
44		talking about when we get farther from the nucleus.
45	Larry:	It's such a, such, such a, such a simple system. I think it's okay to
46		consider farther meaning... or sorry uhh what does it say? 'Higher
47		energy states when the electron in farther'... further away.

48 Karen: I'm trying to [think –

49 Larry: //Wait... higher energy doesn't necessarily mean a higher potential

50 ((turns to Ed))

51 Ed: Yeah. Like when it's closer it would have more...

The interactional patterns seen in the last section continue. We see differentiation of the social conflict associated with these patterns, in that Karen and Ed are interactionally positioned with fewer opportunities to contribute, while the main contributors (Al and Larry) continue to interact weakly. In order to make a better-warranted case that access to participation is inequitable, we tabulate each student's contributions to the discussion (in Segments 1 and 2 of Episode 3) in the table below.

Types of Contributions	Karen (lines)	Al (lines)	Larry (lines)
Short affirmations	**2** (13, 17)	**2** (24, 28)	**0**
Cut-off by others	**3** (10-11, 37, 48)	**1** (5-6)	**4** (7-9, 15, 29-32, 41-42)
Extended	**1** (10-11)	**6** (5-6, 16, 18-20, 33-36, 38-20, 43-44)	**8** (7-9, 15, 21-23, 25-27, 29-32, 41-42, 45-47, 49-50)
Total number of turns of speech	**5** (10-11, 13, 17, 37, 48)	**8** (5-6, 16, 18-20, 24, 28, 33-36, 38-20, 43-44)	**8** (7-9, 15, 21-23, 25-27, 29-32, 41-42, 45-47, 49-50)

TABLE 2.1: Breakdown of types of contributions made to the discussion by Karen, Al, and Larry. Number of contributions, per type, is shown in bold, followed by the associated line numbers in the transcript. "Short affirmations" were contributions of the form, "yeah" and "sure," or were brief

repeats of another person's utterance, such as Karen's "not necessarily," (line 15). "Cut-off by others" indicate lines where the speaker was interrupted by another. 'Extended' utterances are turns in which the speaker was able to connect at least two ideas together, or finish a complete thought.

In table 2.1, we can see that Karen had fewer opportunities to make extended utterances, (1 in comparison to 6 for Al and 8 for Larry). Half of Larry's utterances were cut-off, and almost all of Karen's. Even with the variance associated with the number of contributions, what stands out is that Al and Larry are simply allowed to contribute more before being cut-off, if at all.

The positioning of Ed as peripheral member also becomes more apparent. In Segment 1, Ed contributed several times, but seemed to fade out as the conversational dominance of Al and Larry became more cemented. The current excerpt begins with no contributions from Ed. Towards the end, however, Larry's interruption of Karen in Line 49-50, functions as taking away Karen's opportunity to participate while giving the floor to Ed.

Additionally, we see continued evidence of the social and conceptual conflict associated with the two weakly interacting conceptual lines of reasoning (Rochelle & Teasley, 1995). Al spends three utterances, (lines 33-36, 38-40, 43-44) explicating his position, with Larry only acknowledging those contributions with "yeah" and minor, yet non-engaging, disagreements (line 41-42). Al seems to feel the interactional and conceptual conflict, in that his tone becomes more forced through this segment and he feels the need to further demarcate his position through the preface of “what I’m saying is,” (lines 33, 43).

Segment 3/4: Status negotiations among members and their contributions

52	Larry:	You're right there's the hydrogen [atom thing --
53	Karen:	//But the Fermi sphere is in n-space, it's not in, in like k-space. It's
54		not in, like length.

Larry: Are you talking about this graph? Like this? ((*Karen looks over Larry's paper as he draws*))

Karen: No:. No, that's not what I'm talking about.

Larry: You know what this is, right? ((*Larry points to what he drew*))

Karen: No:. ((*Karen laughs*))

Larry: This is the, umm... this is like the... Actually I don't quite know, so we shouldn't use it.

Ed: I think we may be overthinking this. ((*Karen laughs*))

Ed: I would say that it just doesn't necessarily mean it's in a higher energy state, because, just because the potential's greater doesn't mean that it's in a higher energy state necessarily. (?)That could be.

Al: Well think about... the potential's pretty simple. The further you get away.... It's just like um... It's the same... the potential that the electron is under is the same as this guy ((*points to previous prompt*)). And so the further you get away, you're gonna have a, hi:gher potential energy. Right?

Larry: Mechanically, yes.

Al: Yeah.

Larry: Classically, yes.

Al: Right. So it's under [that same potential. But, but, if you're --

Karen: //That's, that's the next question...

Larry: Oh is it? Okay. Electron can only exist in specific levels, that's why we do quantum mechanics. [Always sphere ((*gestures sphere*)) --

Karen: //Right, that's like… That's like the Fermi sphere. Because--

Larry: What's the Fermi sphere? ((*shakes head, hand up in questioning gesture*))

Karen: Oh, you guys haven't learned that yet?

Larry: No, we haven't.

Karen: Oh it's just a way of thinking about this exact problem ((*pushes paper towards center, draws diagram*)) where pretty much you have different energy levels, and um you kind of think of each electron as filling one state. And so the electron can't, because of the Pauli Exclusion Principle, they can't occupy the same state.

Larry: Right.

Karen: So like, it's like, when you excite one of these electrons, it'll mov and then the next one can move to fill that spot ((*glances back ar forth between her drawing and Larry*)).

Larry: So this acts as whatever ((*mumbles*))….

Karen: That's just it, it's k space, it's not uh, it's not radial.

Larry: Oh, it's not like anything related to that?

Karen: Ye:ah, I didn't realize you guys hadn't, hadn't learned that yet.

97 Larry: Yeah, I don't know. I don't think, I don't, I don't think it has to do
98 with... higher positions.
99 Al: I would say--
100 Larry: Or higher--

Within the continuing interactional patterns of weak interaction (until the conversation settles into a discussion of the Fermi sphere, lines 79-100), status negotiations among peers become more explicit and forceful.

Karen first introduces a new conversational topic, the Fermi sphere (line 53-54). However, Larry’s response, instead of building on or continuing that thread, physically and conceptually draws joint attention back to the graph he drew. His next utterance serves to challenge Karen’s understanding with “you know what this is, right,” positioning himself as more knowledgeable than Karen (Sullivan & Wilson, 2015). Through this interaction, Karen seems to experience growing discomfort, which she expresses through nervous laughter, elongated “no’s” and blushing. It’s evident that Al also experiences the growing tension between Karen and Larry, as he closely follows the back and forth, smiling when Larry asks Karen, “you know what this is, right?” Ed also attends to this interaction, looking back and forth between Karen and Larry as they speak. Although we cannot infer what Ed is feeling in response to the exchange, it is apparent that he is noticing it. In any case, Ed offers a possible resolution to the question (line 63) after which we don’t see him making further utterances here, even though the discussion continues for several more minutes.

After Larry's fairly didactic utterance explaining "why we do quantum mechanics" (line 76-78), Karen reignites her conceptual contribution of the Fermi sphere. While explaining it, Karen notes twice that she didn't realize the "guys haven't learned that yet," explicitly positioning her background against those of the other group members. She pushes her paper towards the center of the group, and looks between the paper and Larry as she speaks, giving a sense that the explanation, and possibly also the positioning moves, are really intended for Larry. Through these embodied actions, she physically and relationally vies for more leverage, perhaps unconsciously, to 'buy' her way into meaningful participation in the discussion. However, Karen's burst of participation is short lived, as Larry in turn positions her idea as irrelevant (lines 96, 97), noting that it has nothing to do with their joint query of "higher positions." More importantly, he also physically positions her and her ideas by turning his body and gaze away from her and to the rest of the group as he finishes speaking, again putting her to the periphery of the conversation.

These status negotiations between Karen and Larry are not lost on the other members, Al and Ed. Al smiles as Karen comments about what the others haven't learned. Ed moves from looking down and away to quickly looking between Karen and Larry around the same time.

As before, we see tendrils of epistemological and conceptual conflict emerging from largely social conflict and becoming more striking with time, with Karen bidding for her knowledge to be considered authoritative but with the group continuing to privilege certain members' conceptual contributions over others. The emergence of multidimensional conflict brings a layer of tension, as the students witness and react to the conflict unfolding in the group.

Segment 4/4: Agreeing to disagree

101 Al: I would say, I think we should say, in general yes. [Because
102 when you see…
103 Larry: //We don't, we don't have to say the same thing.
104 Al: That's true. That's true.
105 Larry: So. We're gonna split on this one?
106 Karen: Ye:ah.
107 Larry: Alright.

Ultimately, the tension associated with Al's and Larry's two weakly interacting lines of reasoning, which is correlated with the epistemological conflict associated with vying for epistemic authority (Greer, Jehn, & Mannix, 2008), appears to win out over striving for conceptual resolution. Al begins the excerpt with a bid for joint sense-making; he notes what "we should say," and goes on to explain with "because when you see." However, Larry talks over him and cuts him off with his bid to close through re-framing their discussion as one that doesn't need group consensus.

Lines 103-104 constitute the adjacency pairs that linguistically mark the close of the conversation. In line 103, Larry asks the group if they will go along with the close. Karen's drawn out "ye:ah" shows some finality in her response. Larry's last line acknowledges Karen's agreement (Al has already agreed and Ed is no longer involved), and aims to serve as the final turn in the conversation. So, as in the Lampert et al. (1996), example, "agreeing to disagree" serves as an escape hatch; changing the rules about what counts as an acceptable answer, with consensus no longer required for acceptability, sidesteps resolving the conceptual disagreements, renders moot the negotiations over epistemic authority, and relieves the tension.

Discussion and Instructional Implications

Entanglement of cognitive and social tensions

The literature review above showed that, while many studies focus on a particular kind of conflict in student interactions (e.g., social, epistemological, emotional, conceptual), few studies simultaneously focus on multiple kinds of conflict and the relations between them. Building on work from organizational studies showing correlations among different kinds of conflict and on the small body of work in science and mathematics education showing the simultaneous presence of multiple kinds of conflict in small-group work, our three episodes support the conjecture—suggested by but not foregrounded in prior studies—that the different types of conflict are not merely co-present but deeply entangled at a fine time scale. In particular, group interaction can become tense as conceptual, epistemological, and social conflicts emerge and build in group discourse.

This entanglement has a couple of implications for researchers. First, the ease with which conceptual, epistemological, social, and emotional dynamics feed on each other invites close attention to learning environments and interactional patterns that prevent such entanglement—specifically, that allow conceptual and epistemological disagreements to emerge (and to be addressed) without overwhelming social conflict or tension. Of course, experienced facilitators of small-group work already know to be on the look-out for unproductive group dynamics and to try to set expectations for productive argumentation (e.g., "accountable talk" moves (Michaels, O'Connor, & Resnick, 2008) emphasized in some classrooms). We suggest that the ease with which different types of conflicts can become entangled can help explain why structures such as accountable talk moves don't always "work," and hence, why other pre-emptive and in-the-moment interventions—as discussed below—might sometimes be needed. Second, even if a researcher is ultimately interested in one type of conflict (e.g., epistemological), the degree of entanglement we documented among the different types of conflicts suggests that deeply understanding the genesis and nature of one type of conflict will require at least some research attention to the other types. For instance, in the "because math" episode, we simply can't understand how the epistemological conflicts arose, sustained, and then ebbed without also attending to the social conflict and tension with which the epistemological conflict was coupled.

Escape hatches: a tension-relieving interactional achievement

Our episodes start to chart the landscape of escape hatches— resolutions of tension interactionally achieved through closing a conversation without resolving the conceptual conflict that helped to produce the tension in the first place. To be clear, we did not “discover” escape hatches: Lampert et al. (1996) focused on how Sam and Connie decided to “agree to disagree” to escape their tense conversation. And instructors probably notice similar phenomena. Our contribution is to recognize “agreeing to disagree” as a special case of a more general “escape hatch” phenomenon that is also achieved in other ways and to begin exploring the commonalities and differences among different instantiations of escape hatches.

A key commonality we documented is that, perhaps partly because different types of conflict are entangled, the escape hatch relieves all the types of conflict instead of just the tension. We also documented how adjacency pairs, a conversation-closing structure documented by sociolinguists across a broad range of interactions, play the same “closing” role in an escape hatch.

The differences in instantiations of escape hatches we documented—agreeing to disagree vs. epistemological humor about the explanatory power of math (“because math”) vs. exploiting the wording of the tutorial question—likely only scratch the surface of the multiple ways in which escape hatches could play out. For now, our point is that it can do work for instructors to see these (and yet-to-be-documented) disparate patterns of turn-taking (or not), gesturing, and adjacency pairs as different instances of *a* thing, an escape hatch. We now take up this point.

Instructional implications. Although we’ll discuss specific instructional moves below, we do not think these moves are our main take-away. Instead, first and foremost, we believe that our work can help instructors develop an “escape-hatch lens”—an attunement to noticing when, how and why students might be taking an escape hatch. This attunement, we suspect, consists partly of knowing about specific analytical tools such as adjacency pairs (for noticing escape hatches) and markers of escalating tension such as extended gestures, changes in paralinguistic features, students “ganging up” on another student, discussions lacking mutuality, relative positioning of students and their ideas, failures to come to shared understandings, repetition of one’s own idea, and so on. But even more central to the escape hatch lens, we suspect, is a holistic, intuitive sense of how conceptual, epistemological, and social can emerge and give rise to tension. This tension and multidimensional conflict may be relieved in a way that doesn’t address the conceptual issue at hand. As researchers, once we became attuned to conflict escalation, tension, and escape hatches, we started seeing them everywhere, including in prior literature such as Lampert et al. (1996). We suspect the same will be true for instructors who regularly facilitate small-group work.

An escape hatch lens includes not only noticing but also interpreting the socio-cognitive phenomena leading up to and constituting an escape hatch. For instance, consider conversation closer “because math” (episode 1). An instructor listening in might think that “because math,” though humorous, represents a consensus epistemological view that a mathematical answer to the question is sufficient. Or, foregrounding the humor, an instructor might think that “because math” is nothing more than joke. However, an escape hatch lens suggests an interpretation in which the epistemological stance toward mathematics is both half-serious and half-joking and serves to relieve tension to allow the group to move on.

Similarly, when the group in episode 3 decides to answer the question "*Can* we define," an instructor might think the students are reading the tutorial too literally and might therefore suggest that they instead address "*How* can we define…" An escape hatch lens suggests, by contrast, that the literal reading was a tension-relieving move, not the group's original or default reading of the tutorial question, and that telling the students to continue the discussion would likely lead to a renewal of the conflict that gave rise to tension and motivated the escape hatch.

Indeed, from our own (mostly minimalist) facilitation of episode 3, we observed what can happen when an instructor directs a group to return to the conceptual conflict from which they escaped. The facilitator did notice the discussion becoming quite tense and the escape hatch that the students took (agreeing to disagree). However, the facilitator perceived the conceptual idea of potential energy as a sticking point for the group, impeding their progress and resulting in their inability to come to a consensus. As such, the facilitator intervened with "So I think we're not, concerned as much with potential energy here, just if my electron has more total energy, it's in a higher like n, does that mean it's further from the nucleus?" However, the intervention was largely unsuccessful. The group obliged the implicit instruction to continue talking, but they did so half-heartedly. Larry seemed sarcastic, Karen provided an answer of "not necessarily" but provides no explanation upon request. Ed simply said "not sure." An escape hatch lens makes this result unsurprising and suggests an instructional need to address the sources of escalating tension, which is coupled with why the conceptual space took the twists and turns that it did. So while the intent of the facilitator was to turn the students back to their ‘conceptual’ discussion about potential energy, the move also turned the students back to an emotional tense conversation about potential energy.

Productivity of conflict and escape hatches in group discourse. Although we have spent a good deal of this paper arguing for the entanglement of different types of conflict, we do not aim to suggest that any/all conflict is bad. Indeed, an underlying premise of collaborative group-work often assumes that some sort of cognitive conflict will emerge in argumentation, potentially leading to a deeper understanding of the material being discussed. The point we are trying to make is that different types of conflict can be deeply entangled, and so emerge together in group discourse. So while a tutorial may encourage some conceptual conflict among participants, it is also possible that tension as well as different types of conflict will emerge with the conceptual conflict. We have provided some means of noticing escalating tension, different types of conflict, and their entanglement. However, it is up to instructors to decide if the tension and conflict they see in group-work is productive, or worthy of an instructional intervention. Determinations of productivity will likely involve considerations of local and global instructional goals. For example, is it more important, in the moment, for the students in episode 3 to understand why the Fermi sphere is irrelevant to the situation or is it more important to help the group establish more equitable discussion norms? Both are certainly possible. It’s up to the instructor to decide whether the different conflicts emerging need instructional resolution.

The same line of reasoning goes for when instructors are considering the productivity of students taking escape hatches. We don't want to suggest that instructors should always push back against escape hatches. In episode 1, the "because math" escape hatch relieved tension and opened space for the group to have a collaborative, conceptually rich discussion addressing the tutorial question. By contrast, in episodes 2 and 3, the escape hatches represented a more permanent escape from resolving the conceptual issue.

Curricular design implications. The "Can we define" escape hatch highlights that curricular designers should be aware that certain question wordings can invite escape-hatch responses. We suspect that such invitations are ubiquitous. For instance, instructors of large undergraduate lecture classes often pose multiple-choice "clicker questions" for students to discuss in small groups before "voting" individually on their answers. These clicker questions, sometimes adapted from conceptual diagnostic instruments such as the Force Concept Inventory (FCI; Hestenes, Wells, & Swackhamer, 1992), often include options such as "not sure," or "not enough information to tell." The instructor has no way of knowing whether students chose these answers for sound intellectual reasons or as an escape hatch from a tense discussion.

Going meta on emotions

Influenced by our own instructional experiences and by studies of the role of metacognition and empathetic listening in small-group problem solving (e.g., Schoenfeld, 1992; Blumenfeld, Marx, Soloway, & Krajcik, 1996), we can quickly rehearse "standard" instructional moves to respond to escalating tension and escape hatches. These include having group members articulate each other's ideas, verbally or in writing; pausing the discussion for a few minutes to allow everyone to collect and write down their thoughts; making space for non-participating group members to share their ideas; and having students "go meta" on the source of their conceptual or epistemological conflict. These types of interventions generally function to provide some kind of awareness to students, although the awareness is generally limited to content concerns and relative amounts of participation. As instructors, we routinely help students become aware of conceptual ideas and epistemological approaches. But this can leave the emotional aspects of interaction unaddressed.

And so we ask, why not help students become aware of their own emotions in sense-making and group work? This is especially important given the entanglement of emotional and cognitive dynamics in students' sense-making as we showed above. In fact, epistemological "shifts"—reinterpreting the tutorial question to redefine what counts as an answer, agreeing to disagree, joking about the adequacy of a mathematical response—seem to be recruited to manage the emotional content of interactions, making emotion the driving factor and epistemology the tool to manage emotion. With this in mind, we have sketched a general intervention that might work well to head off or retroactively address escape hatches. The intervention is simple: When a facilitator notices an escape hatch in action, or tense interactional patterns, the instructor asks the students to take a moment to individually write how they are feeling in that moment. Having done so, the students are then asked to think about what conceptual ideas they are hearing and how they are deciding between them. This reflective intervention prioritizes what students are feeling emotionally and provides space for the students to connect their emotional states to the conceptual content. This intervention may also provide a natural "re-set," potentially disrupting problematic conversational dynamics in the quiet-time. However, if those patterns re-emerge after the intervention, an instructor might see that as an indication that a more explicit intervention is needed.

Interactional dynamics can also develop more stable patterns over of the course of a semester. This may mean that students settle into particular roles within their group and that certain interactional histories develop between the instructor and student(s). Judgements about how to intervene in group dynamics may therefore change significantly over the course of the semester as the instructor and students learn more about each other.

Conclusion

So far in this article, we have implicitly adopted the standard reason for attending to emotions and social dynamics in group work: the type and degree of conflict in group work can encourage or hinder cognitive development and shared understanding (Van den Bossche, Gijselaers, Segers, Woltjer, & Kirschner, 2011; Buchs, Butera, Mugny, & Darnon, 2004). As our parting shot, we advocate a more radical take on the importance of escape hatches and related phenomena.

In students' lives, managing tension and conflict in group problem-solving—at home, at school, at work—is likely to be more important than their understanding particular science concepts such as the relation between energy levels and proton-to-electron distances in the hydrogen atom. We therefore propose that helping students learn to notice tension, conflict, and other group-work-related emotions and interactional patterns, go meta on them, and ultimately manage them, be a primary goal of instruction, not just something we do in the service of helping students engage in productive sense-making. In other words, we advocate re-conceptualizing the classroom as a space where students learn how to grapple with disagreements and the build-up of emotion in the face of those disagreements. With this work, we hope to help support classrooms where students can deal with these emotionally-charged disagreements without disengaging or truncating the discussion before reaching a point of more clarity. For instructors to accomplish this broader goal, they need to frame the classroom expansively, so that the conversational and emotional skills can span beyond physics classrooms and include everyday and socio-political conversations with friends and family. Now, more than ever, there is the need for our classrooms to help students acquire the tools to engage with conceptually, epistemologically, socially, and emotionally difficult issues in the world.

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Chapter 3: Learning with and about toy models in QM

Introduction

In this chapter, I study student sense-making with toy models in undergraduate quantum mechanics. Toy models are highly simplified systems that elucidate some physical mechanism underlying a phenomenon. For example, the finite potential well model helps demonstrate tunneling of a quantum particle into classically-forbidden regions. Toy models are also important in developing models of potentially more complicated systems; aspects of formalism associated with a toy model may also be important building blocks or serve as validity checks. For example, raising and lowering operators in the context of the quantum harmonic oscillator can be used to develop a coherent set of energy eigenstates in a model of cavity-coupling to a two-level system (Nanda, Kruis, Fissan, Behera, 2004). For physicists, toy models are not just something to know about to understand fundamental quantum behavior, they are also tools for reasoning about and modeling new situations. I argue that instructionally, toy models are already well positioned as a collection of important models to learn about. However, they should also be treated as tools for learning with (Greeno & Hall, 1997).

An important aspect of toy models in QM is that they are often associated with particular iconic representations. By iconic representations I mean pictures or graphs that are closely associated with the toy models. These pictures/graphs are iconic in the sense that they frequently show up in experts' reasoning with and about the toy models. They are ubiquitous in textbooks, curricula, and other classroom artifacts.

In this chapter, I focus on students' use of iconic graphical representations associated with the infinite and finite well potentials and energy eigenstate wavefunctions of quantum entities trapped in these wells. The focus on student use of analogy through these iconic representation comes from an instructional interest on being able to sketch wavefunctions. Wavefunctions are important for describing all observable behavior of a system. Being able to qualitatively reason about the behavior of quantum entities in a given system seems like an important first step in developing an intuition about the system at hand.

Quantum physics has been one of the topics of physics explored within physics education research. This collection of work and resources suggests that fundamentals of toy QM models should be mastered before giving students opportunities to use the models to understand novel situations. This work includes curricular development (Singh, 2008; Zhu & Singh, 2012, Tutorials in Physics: Quantum Mechanics[3], QuILTs[4], Intuitive Quantum Physics[5]), research on student thinking (Bailey & Finkelstein, 2009; Bailey & Finkelstein, 2010; Emigh, Passante, & Shaffer, 2015; Gire & Price, 2015; Johnston, Crawford, Fletcher, 1998; Kalkanis, Hadzidaki, Stavrou, 2003; Ke, Monk, Duschl, 2005; Lin & Singh, 2009; Mannila,

[3] https://depts.washington.edu/uwpeg/tutorials-QM
[4] https://www.physport.org/examples/quilts/index.cfm
[5] http://umaine.edu/per/projects/iqp/

Koponen, Niskanen, 2001; Marshman & Singh, 2015; McKagan & Wieman, 2006; McKagan, Perkins, & Wieman, 2008a; McKagan, Perkins, & Wieman, 2008a; Morgan, Wittman, & Thompson, 2003; Passante, Emigh, & Shaffer, 2015; Petri & Niedderer, 1998; Singh, 2001; Singh & Marshman, 2015; Steinberg, Wittmann, Bao, & Redish, 1999; Wittmann, Morgan, & Bao, 2005; Wittmann, Steinberg, & Redish, 2002), and conceptual surveys and inventories (Sadaghiani & Pollock, 2015; McKagan, Perkins, & Wieman, 2010; Cataloglu &Robinett, 2002). This work has a strong focus on patterns of student difficulties. This work also includes significant research on student difficulties on concepts which seem essential to understanding QM concepts (Sadaghiani, 2006; Bao & Redish, 2002). Some researchers have further argued for the necessity of disrupting difficulties with classical mechanics before moving onto quantum systems that build on classical concepts (Steinberg, Wittmann, Bao, Redish, 1999).

I provide a complementary perspective. Through my analysis, I find that **1) students are capable of applying these models before having developed a sophisticated understanding of model, 2) the process of adapting the toy model can lead to new understandings of the toy model and the systems represented with the toy model.** First modeling of these situations is often drawn directly from these toy models (in various ways). It's not always clear why, for students, they intuitively draw on these models. Even though sometimes incorrect, immediate adaptation of the toy model often directs student sense-making to areas or features in one or both systems. Conceptual development then happens as students attend to additional features and regions in their representations. Sometimes this results in a divergence of the student's conceptual understanding of the two systems, i.e. students develop an understanding as to why their initial, intuitive modeling of the situation needed some refinement.

Background: Why create opportunities for adaptation of toy models and iconic representations?

In this section, I argue that there is a need within quantum physics instruction for creating opportunities for students to adapt toy models and iconic representations towards understanding novel quantum physics scenarios. I structure this argument through four sub-points:

1. Expert physicists adapt toy models to understand more complex quantum physics scenarios
2. Adapting fundamental forms, concepts, and methods to understand novel situations is a core aspect of disciplinary expertise
3. Current quantum physics instruction is inadequate in providing opportunities for adapting toy models and iconic representations in QM to novel situations, and
4. Current physics education research has underexplored student reasoning in the context of adapting toy models and iconic representations in QM to novel situations (which ties in with and reinforces #3 above).

I briefly explore each of these points below.

Physicists adapt toy models to model more complicated quantum physics scenarios

In this section, I provide three examples of professional physics practice. In each case, toy models are an essential means for developing new formalism. These three reference cases include: the Jaynes-Cummings model, electron turnstiles, and models of size-dependent band gaps in semiconductors.

The Jaynes-Cummings model describes coupling between a two-level atom and a quantized light field. In more traditional, semi-classical treatments of cavity coupling, the atom is treated as having quantized energy levels and the light is treated classically. Developing the formalism of the Jaynes-Cummings model normally starts with mapping out the coherent states of the system through reference to the quantum harmonic oscillator (Daoud & Hussin, 2002). The harmonic oscillator provides an ideal starting point of the formalism because the energy eigenstates are also eigenstates of the lowering operator. The harmonic oscillator also ends up providing a validity check in developing the Jaynes-Cummings model. For Daoud & Hussin, there are points in their derivations that they take limits of a cavity coupling parameter. Taking the limit is a means of showing they can recover aspects of the harmonic oscillator formalism and therefore prove that their formalism agrees with quantum cannon.

The goal of developing the Jaynes-Cummings model is to see any peculiar quantum effects from treating the light field as quantized. In the semi-classical theory, Rabi oscillations eventually decay. However, the Jaynes-Cummings model shows quantum revivals in these oscillations that arise from the discreteness of the photon energy spectrum (developed through adaptation of the harmonic oscillator spectrum).

Another example comes from how modeling electron transport systems draw on and adapt the finite quantum well toy-model. An electron turnstile is a means of manipulating a single elementary charge. Ono, Zimmerman, Yamazaki, and Takakashi developed a model of turnstile behavior in a single-electron transistor (Ono, Zimmerman, Yamazaki, and Takakashi; 2003). To develop the model Ono et al. piece together two finite wells as models for the source and drain gates. The finite wells are then dynamically adapted; the "floor" voltage of the gates are modulated to increase the probability of an electron hopping from one gate to the other. The current can then be tuned through adjusting the modulation of the finite-well "depths."

Band gap approximations by Nanda, Kruis, and Fissan provide a similar example to the electron turnstile. This group also uses the finite well to develop a model of a more real-world situation. Nanda et al. study the size-dependent band gap of PbS and CuBr nanoparticles (Nanda, Kruis, and Fissan; 2004). To do so, they model the electrons in the semiconductor as having an effective mass in a finite-depth square well. The width of the potential comes from the size of the semiconductor and its depth comes from the band gap energy(s) of the semiconductor. The band gap energies that emerge from this model agree well with experimental measurements, showing the importance of relying on the finite well toy model. This method stands in contrast to those using effective mass approximations with no potential approximation, or with an infinite well approximation. The finite well toy model has more utility than the infinite well in describing the behavior of the materials of interest to Nanda et al.

In summary, toy models are meant to be adapted to make sense of situations that are arguably more complicated than the situations they represent. Complicated can mean: modeling different types of entities (e.g. lattices), piecing together multiple toy models, systems that involve interactions between entities, and dynamic models. The ways in which toy models are used to make sense of these systems varies. In some cases, the toy model is a way to make sure new developments reproduce expected or empirical results. In other cases, aspects of the toy model become explicitly involved in building new formalism and predicting new experimental outcomes/results.

Problematizing toy model adaptation in QM instruction

As the examples above show, toy models are useful for helping develop understanding in situations beyond those in which they were conceived. If instruction in physics classes strives for authenticity, then students should be given ample opportunities for adapting toy models. The question then becomes when is it appropriate to position toy models as tools to learn with (adapt to new situations), not just as representations to learn about.

In looking across common instructional materials for introductory QM (Griffiths, 2016; Serway, Moses, & Moyer, 2004; Liboff, 2003; Singh, 2008; Zhu &

Singh, 2012, Tutorials in Physics: Quantum Mechanics[6], QuILTs[7], Intuitive Quantum Physics[8]), I find the materials to be providing few opportunities for students to adapt toy models and their representations to new situations. Most problem-solving opportunities ask students to reproduce or minimally expand on things they have already encountered. When opportunities do arise for inventiveness, they require almost entirely equation-based modeling. These opportunities also typically come a significant time after the toy model has been introduced. In the section below, I will argue that adaptation of representational forms can be a means of learning. This implies that instructors should consider a "learning-with" framing, an adapting-toy-models framing, early on in the learning process.

Adaptation as a form of learning

Leveraging known models and representations, in their entirety or in a piecewise manner, can be seen as a type of adaptive expertise. This type of sense-making balances some dimensions of efficiency and innovation when modeling a new situation (Schwartz, Bransford, & Sears, 2005; Rebello, 2009). An adaptive expert flexibly utilizes aspects of known models that apply to the context in accordance with local constraints to develop a runnable model (Rebello, 2009). Adaptive expertise is characteristic of professional practice and is therefore an important target of instruction (Hatano & Oura, 2003; Bransford & Schwartz, 1999).

[6] https://depts.washington.edu/uwpeg/tutorials-QM
[7] https://www.physport.org/examples/quilts/index.cfm
[8] http://umaine.edu/per/projects/iqp/

Schwartz, Bransford, a Sears (2005) argue that there may be some "hidden efficacy" in giving students early opportunities for adaptation and invention. Such opportunities better position students to learn later on. To support this, they cite two of their previous studies utilizing double-transfer experiments (Schwartz & Bransford, 1998; Schwartz & Martin, 2004). In these studies, students are essentially divided into two conditions. One condition has student invent a method of data analysis. In the other condition students are given and practice a canonical method of analysis. Both groups then receive a common resource to learn from and are assessed. In both studies, students who were in the invent-a-method condition performed significantly better than students in the tell-and-practice condition. They cite some perceptual learning literature as providing a possible explanation for these results. Analyzing the two cases side-by-side allows learners to perceive differentiating features of the cases (Gibson & Gibson, 1955). Attention to these features may then help guide student learning from more traditional resources like book chapters or lectures.

I see using these toy models of QM as a particularly rich site for helping students develop the professional vision I associate with seeing the world from not only a physics perspective, but a physics perspective that deals with especially small-scale phenomena. These models become a means of highlighting and representing situations to make mechanistic, causal relations about particular entities more apparent. In this view, learning with and about a QM toy model or iconic representation entails more than simply learning how to reproduce and/or point to important features/aspects of a representations. Learning here entails using the toy model structure to change the way one sees new situations and interacts with the world while "grappling with the core ideas of a discipline," (Rosebery, Ogonowski, DiSchino, and Warren, 2010).

Current physics education research has underexplored student reasoning in the context of adapting toy models and iconic representations in QM to novel situations

There has been some focus in physics education research on student understanding of toy QM models and their iconic graphical representations. The focus tends to be on what students have learned (or not) about the particular models (Bao, 199; Sadaghiani, 2005; Steinberg, Oberem, & McDermott, 1996; Muller & Wiesner, 2002; Olsen, 2002, Domert, Linder, & Ingerman, 2004; Ambrose, 1999; Singh and Marshman, 2015; Singh, 2008). There is a smaller portion of the literature that touches on what ways, if at all, toy models can be tools for students to learn with (McKagan et al, 2010; Cataloglu and Robinett, 2002; Morgan, Wittmann, & Thompson, 2003; Petri & Niedderer, 1998). However, the focus of half of these works is still mainly understanding student difficulties (McKagan et al, 2010; Cataloglu and Robinett, 2002; Morgan, Wittmann, & Thompson, 2003)

In introductory quantum mechanics, iconic representations are an early target of expertise for students. Traditional instruction tends to focus heavily on these models and representations early on. This seems designed to get students to master the fundamentals of these representations and models before moving on to more complex systems like the hydrogen atom (Griffiths, 2016; Serway, Moses, & Moyer, 2004; Liboff, 2003). Accordingly, there has been a good deal of research and curriculum development around student use and understanding of various iconic representations (Bao, 199; Sadaghiani, 2005; Steinberg, Oberem, & McDermott, 1996; Muller & Wiesner, 2002; Olsen, 2002, Domert, Linder, & Ingerman, 2004; Ambrose, 1999; Singh and Marshman, 2015; Singh, 2008). This work tends to catalog student difficulties observed by researchers.

The collection of research on student difficulties also documents difficulties with classical or statistical physics that may influence student understanding of quantum mechanics, such as difficulties with probability (Sadaghiani, Bao, 2006; Bao, Redish, 2001; Domert, Linder, & Ingerman, 2004) or the wave model of light (Steinberg, Wittmann, Bao, Redish, 1999). This work indicates that some difficulties in QM may result from a lack of understanding of more classical models or notions, such as probability. Implicitly, this work suggests that students need to master the fundamental concepts and the toy models before engaging in adaptation.

While there is a large body of research seeking to understand how students deal with the toy models and iconic representations themselves, a smaller group have started to investigate whether, and how, students adapt what they have learned about iconic representations and toy models to more distant contexts. This work includes the development of the Quantum Mechanics Visualization Instrument, which is designed to test student ability to reason beyond more traditional quantum mechanics problems (Cataloglu and Robinett, 2002). With several courses having taken the inventory, Cataloglu and Robinett found that students seemed reasonably capable of generalizing the 1D infinite well to 2D. In generalizing, students were able to correctly identify the algebraic and graphical forms of the energy eigenstates. However, students seem less consistent in generalizing wavefunction forms of the 1D infinite well to a spherical potential. Similarly, McKagan et al. (2010) in their design and validation of the Quantum Mechanics Conceptual Survey, McKagan et al. report on a collection of responses to a problem asking students to reason about a higher order wave function in a slanted well[9]. In a course where the instructor had a particular focus on discussing how the kinetic energy “encodes the curvature” of the wavefunction, the students were better able to reason about the correct shape of the wavefunction (McKagan et al, 2010). Also, McKagan and colleagues noted that students who answered this question correctly were transferring knowledge to the situation. In particular, one student transferred representational knowledge about the wavefunction of a particle in a step potential. Additionally, Morgan, Wittmann, and Thompson (2003) found that a small sample of students were able to correctly reason about the shapes of wavefunctions in novel contexts. These students adapted sinusoidal and decay pieces depending on the relative value of the particle’s energy and potential in the region of interest.

The work reviewed above (McKagan et al, 2010; Cataloglu and Robinett, 2002; Morgan, Wittmann, and Thompson, 2003) shows that students can take aspects

[9] See below for a description of the problem.

of iconic representations from toy models to represent different situations. What these works do is provide evidence of the "end products" of such application. However, these works fail to provide any fine-grained account of how toy models can be tools for students modeling these new situations. Such accounts require analysis of the process of sense-making, not just the outcome. Our work will help show some of the ways students tend to adapt the toy models in order to help complement existing literature and to make a case to instructors for adaptation early on in a course.

It may seem that, given the wealth of research showing students have difficulties with fundamental quantum concepts and representations, instruction should focus on these first. However, it may be that the research community's major focus on student difficulties overly colors our perception of what students are capable of doing and where instructors should focus their attention and resources. I hope to add to work by others in the field, who are going beyond a focus on student difficulties (e.g. see Dosa & Russ, 2016; Hammer, 2000; Smith, Disessa, & Roschelle, 1994).

A similar issue arose in mathematics research education. Early studies of students' algebraic reasoning focused strongly on difficulties (Schliemann, 2007). This reinforced a curricula set-up that kept algebra later in the curriculum and introduced transitional courses like pre-algebra, all towards making sure algebra came at a time when students were ready and able. Proponents of early algebra argue that "a deep understanding of arithmetic requires certain mathematical generalizations," which may be afforded through algebraic notation (Schliemann, 2007 citing: Brizuela, 2004; Carraher, Schliemann, & Brizuela, 1999, 2000).

Methods (Analytical Flow)

How we created opportunities for adaptation of toy models and iconic representations

I wanted to create opportunities for adaptation to study how students use toy models as tools to learn with. Students learn to construct and interpret representations in disciplinary authentic ways by engaging in the practices of sense-making and communication involving representations (Greeno & Hall, 1997). These include discussions around conventions of interpretation of toy models, the affordances and constraints of disciplinary representational norms. Engaging students in these types of discussions in the context of content questions helps reveal different ways of talking, thinking, and reasoning that are well-grounded in problem-solving experiences. The goal of our interview design was to create situations where I might be able to study whether and how toy models become tools for learning with.

These two principles helped guide task design:1) Continuity in content: The researchers saw continuity between the new situations being modeled and various toy models. Toy model prompts are immediately preceding the prompts designed to promote adaptation to help prime or cue those models.

2) Interactional space: The space intended to be filled with a lot of student talk, moves by the interviewer are meant to make student thinking more visible. In

reasoning out loud with the interviewer students are also justifying their constructions against disciplinary standards and norms.

Data Context

My data set was selected from a collection of 14 hour-long interviews with a mix of physics and engineering students. Most students were of junior or senior standing. In the interviews, the students were given tutorial-style problem sets and were asked to think aloud with the interviewer as they went through the problem sets. If there were moments of silence, the interviewer would check in. Questions directed to students were meant to make the students' thinking more explicit.

The interviews were taken in two sets, with similar interview protocols for each set. On the first set of interviews, the protocol included a problem on an infinite well with a slanted problem, which I refer to as the “slanted well” problem (described in more detail below) (McKagan et al., 2010, Cataloglu & Robinett, 2002). When viewing these interviews, I noticed a relatively consistent pattern of students finding ways to adapt the representation of the infinite square well to reason about the shape of the slanted well wavefunction. I then designed an interview protocol explicitly around creating opportunities for this type of sense-making. In the design of the second interview protocol, students first encountered problems on the particle in a box; these included drawing energy eigenstates, describing the energies, and thinking about the speed of the particle. The next problem is on a classical analog of the particle in a box, the “classical well” problem (described in detail below). This is followed by several problems on explaining quantum systems to peers, the slanted well problem, and several problems on tunneling. Evidence of students adapting iconic representations occurred on the classical and slanted well problems. However, not all students were able to get to these questions in the protocol. Across both interview sets, I had a total of 17 episodes of students reasoning about the slanted and classical well problems.

Classical Well Problem

Problem statement: Suppose you had a classical particle in a physical situation analogous to the quantum particle in the box. Consider a bead on a string, and the string is knotted at x=0 and x=L so that the bead is confined between 0 to L, and can move smoothly and freely between these bounds. The bead has some energy E, and can bounce elastically at the knotted ends. Sketch the wavefunction of this classical particle.

Solution: An anticipated solution would be a flat wavefunction, as the particle spends equal time between the knots and can therefore be found at each position with equal likelihood. Some students attend to the behavior at the edges, either suggesting the bead cannot be found on the knot or modeling the acceleration of the bead at the edges.

Slanted Well Problem

Problem statement: Consider a quantum system with $V(x) = \infty$ for $x=(-\infty, 0)$ and (L, ∞), and $V(x)= Ax$ for $x=(0, L)$. Sketch the wavefunction for the first allowed state, or ground state, of the particle.

Solution: The canonical solution for the ground state is a first-order Airy function. On this problem, I only expected students to reason about the shape of the wavefunction.

Identifying Relevant Case studies for Analysis

The goal of my analysis is to understand how toy model representations are used to make sense of new contexts. Previous work has suggested that adaptation can happen (McKagan et al, 2010; Cataloglu and Robinett, 2002; Morgan, Wittmann, and Thompson, 2003), but does not elucidate the process and mechanism behind such change. Here, the orientation of microgenetic analysis provides guidance in studying processes of change (Siegler &Crowley, 1991). In providing accounts of cognitive development, 1) observations should occur throughout the process, 2) the density of observations should be large with respect to the rate of change, 3) trial-by-trial analyses should be used to model mechanism (Siegler & Crowley, 1991). Video data allows such close, detailed observation. It also enables the data to be viewed by many people across time, thereby allowing trial-by-trial analyses to develop through repeated viewing of the data.

My analysis begins with preliminary clipping of the video in order to hone in on places where I think I see toy model representations (and associated concepts) being applied to new contexts. This preliminary analysis included content logging the interviews for general observations and moving between collective and individual viewing of the data to reach some consensus on there is evidence of reasoning with and about toy model representations in the highlighted sections of the data (Jordan & Henderson, 1995; Derry, Pea, Barron, Engle, Erickson, & Sherin, 2010).

Analytical Methods

Orientation

I closely study interaction to understand knowledge-in-use because I see cognition and learning as inherently embedded in interaction with socio-material settings. My work is guided by others who do fine-grained analyses of the assembly of representational states through the coordination of different semiotic channels (diSessa, 1991; Stevens and Hall, 1998). In studying this assembly, I aim to understand how speech, gesture, and material structure are coordinated to generate a representational state. This includes understanding how coordination evolves throughout the interaction, i.e. how the meaning/utility of a representation may be changing.

Researchers studying knowledge-in-use in interaction between people and their settings have taken different views of the ontology of knowledge. The studies mentioned above study knowledge-in-use, but take different ontologies of knowledge and units of analysis. For example, diSessa (1991) uses individual, internal cognitive processes as units of analysis, whereas Stevens and Hall (1998) more so consider the group interaction as a unit. Unlike diSessa, I refrain from doing a full-scale knowledge analysis of the interaction. Instead, I use tools from interaction analysis (described in more detail below), and my membership in the physics community, to look at an interaction and describe the informational content of the interaction (Hutchins, 2000). Because of this orientation, my analysis describes internal transformations only when such transformations are not explainable from looking at the details of interaction.

Tools from Conversation Analysis

Tools from conversation analysis allow one to frame an interaction as a sequence of organized events, where one event follows sequentially from the previous (Sidnell, 2011). The aim of this analysis is then to describe how each event is shaped by the previous event, and how the current shapes the next, and so forth. Tools from interaction analysis allow me to categorize and make meaning from the organization of a conversation. It also allows me to identify entities helping organize the interaction and understand how representations become imbued with conceptual meaning. For example, the coordination of saying “this wall” while tracing a potential wall in a representation, reveals a potential wall construct as a relevant entity.

Below, I list some of the ways that people coordinate speech, gesture, and material resources:

- Gesture; all forms of gesture can add semantic meaning to interaction.
    - Environmentally-coupled; gestures that mark something in the speaker's environment. This may be in the form of tracing a drawing or pointing to something.
    - Discourse-coupled; gesture that does not clearly mark anything in the speaker's environment but unfolds with speech. i.e. Shrugging shoulders when saying "I don't know."
    - Stand-alone; gesture that does not pair with speech. i.e. Shrugging shoulders
- Organization and structure of speech; these tools help me understand what meaning is being generated between participants through how the conversation it structured. This includes understanding how particular ideas in speech are meaningfully connected to each other (Sidnell, 2010; Schegloff, 2007).
    - Turn-taking; individual turns at conversation
    - Repair; attempts to alleviate conversational trouble or breakdowns in mutual understanding
    - turn construction; conversational turns are structurally comprised of turn-construction units, which may be single words, clauses, questions, etc.

- adjacency pairs (Sacks and Schegloff, 1973); distributed conversational sequence of two utterances, where the first-pair part mutually constrains second-pair part
- Preference; some second-pair parts are organizationally “preferred” in the sense that some second-pair parts make more significant progress towards the joint enterprise underway
- Progression; on a larger scale than preference, there is a sense that the conversation should move towards accomplishing the mutually determined purpose
- Discourse markers (Schiffrin, 1988; Bolden, 2009); words or phrases that help organize conversation into segments, and suggest meaningful connection between those segments.

Example Analysis: interview with Quinn on the classical well problem

I present a short piece of transcript to provide an example of how I am analyzing the data. The table 3.1 is split into three columns. The first column gives the transcript. The second column describes what aspects of the transcript I am attending too. The third column shows conclusions I make from what I notice in the data. In the snippet, Quinn is working on the classical well problem. Immediately before the transcript shown below, Quinn was reasoning about the system as if the bead and string are both free to move. The interviewer then suggests adjusting the situation being considered to one where the string is held taut.

FIG. 3.1 Quinn’s Bead representation: Quinn’s drawing of the bead on a string. Arrows show that the bead can move back and forth along the string.

Transcript	Noticing → Meaning	Analysis
QUINN: Ok, if that's the case like ((draws bead representation)), so it's just moving back and forth? ((gestures motion of the bead over the representation, within bounds shown in the representation))	Ok (Schiffrin, 1988) → discourse marker as check on shared understanding “If that’s the case” → conditional statement following a reframing move from the interviewer So → marks another turn in the conversation (Bolden, 2009), showing conditional	Quinn first generates a representation of the bead (see Fig. 3.1), with which she physically represents the motion through her speech “it’s just moving back and forth” and simultaneous environmentally-coupled gesture. Given that this interaction between Quinn and the material structure she has created occurs

	statement not completed It’s just moving back and forth? ((gestures over bead representation in Fig. 3.1)) → Coordination between gesture, speech and representation creates an embodied representation of the bead’s motion. We need all channels to get the fullest understanding of the motion; the gesture shows smooth, linear motion that’s bounded “by” the edges of the string marked in the representation.	immediately after the interviewer has reframed the situation being discussed, Quinn prefaces her statement with “if that’s the case”, and that she ends her conversational turn with a question, this suggests that Quinn is using her turn to check in to see if she is conceptualizing the motion of the bead correctly. So, not only is Quinn coordinating material, embodied, and discursive (“moving back and forth”) to generate a representation of the bead; she is also generating a representation in a way that is apparent to the interviewer, as a means of checking for

		mutual understanding.
INTERVIEWER: Yeah, bouncing at the knotted ends ((gestures back and forth motion over table)).	Completion of question-answer pair → affirmative to Quinn Interviewer utilizes speech and gesture to generate her own, similar representation of the bead's motion following question-answer	Following Quinn's attempt to reach a shared understanding of the bead's motion, the interviewer starts her turn with an affirmation. She then includes a rephrasing of the description of the motion ("bouncing") along with an extension of the description (the motion turns back at the "knotted ends" with a simultaneously gesture, which closely mirrors that of Quinn in the previous turn. The boundedness of the motion was previously perceptually available in the interaction in that Quinn's embodied representation of

		the motion ended at the knotted ends. This notion becomes discursively available as the interviewer both notes this verbally, and in her gesturing of the beads motion, which shows the motion punctuated at particular points. The affirmation at the beginning of the turn, the rephrasing and extension of the representation of the bead's motion, along with the back-and-forth gesture being used by both participants suggests the two have reached a shared understanding of the bead's motion.
QUINN: Oh, ok. So,	Oh → discourse marker	Quinn's opening statement of

I think we're still gonna have the same condition as that one ((gestures to previous prompts[10])).	marking receipt of information Ok→ discourse marker as reference to action of the base pair (question-answer). In this context, it seems to register acknowledgment of shared understanding of the bead's motion. So → discourse marker showing turn in the conversation following acknowledgement of shared understanding of motion. The connection also seems to imply some causal linkage between motion and "condition" i.e. motion implies condition	"oh, ok. So," followed by a focal turn in conversation, moving from talking about the motion of the bead to "conditions," suggests Quinn sees a shared understanding of the situation. Her follow-up, saying the "condition" of one system is the same as what she experienced while working previously. Context of the prompts (previous and current) suggest that she is referring to the wavefunction as the "condition."

[10] Previous prompts were questions on the infinite well.

INTERVIEWER: Which one? QUINN: As the, the particle. INTERVIEWER: The infinite box? QUINN: Yeah. The infinite box ((gestures to previous prompts)).	Question-answer Question - answer → Repetition of the question implies a non-satisfactory answer with "the particle" … asking about what is the "one" Environmentally-coupled gesture coordinated with "the infinite box", follow-up turn unit used to the answer in the question-answer pair is a rephrasing or renaming (shows continued search for shared understanding of referential system)	In her previous turn, Quinn elucidated "that one" through gesture to previous prompts. However, the interviews next turn, *"which* one?" (Emphasis added), shows to which system Quinn is referring is unclear to the interviewer. Quinn's response is not enough to make the clarification for the interviewer, as when Quinn says "the particle," the interviewer again asks a clarifying question, asking whether it is the "infinite box" that she is referring to. Quinn starts her turn with an affirmative "yeah," then takes on the interviewer's language to reference the particular

		system "infinite box" as she again gestures to the previous prompts. These coupled actions show that Quinn and the interview share a sense of which system she is borrowing a wavefunction ("condition") from the infinite box.

Table 3.1: Example analysis of episode in interview with Quinn

1. The example above showcases how I am engaging in a microgenetic analysis and in doing so draw on multimodal analysis and tools from conversation and interaction analysis.
2. In the story so far, Quinn has drawn on the infinite well toy model and representation to reason about the bead-on-a-string. In her reasoning, she has drawn on the infinite well to model the movement of the bead and the string.
3. I pick up the story below in the Results section. In the next snippet, the interviewer reframes the situation to considering to one in which the string is held taut.

Results

Preview

I find that students frequently do see continuities (and discontinuities) across situations involving toy models. The different continuities that individual students notice help shape the way the student sees, highlights, represents, and reasons in the new situation. I try to differentiate these interactions taken by students some by pointing out what aspects of the iconic representational form are used in modeling the new situation. In some cases, students may see the entire physical form (with some conceptual meaning layered) as applicable, in other cases students may see smaller, modular portions of the form being applicable. For example, in looking at a problem that involves some tunneling, a student may reference the finite well representation, repurposing the representational form and conceptual notion of decay. In other cases, students may see the entire form as applying with some transformation of that form, or pieces within. For example, in borrowing the decay form from the finite well, a student may note that the piece actually needs to "decay sharper" than what's experienced in the finite well situation because the potential of the current problem grows, as opposed to staying constant.

My analyses show that students drag pieces of toy model representations to incorporate into new representations. This process helps shape and focus students' sense-making. In some cases, the iconic representation provides a pathway to comparing and contrasting fundamental behavior. In my analyses, I show that learning with a toy QM model often provides a means of focusing sense-making to particular regions in the toy model (and referent physical set-up) or regions that emerge from sense-making.

Quinn on the classical well problem (continued)

Infinite well representation as a first model for the bead-on-a-string

In the illustrated analysis in table 3.1, I showed how from the outset, Quinn sees continuities between the infinite well and the bead-on-a-string. I now pick up this story in this section. The interviewer now attempts to reframe Quinn's understanding of the set-up by suggesting that she consider a situation in which the string cannot move. In considering this new situation, Quinn still sees continuities between the infinite well and the bead-on-a-string. Seeing the bead through the lens of the infinite well directs Quinn's sense-making to the center of the bead's motion and the center of the infinite well representation.

Fig 3.2. Quinn's Bead representation: Quinn's drawing of the bead on a string. Arrows show that the bead can move back and forth along the string.

QUINN:	Oh ok. So, I think we're still gonna have the same condition as that one ((gestures to previous prompts)).
INTERVIEWER:	Which one?
QUINN:	As the, the particle.
INTERVIEWER:	The infinite box?
QUINN:	Yeah. The infinite box ((gestures to previous prompts)). Because, so this is like the x-axis right? ((Gestures to horizontal axis in bead representation)). And the particle is moving, and we're seeing where it's gonna be ((gestures back and forth over bead representation)). So, when it moves this way and comes back ((gestures back and forth over bead representation)), it will reach the middle one twice ((points to center of bead representation)). So, there is a bigger probability of seeing the particle there ((points to center of bead representation)). So, in that sense it might look like this, this being the x-axis and this being the probability ((draws probability representation)).

FIG 3.3. Quinn's Probability Representation: Quinn's representation of probability versus position of the bead-on-a-string.

Quinn uses the infinite well representation as a means of representing and sense-making around the bead-on-a-string. In doing so, she reasons about the most likely location of the quantum particle and the bead.

Seeing the bead-on-a-string through the lens of the infinite well helps direct Quinn's sense-making. Quinn reasons *only* about the center point of the infinite well wavefunction and the bead's motion as a means of inferring a wavefunction for the classical bead. Quinn describes that "we're seeing where it's gonna be" as she represents the motion of the bead with her pencil. With joint attention on the represented motion of the bead, Quinn highlights the center point of the motion. She then infers that the probability will be larger at this point because this point gets "hit twice." She moves down the page and draws a representation of the probability versus x-axis. The wavefunction she draws is also the ground state of the infinite well.

Through her highly localized sense-making, Quinn develops a sense that both the classical and quantum particles have the same probability distributions. However, these two systems should not have the same probability distributions. At this point, Quinn's drawing on the infinite well may look unproductive towards generating a correct solution for the bead-on-a-string. In the coming sections, slight shifts in Quinn's perceptual attention to the situation helps Quinn develop a deeper understanding of both the infinite well particle and the bead-on-a-string. This deeper understanding very much relies on Quinn having taken this "incorrect" step in her modeling.

In the next sub-section below, I continue the story of Quinn's sense-making about the bead-on-a-string.

Intervention: Attention to new regions shifts Quinn's modeling of the bead-on-a-string

In the previous section, Quinn had only reasoned about the center point of the bead's motion and the infinite well wavefunction. Next, Quinn broadens her attention and sense-making to consider new regions in both representations. Doing so helps Quinn develop a deeper understanding of the infinite well wavefunction and the bead-on-a-string.

> INTERVIEWER: Can I ask you if I have two points like here and here ((points to two points on probability representation)).
>
> QUINN: Two points here and here ((points to two points above curve in probability representation))?

INTERVIEWER: Along the curve ((points to two points on probability representation)), um and that corresponds to two points here and here ((points to two points on bead representation)). This tells me that this point is more likely than that point ((points to two points on probability representation)).

QUINN: Ye:ah.

INTERVIEWER: Is that right?

QUINN: No.

INTERVIEWER: Why do you say it's not right?

QUINN: Yeah. Because it's gonna be uniform. The particle, it’s the same particle, and it's moving this way, you can see it the all the way and coming back ((traces over probability representation)). So it's definite that it's going to reach this point twice and this point twice ((points to two points on probability representation)). And I think the probability will be uniform.

INTERVIEWER: So what would the wavefunction look like do you think?

QUINN: In that sense, I think it would just be like ((draws flat line wavefunction in refined probability representation))

INTERVIEWER: A flat bar?

QUINN: Yeah.

INTERVIEWER: So everywhere is be equally likely?

QUINN: Mhm.

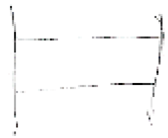

FIG 3.4. Quinn's Refined probability Representation: Quinn's representation of probability versus position of the bead-on-a-string.

Attention to new regions in her two representations, and conceptual coordination of those regions, helps Quinn deepen her understanding of the two systems and their probability distributions.

Quinn's speech shows that she sees the differential likelihood encoded in the infinite well representation and that the differential likelihood of those two locations may not apply here ("ye:ah"). Quinn essentially repeats her sense-making around the motion of the bead. In doing so, she again simulates the motion of the bead and reads-out where the particle hits twice. This time, however, her sense-making also attends to the two points of the infinite well and the bead's motion. Like the center point in the previous simulation, these points are also reasoned as getting being reached twice. Though her sense-making is not substantially different, directing her reasoning to these new regions in her representations leads to a new understanding of the bead's physical behavior and its associated wavefunction. Quinn now concludes that the probability of the bead should be uniform across the string.

Her expanded sense-making leads her to realize that the bead-on-a-string and the infinite wells have different probability distributions. The physics of the bead-on-a-string and the quantum particle are very different. This is in sharp contrast with her modeling of the situation, where she explicitly stated that the behavior of the two systems is the same. The question is, will she notice this foundational concept that has emerged from her own sense-making?

Reflections on the differences in ontology of the infinite well particle and the bead-on-a-string

Quinn does notice the profound consequences of her reasoning. Her reflection on the differences in the wavefunctions leads to an insight into the ontological differences of the two entities.

> QUINN: I'm wondering why the particle in quantum mechanics have this kind of wavefunction, because-- yeah I guess it makes sense. Maybe it's because it's not a particle particle, it's a wave. So it would disperse and change its shape. Yeah but this one is a particle and if we see it like that, it has to look like something flat.

Quinn’s rejection of the infinite well representation as also a representation for the bead enabled her to reflect on the ontological distinctions of the quantum and classical particles. Careful ontological distinctions of entities is a crucial element of developing expertise in quantum mechanics (Brookes & Etkina, 2007).

Discussion of Quinn

Recounting Quinn's sense-making: how Quinn's reasoning developed through attention to new representational features

Quinn began the episode by immediately recruiting the infinite well representation as a means of explaining the wavefunction for the classical bead. The infinite well was a relatively stable means of seeing, representing, and sense-making about the bead-on-a-string. As the episode progressed, Quinn reasoned about additional features of the infinite well representation which led to additional sense-making in new regions of the physical setup. Attention to new areas in both contexts lead to a distinction between the wavefunction distributions of the classical and quantum particles. Quinn ultimately found a limit for the infinite well model and rejected the representation as appropriate.

Opportunities for gaining conceptual insight: 1) Coordinating sense-making around new representational features, and 2) Deeper understanding of ontological differences

I discussed two ways in which Quinn may have gained some conceptual insight in this episode. First, in her reasoning, Quinn began to sensemake around additionally relevant features of the infinite well wavefunction. She went from reasoning about the most likely location to differences in probability along the horizontal axis. As she reasoned, new features of both spaces became relevant and provided the mechanism for the eventual separation of the classical and quantum models. Highlighting or attending to new visual features, and coordinating subsequent action or sense-making around those features, is fundamental to learning (Parnafes, 2007; Goodwin, 1994).

Second, the prompt provided an opportunity for Quinn to reflect on the ontological differences between quantum and classical particles. Both threads may help Quinn develop a deeper understanding of the behavior of quantum particles. Previous prompts in the interview were on the quantum infinite well. These included questions asking students to draw the first few energy eigenstates and find the most and least likely locations of the particle in the n=2 state. On the latter, Quinn correctly drew the wavefunction and identified the least likely locations shown. However, she said that “we might not find the electron there.” The interviewer asked if this meant that it was impossible to find the electron there but she was unsure. It seems likely that a developing intuition around the quantum particle as a wave may help Quinn flesh out her uncertainty around the physical meaning of the nodes in the quantum wavefunction.

Oliver on the classical well problem

Adding an offset to the infinite well representation to model the bead-on-a-string

Oliver begins by drawing a picture of the bead-on-a-string (fig. 3.5) and then starts reasoning about the bead's motion. In doing so, he compares the bead-on-a-string to the infinite well. Discontinuities that Oliver perceives between the two systems help shape Oliver's sense-making of both systems.

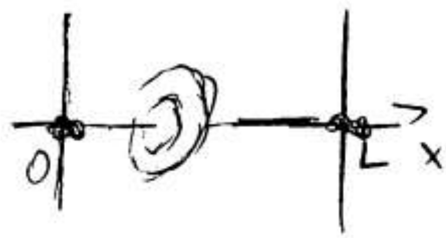

FIG 3.5. Oliver's Bead-on-a-string representation

After drawing a picture of the bead (fig. 3.5), Oliver reasons about the velocity of the bead across the string:

> OLIVER: It'll have a like, its velocity will peak in the middle and then be at a minimum towards the edges ((vertical hands gesture edges)). And. That technically is the same, I would say, for its position as well. Just because, it wouldn't, I guess in this case though, you wouldn't have a probability of zero, at the knots. Because uh, the bead will just bounce back off. Like you could still catch the bead on the knot.

Subsequent turns will reveal Oliver is making a comparison to the infinite well ("in this case though"). It's through the comparison to the infinite well that Oliver moves from sense-making about position and velocity of the bead, to inferring what this may mean for the wavefunction. His sense-making is focused on comparing the edge-behavior of the two systems because this is where he sees a discontinuity between the two systems.

Oliver says that "in this case though, you wouldn't have a probability of zero, at the knots." The wavefunction in this case is non-zero, but the infinite well wavefunction *is* zero at the edges. Oliver goes on to connect the physical behavior of the bead to the nonzero probability. He notes that the "bead will just bounce back off", which for him implies that you can find the bead on the knot. In this way, adapting the infinite well representation to the bead-on-a-string is focuses Oliver's physical sense-making about the two situations.

After reasoning about the bead's position and velocity through comparison to the infinite well, Oliver draws a wavefunction. He goes on to expand on the discontinuity he perceives between the wavefunction for the classical and infinite wells:

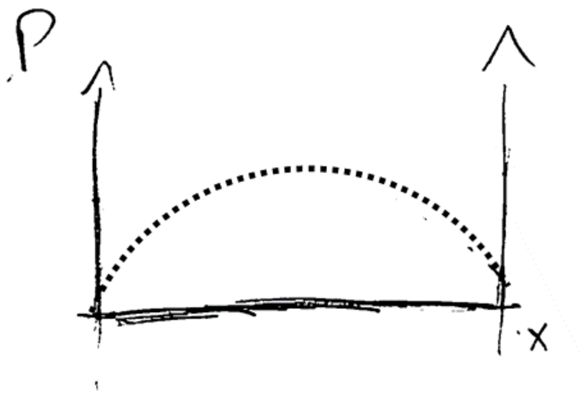

FIG 3.6. Oliver’s Probability Representation: This is a recreation of Oliver’s first attempt at a wavefunction for the bead-on-a-string. Later in the episode, his sense-making leads him a new conclusion and so he erases his wavefunction in the graph and redraws it. For this reason, I recreate his original drawing.

> OLIVER: This is basically the wavefunction for the bead. And, the difference that it has with, like a particle in an infinite well, is that it doesn't touch the edges, like there's still a little bit of distance between the minimum values of the wavefunction and I guess the zero of the, or just the ground level. Or V=0.
>
> INTERVIEWER: So physically, what does that mean?
>
> OLIVER: Physically, this just means that you can find the bead AT the knot. As opp-, you can have the bead at the knot, as opposed to the quantum particle, you can't really have the particle on the wall, because it will bounce off.

Oliver continues his comparison to the infinite well, which also continues to direct his sense-making to the edge behavior of the two systems. The comparison also leads to sense-making about wavefunction representations more generally.

The wavefunction he has drawn is the ground state of the infinite well with an offset, which emerges from being unable to find the quantum particle at the edge. Oliver then reasons more deeply about the offset: What is the offset actually measuring? The bead's edge probability is nonzero, but nonzero with respect to what? Here, Oliver makes several repairs while attempting to articulate what the distance describes. However, uncertainty around how to describe the offset is not stopping Oliver from engaging in some mechanistic reasoning around how the two entities are behaving at their boundaries, and how that physical behavior leads to different probability distributions. i.e. The difference in probability at the edges arises from being about to find the bead, but not the quantum particle, at the knot. And so, even though Oliver could be uncertain about how to describe the offset at the edge, this does not stop him from reasoning about its physical significance.

Intervention: Attention to new regions leads to uncertainty in modeling the bead-on-a-string as the infinite well with an offset

Oliver's interview proceeded with roughly the same intervention as Quinn. The interviewer first pointed out the differential likelihood of two off-center points implied by Oliver's representations and asked if that interpretation sounded right. Oliver agreed and went on to explain why. Oliver reasons about the bead's position to show why the differential likelihood applies to the bead. His reasoning leads him to new understandings and new areas in his representations.

> INTERVIEWER: So can I ask you why like, maybe two positions, here and here, a little off center ((points to two points on wavefunction in probability representation)), like here versus here ((points to two points in bead-on-a-string representation)). Why, I think what this ((points to wavefunction in probability representation)) shows me is like, this position ((points to point on bead-on-a-string representation)) closer to the center, is more likely than something like right next to it. Does that, it, does that sounds right?
>
> OLIVER: Ye:ah. So, essentially, ummm. Similar to how, with a classical particle, it bounces off the barrier, so to speak, it always hits, every point. Well. Saying that you would travel, starting from zero ((left pointer into table)) to L ((right hand into table)), and then back to zero ((left pointer into table)). You're hitting every point twice ((points to space between where his hands were)). So to speak. But. Hm. Actually. Hmmm, I don't know. Hmm.
>
> INTERVIEWER: What are you thinking now?
>
> OLIVER: Uhhhh, I'm not sure, well. In a sense, I guess I'm trying to, compare this to, the quantum particle. But then again, this. The physics don't work the same.

In response to the interviewer's question, Oliver introduces a new line of reasoning that also incorporates attention to new regions in the bead-on-a-string's motion to think about the differential likelihood in the infinite well. He gestures the motion of the bead and reasons about what points get hit twice. This is in contrast to Oliver's first line of reasoning (momentum and velocity), where Oliver only reasoned about the edges and center. His new line of reasoning is not differentiating the different points along the string. Instead, his reasoning suggests that every point is the same because every point gets hit twice. Oliver seems surprised by this new understanding, suggesting that he may now be unsure of his earlier reasoning. At the interviewer's prompting, he steps back to express his lack of surety through explaining what he's *trying* to do: compare to the "quantum particle." He goes on to stress that there are fundamental differences in the physics of the two situations.

Oliver holds onto sinusoidal wavefunction for the bead-on-a-string even though his reasoning does not back it up

Oliver continues reasoning about what points get hit twice. It's clear that he wants to hold onto the sinusoidal wavefunction shape but his developing reasoning, which considers more areas of the bead's motion, is leading to other conclusions.

> OLIVER: Like it always ends up hitting the middle point twice ((pen into table)). Or the middle points twice ((gestures range with both hands)), more

than it hits the edges. But that still doesn't really, it doesn't really all the way explain why, you're not, why the middle points aren't counted equally ((gestures range with hands)). So like why it does, why isn't it just a step function as opposed to a sinusoid. Hmmm.

OLIVER: This is. Well. mv is the momentum. Its mass is the same. The only thing that's changing is the velocity. And velocity in terms of x. So.

Oliver is now more explicit about how his reasoning about what points get hit is yielding a deeper understanding of the bead's motion that makes the infinite well's sinusoidal shape a now questionable model for the bead. His reasoning is leading to the new conclusion that middle points should be counted equally. But he seems reluctant to let his initial sinusoidal representation of the probability go.

Attention to new regions in velocity representation finally disrupts Oliver's initial modeling of the bead-on-a-string

Oliver has returned to his initial entry point to the problem, reasoning about momentum and velocity of the bead. The interviewer asks for a representation of the velocity so that they can better sensemake around it.

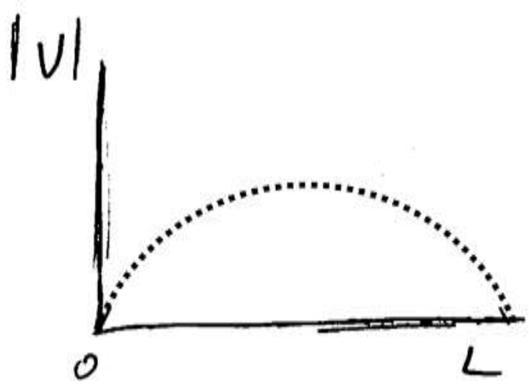

FIG 3.7. Oliver's Velocity Representation: This is a recreation of Oliver's first attempt at graph for the velocity of the bead-on-a-string. His sense-making later in the episode leads to a more nuanced understanding of the velocity and so he erases his first velocity curve. I recreate his original.

Oliver draws the "first half of a sinusoid," and again mentions that velocity will "maximize" in the center. The interviewer then directs Oliver's attention to half of his velocity plot and draws some implications from Oliver's reasoning and representation. Doing so leads to a new understanding of the forces, acceleration, and velocity of the bead, which helps him understand the probability of the bead to be uniform.

> INTERVIEWER: So can I ask, in the first, like half, what is causing the bead to accelerate ((points to right half of velocity representation))? And then decelerate?
>
> OLIVER: Ummm. Hm

OLIVER: This is true. Technically, there isn't anything decelerating the bead. Whatever initial force kind of, hits the bead, it'll shoot to the velocity it's supposed to and then drop back off. ((erases velocity representation))

OLIVER: Which probably ends up meaning that, the:.... the same thing holds for the wavefunction as well. Since, ((erases wavefunction in probability representation)) the velocity is a constant, its position is also, going to be, well it, the position, the position itself is not going to be constant. But, in terms of, predicting where it's going to be, you know that it's just going to be somewhere in between the two knots equally, somewhere between those knots ((draws flat wavefunction in his probability representation)).

Reasoning around half of the velocity plot leads to some consideration of the relationship between forces, acceleration, and velocity of the system. Coordination of these constructs across the string (“between those knots”) allows Oliver to conclude a flat wavefunction for the bead.

At this point, I end my narration of the interview to discuss how Oliver’s story connects to the main claims of this chapter.

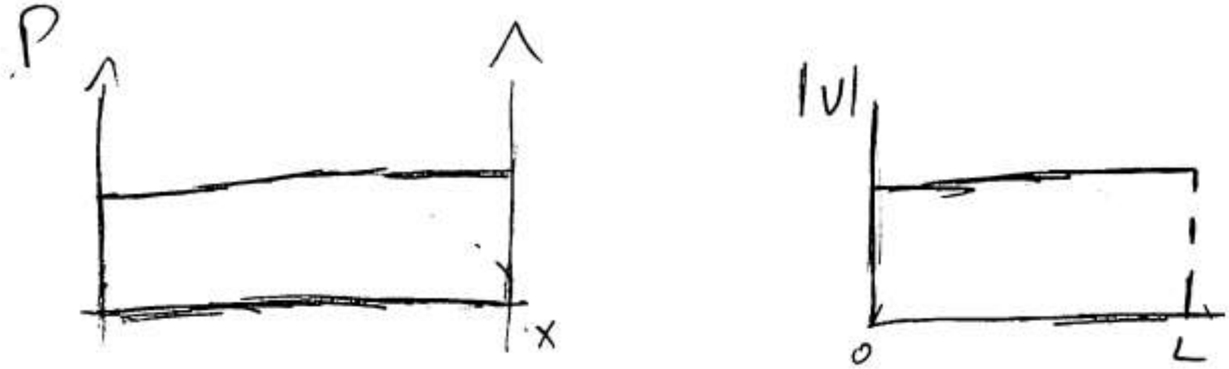

FIG 3.8. Oliver’s Refined Probability and velocity Representations: Oliver’s refined representations of probability versus position of the bead-on-a-string and velocity versus position.

Discussion of Oliver

Adapting the infinite well toy model to the bead-on-a-string explicitly helped shape Oliver’s sense-making around the bead-on-a-string and the infinite well. Oliver saw a discontinuity between the two situations in terms of the probability of finding the two different entities at their boundaries. This helped focus Oliver’s sense-making to the edge behavior of both situations. Like Quinn, Oliver’s adaptation of the infinite well representation to model the bead-on-a-string leads to an incorrect result, a sinusoidal wavefunction as opposed to a flat wavefunction.

Also like Quinn, attention to new regions in his representations, and new conceptual coordination around those regions, helps Oliver find his way to a correct understanding of the probability distribution of the bead-on-a-string. Getting to this understanding involved the necessary conceptual steps of 1) developing a deeper understanding of the velocity of the bead, through sense-making about particular regions of the bead's movement, 2) sense-making around the differential likelihood of the probability distribution of the infinite well particle.

Chad on the slanted well problem

Finite well: a representational space for reasoning about how potential walls affect the wavefunction through "decay"

I now turn to Chad, specifically the place in the interview where Chad and Erin are discussing the slanted well problem. After reading the problem, Chad begins by noting that the slanted well was a problem on a final exam, although it was not something they had talked about in class. However, in the space of the interview, we do not see Chad reproducing his work from the final exam. Instead what we see is in-the-moment sense-making about the slanted well as Chad assembles a solution from the coordination of two toy model representations.

Chad draws on the finite well representation to explain how the walls of the potential well will have an effect on the wavefunction of the slanted well. In doing so, Chad focuses his sense-making on how potential walls and regions shape the wavefunction of the finite well.

> Chad: We talk about how the potential walls affect it ((traces vertical wall on slanted well representation)). And how it would be uhh... ((re-traces vertical

wall on slanted well representation)) Yeah 'cus we talked, yeah if you talk about, uhh finite regions ((starts drawing finite well representation)), you have the wavefunction in here, it doesn't go to zero here ((starts drawing wavefunction in finite well representation)), it goes to the points that it does, then exponentially decays in it ((finishes drawing wavefunction in finite well representation)).

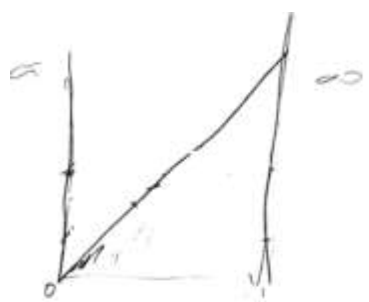

FIG 3.9. Chad’s Slanted well representation: Chad first draws the potential of the slanted well.

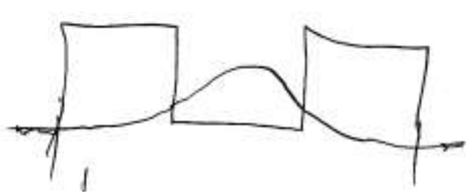

FIG 3.10. Chad’s Finite well representation: Chad’s completed drawing of the finite well representation

Chad begins with the claim that the “potential walls affect it.” However, his explanation stalls after this point, with three clauses showing failed attempts to continue justifying how potential walls affect the wavefunction. With the generation of the finite well representation, he is able to continue. In doing so, he talks about the wavefunction in the finite well and focuses on the behavior in the classically-forbidden region. He describes the behavior as “exponentially decay[ing] in it.” He also describes the values at the boundaries, which are non-zero at the inner boundary (above), and “if they’re tall enough, you get tunneling,” (subsequent conversational turn).

Chad goes on to draw the wavefunction for the slanted well, and in doing so, recruits the conceptual and representational notion of decay from the finite well to describe the right side of the slanted well wavefunction.

> Chad: For this one it would just be, uhh it starts off like it, and then it decays ((draws ground state as he speaks)).

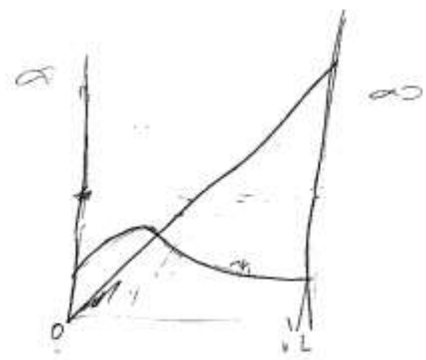

FIG. 3.11. Chad's Slanted well representation: Chad draws the wavefunction in the slanted well potential.

Recruiting the finite well representation helps Chad reason about potential wall's effect on the wavefunction. Sense-making around different regions in the finite well then helps shape Chad's compartmentalized sense-making around the slanted well wavefunction. Chad has essentially partitioned the construction of the wavefunction into two halves, where the halves are being explicitly recruited from other systems. The right half of the wavefunction is taken from the classically-forbidden region of the finite well representation. In Chad's next turn, he makes it clear that the left-half of his construction is recruited from the infinite well representation.

Infinite well: a representational space to further explore how potential walls affect the wavefunction

Chad goes over his construction of the slanted well wavefunction. This time, he draws on the infinite well representation to reason further about how the potential walls affect the wavefunction. Chad's deepening reasoning leads to the emergence of a new conceptual and representational feature in his wavefunction.

> Chad: So it's, you can kind of take it as perturbation upon the particle in the box ((begins to draw infinite well representation)). So it's going to be essentially particle in the box ((draws n=1,2 in infinite well representation)) but then, uhh what's happening is as the potential increases ((draws potential slant)), it reduces the probability of being in that region. Which means that if you still normalize it ((traces n=2 in infinite well representation)), it would have to follow, it would have to follow the same energy, stepping, where it's going by nodes added, but it will reduce the probability of this region ((circles right hand side of n=2 in infinite well representation)), linearly.
>
> Interviewer: Uhh, this region? Is that... ((Interviewer points to right side of n=2 in slanted well representation))
>
> Chad: Yeah, this is the ((shades under slant in slanted well representation))... but, so it will follow essentially it ((traces left half of ground state of infinite well in slanted well representation, leaving small line showing where two wavefunctions deviate)).... I think I made it too big for my waves to look right. But it will go into it generally like that ((points to n=1 in infinite well

representation)). But it will also decay ((traces remainder of ground state in slanted well representation)) after it enters the region.

FIG 3.12. Chad's Infinite well representation above the slanted potential representation: Chad draws diagonal line (slanted potential representation) directly below his infinite well representation. Lettering below the slanted well representation occurred later in the episode and so should be ignored.

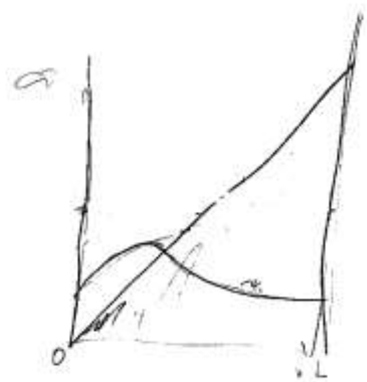

FIG 3.13. Chad's Slanted well representation: Chad has traced over his wavefunction for the slanted well. In tracing over, Chad marks a mathematical/representational turning point; the point where the wavefunction of the particle begins to deviate noticeably from the wavefunction of the infinite well.

Through the introduction of the infinite well, Chad can reason more deeply about the slanted well wavefunction and the relationship between the probability and potential. In the first section, the slanted well wavefunction was treated as the juxtaposition of the left half and the decay portion. With Chad's perceptual and conceptual coordination between the infinite well representation and potential slant representation, Chad relays a more nuanced understanding of the probability and potential relationship; the probability and potential being inversely related. This also reveals a more nuanced understanding of the slanted well wavefunction. As opposed to a simple juxtaposition, his explanation now positions the slanted well as one influence (potential slant) modulating the other (infinite well). This in turn gives rise to a new conceptual and representational feature of interest: a representational turning-point where the modulation from the rising potential causes a noticeable deviation of the slanted well wavefunction from the infinite well wavefunction.

Discussion of Chad

The introduction of the finite well helped focus Chad's sense-making through the coordination of his reasoning on how potential walls and regions affect the wavefunction. To do so, Chad reasoned about different regions in the finite well representation, focusing on how the values of the potential at walls or within regions affected the wavefunction. And so, Chad's adaptation of the finite well led him to sense-make about it in this particular way.

Coordinating his reasoning with the infinite well helped Chad flesh out his slanted well wavefunction model as two competing influences. This led to the emergence of a turning-point marking where the decay influence 'wins' over the infinite well influence.

Conclusions

Seeing continuities across situations

With this paper, we hoped to show that toy models can be tools to learn with, even if the student has not developed a sophisticated understanding of the model a priori. Above, we described in detail how each episode of adaptation was also a learning process or presented rich opportunities for learning. In this section, we will expand on these claims.

Seeing toy models as tools for learning with means first seeing continuity across situations. However, in this data set, continuity was seen across various grain sizes: from seeing the entire toy model representation as applicable to seeing smaller "chunks" of the representation as applicable. After recruiting a toy model representation to reason with, sometimes fleshing out specific differences between the scenario of the iconic representation and the new application scenario can be helpful in the generation of new meaning, as we saw in the case of Oliver. For example, Oliver saw discontinuity between the quantum and classical infinite well situations in the edge behavior of the particles and their associated representational forms. For each student, continuities and differences helped shape and direct the students' sense-making.

First instincts

Across our data, we find students' first, intuitive modeling of situations often seems to come in terms of the toy models. We say it's intuitive because we often don't know why the student is drawing on the toy model, and the comparison comes almost immediately after the student reads the prompt. Attention to new areas in the representations students draw or new areas in the referent physical system, and conceptual coordination around the areas, then helps the students develop their sense-making through the generation of new meaning. Sometimes the new meaning is finding some conceptual and/or representational means for rejecting or refining the initial, intuitive model. The fact that many of these quick responses come from toy models is pretty significant. Yes, we designed for this. But this might be some hint that toy models might be useful for having students develop intuition around the behavior of quantum entities. Particularly through the refinement or development of their initial instincts. For Quinn, the refinement of her initial orientation towards the problem led to a reflection on very fundamental differences of quantum and classical particles.

Sometimes first instincts towards a situation can be hard to disrupt. As we saw with Oliver, it seemed that his intuition was drawing him towards sinusoidal shapes for the velocity and wavefunction. It was through attention to, and conceptual coordination of, new areas in the both the infinite well and the motion of the bead that Oliver was finally able to appropriately refine his initial intuition. Being able to refine a first instinct in favor of a model that makes more physical sense is an aspect of adaptive expertise (Hatano & Inagaki, 1986).

Coherence seeking across situations

My claim in this chapter is that the process of adapting toy model representations can naturally lead to opportunities to deepen one's understanding of the representation being adapted. In our description of Quinn's sense-making around the classical well problem above, we noted how Quinn's episode ended on her reflecting on the ontological difference between the quantum and classical particles. In looking across the data set, we see that reflections on the adapted representation occurs mainly on the classical well problem, suggesting the importance of task structure. Of the four students who adapted the infinite well to reason about the classical well particle, three of the students sought some sort of coherence between the two systems. Two students found an explanation in a high-n limit, suggesting that the classical particle might have a high-frequency wave function, (possibly due to much higher energy), which the students both considered to be well-approximated by a flat line. Oliver is the only student who moved on without reflecting on the two systems. What's interesting to note is that all four of these students began with a wavefunction explicitly adapted from the infinite well but settled on a flat wavefunction description. It seems that adapting the infinite well representation, and then rejecting that adaptation, provided a nice material (for all students but Oliver, as he erased the infinite well ground state) and temporal juxtaposition of the representations for the two systems for the students to then compare.

Implications

Opportunities for adaptation

Something I have argued for in this chapter is seeing these QM toy models and their representations as means for structuring one's seeing and reasoning in new contexts. This is true even for students who have not yet developed a sophisticated understanding of the toy model. In doing so, I want to help push back against the notion of waiting until students acquire some pre-determined, culturally-sanctioned representational and conceptual understanding before allowing students to reason about more complex situations. Understanding experience (in education and otherwise) to be continuous, experiences with these toy models come to contextualize current experiences for these students. Having this type of reasoning become expected is then a matter of designing learning experiences so that students can make more meaningful connections, as well as opportunities to investigate and reflect on those connections. As researchers, I help create situations that afford continuity through interaction with the student and careful task design.

What if adaptation leads to incorrect results?

One concern for instructors may be that adaptation of toy models can lead to incorrect results. Additionally, it's difficult to predict how stable/robust the new meaning generated in these sense-making episodes will be. For example, even though Oliver reasoned about both the infinite well and the bead-on-a-string, and came to reject the infinite well representation for the bead-on-a-string. Unlike Quinn, Oliver did not explicitly reflect on the differences of the two entities.

This concern leads us back to the arguments of Bransford, Schwartz and Sears, (Bransford, Schwartz and Sears; 2005). In their double transfer studies, they are arguing for early invention as a way to better prepare students to learn from common resources. In their 2004 work, they show that students in the invent-a-method condition and the tell-and-practice condition performed (Schwartz & Martin, 2004). However, assessment of the two groups after a common learning resource[11] showed that the invent-a-method group was better positioned to learn from the

[11] The common resource came in the form of a worked example on an exam.

resource. This suggests that early invention of toy models may not reveal immediate, obvious learning gains. However, those opportunities may better allow students to make sense of lectures and homework problems later on. And so, I implore instructors to allow students opportunities to go past ‘fundamentals’, where they are able to creatively leverage what they’ve learned to make sense of systems that are of real interest to the,

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Chapter 4: Designing Disruptions to Wavefunction Infrastructure for Flexible Representation Use

Introduction

Developing proficiency with representations such as graphs, equations, schematics, diagrams, and free-body diagrams, is central to learning and problem-solving in physics (Kohl, Rosengrant, & Finkelstein, 2007; Ainsworth, 1999; Heller, Hollabaugh, 1992; Fredlund, Airey, & Linder, 2012). However, representation reproduction and use is also often challenging for students (McDermott, 1987; McKagan & Wieman, 2006). My goal is not to solve these difficult instructional issues, but rather, to problematize what counts as proficiency in using graphical representations. Typically, research on instructional interventions deems students proficient with a representation if students can demonstrate the skills of correctly generating the representation and extracting information (reading out) from the representation. Recent research, however, has started to build a more nuanced story of students' use of representations during sense-making, going beyond a "skills development" narrative. For example, Heckler (2010) shows that requiring students to draw diagrams can lead to a conceptual disconnect between the representation and the student's sense-making. Students who construct the diagram unprompted tend to do more effective at integrating the diagram with conceptual reasoning. Others, for example, Gire and Price (2015) and Parnafes (2007), have shown that different physical features within representations may serve different roles in student reasoning. Gire and Price suggest that different algebraic forms may provide different types of support for computation. Similarly, Parnafes shows that attention to particular figural features of a situation or representation is important for coordinating conceptual change. The picture that emerges from these studies is that representation use by physics learners is contextual and tied closely to their conceptual reasoning.

In this chapter, I extend the thread of research on the dynamics of students' sense-making and inscription use. I draw on Greeno & Hall's (1997) framework for describing representational practices students should engage in, which I term flexible representation use for brevity. Drawing on existing literature helps direct my research into fine-grained characterization flexible representation use in the context of introductory quantum mechanics (e.g. Greeno & Hall, 1997; Lehrer, Schauble, Carpenter, & Penner, 2000.) Taken together, flexible representation has the following characteristics: 1) representations play an integral role in communication and problem-solving, 2) construction, invention, or adaptation of representations to serve local needs, 3) representations can co-evolve, or evolve reflexively, with evolving conceptual understanding, and 4) the coordination of different representations.

To study flexible representation use, I draw on the notion of designing disruptions to representational infrastructure to help guide the design of interview protocols (Hall, Stevens, & Torralba, 2002). In those protocols, I "tweaked" the traditional particle in a box scenario in introductory quantum physics to see how students would use representations when sense-making in similar situations (McKagan & Wieman, 2006). I do a microanalysis (Siegler & Crowley, 1991) of two episodes from my interview with a student, pseudonym Chad. Chad, I argue, creates a rich system of representations to serve various coordinating, communicating, and reasoning purposes. Chad generates and uses the representations reflexively, in the sense that his conceptual thinking and his representations change in ways that mutually affect each other. My goal with this work is to begin to describe and characterize flexible representation use of inscriptions, which in this case, challenges traditional notions of proficiency in using representations. These preliminary results will inform future work on analyzing and supporting student sense-making, as well as instructional design of representation tasks.

Greeno and Hall's invocation to teachers

Greeno and Hall (1997) encourage instructors to provide students with a rich variety of representational experiences because through participation, students learn the predominant ways of knowing and learning of that setting (Greeno & Hall, 1997). A broader variety of experiences can lead to a broader understanding of representation use by students. Looking across studies of professionals and students working with representations provides a sense of the breadth of experiences or representational practices that people and communities can engage in.

Unfortunately, some classrooms fail to provide students with such a rich variety of experiences and treat representation use very rigidly. Representations are treated as end-goal in themselves. Here, students may be asked to reproduce or read-out from standard forms introduced in instruction. Even though these practices may have good intentions behind them, they position representation-use as mainly a performance to be evaluated on.

In other cases, instruction can provide opportunities for students to engage in flexible representation use, where students participate in a variety of practices of representation, including opportunities to invent, construct, interpret, and discussions of conventions of use. Participation in a wider range of practices positions representations as integral parts of sense-making and communication. Often a nonstandard form can better serve the needs that emerge during communication and sensemaking. This is because different representations may provide different supports for reasoning (Greeno & Hall, 1997; Ainsworth, 1999; Gire & Price 2015).

In this realm of flexible representation use, inquiry with representations can take on reflexive form:

> "Solving a problem involves an interactive process in which students construct representations based on partial understanding and then can use the representations to improve their understanding, which leads to more refined representations, and so on," pg.365, Greeno & Hall, 1997.

What Greeno and Hall describe is a form of inquiry that is clearly non-linear. As opposed to a process in which increasingly sophisticated ideas come to be inscribed, the process of inscription is a driving mechanism in generating these increasingly sophisticated ideas.

With the rest of this chapter, I use this reflexive notion of inquiry as an orienting characteristic of flexible representation use. It is a characteristic of student sense-making that instructors may value because it supports the generation of a deeper understanding of a situation. And so, in the following sections, I highlight work others have done showing varying degrees of reflexiveness in student sense-making with graphical or pictorial representations. In doing so, I try to highlight what might be supporting or inhibiting this type of reflexive, connected sense-making.

Opportunities for Flexible Representation Use are needed in physics

In this section, I further motivate the need to study and create opportunities for flexible use of representations. To do so, I make the following points:

1. Reflexiveness is a means pattern of sense-making that supports the development of new meaning
2. More rigid use of diagrams and representations may lead to a disconnection between representational and conceptual sensemaking

Current notions of reflexivity

Others have noted that conceptual and representational development often takes on a reflexive characteristic. Kirsh (2010) provides a nice description of this interactive process:

> “By ‘interactive’ I mean a back and forth process: a person alters the outside word, the changed world alters the person, and the dynamic continues,” pg. 440.

Kirsh argues that by structuring the world in which we think, we can change the “cognitive cost” required by my thinking processes. Such structuring makes the thinking processes we perform simpler. Additionally, structuring the world through external representations can also change what types of thinking processes are possible. For example, the range of actions a thinker can perform on a representation increases when the representation is externalized. The range of actions possible can be extended further given that different representational forms can encode information differently. And so, a thinker can “try out different *internal and external representational forms*, the two forms can play off each other in an interactive manner, leading to new insights,” (Kirsh, 2010).

A few others (Lehrer, Schauble, Carpenter, & Penner, 2000; Nemirovsky, 1994) have studied the reflexive and co-development of representations and conceptual understanding. Both works point to the importance of attending to new physical and representational features as important to, and emergent from, developing representational and conceptual understanding.

Nemirovsky (1994) shows the joint conceptual and representational development of the velocity sign by a high school student. In the study, the student is developing ideas about the velocity sign through interaction with a computer program that allows her to move a cart on a track to plot distance and velocity of the cart. At first, the student does not understand how to generate a negative velocity plot. Attending to new physical and representational features, such as the starting and stopping points, became crucial in her developing stories about the car's negative-velocity motion and her development of an understanding of how to generate such representations (Nemirovsky, 1994).

Lehrer, Schauble, Carpenter, & Penner show the reflexive development of conceptual and representational understanding in a third-grade science classroom studying what aspects affect the growth of fast-growing plants. To do so, the students developed a cascade of inscriptions (Latour, 1990) that helped shape their conceptual growth. As their representations took on more dimensions, the students were able to ask and answer different questions about the growth. For example, moving from asking questions about how the height of the plant changes over time with one-dimensional representations to asking questions about how the relationship between height and width changes over time after developing two-dimensional representations.

With this chapter, I hope to add to this small line of work showing co-development of representational and conceptual understanding. In particular, my work provides a complementary perspective by looking for reflexiveness on a finer-grained scale. The development that takes place in the two works cited is much longer than the span of a few minutes, which may be typical of a tutorial or homework problem, where students are may be tasked with generating a single representation. I begin by reflecting on work that, unlike that described above, shows a deep *disconnect* between conceptual and representational sense-making.

Rigid Use of Representations can lead to disconnected Sensemaking

It's common practice in physics to ask or require students to draw a diagram or representation when problem-solving. Many prescriptive problem-solving methods start with having student draw a picture or representation (Heller, Keith, Anderson, 1992; Reif & Heller, 1982; Schoenfeld, 1985). These approaches are based on a progressive translation from the problem situation to increasingly more mathematical descriptions (Heller, Keith, Anderson, 1992). The first prompt often asks students to draw a diagram or representation of the problem situation. However, some researchers have found that prompting for representations may lead to sensemaking that is disconnected from representation use (Heckler, 2010; Lehrer, Schauble, Carpenter, Penner, 2000; Kuo, Hallinen, & Conlin, 2017). Prompting a representation for assessment may fall under Greeno and Hall's notion of "rigid representation use."

Heckler studied a large group of students solving fairly standard introductory mechanics problems. Heckler gave two groups of students identical problems on identifying and modeling forces. One group was prompted to draw diagrams and the other group was not. Analysis of the two groups' problem-solving found that students who were not prompted to draw diagrams were more likely to generate correct solutions. Of the students in the two groups who drew diagrams, the prompted group was more likely to generate solutions that were disconnected from their mathematical modeling. Additionally, the students who were not prompted to draw diagrams tended to use more intuitive, less formal problem solving approaches. Heckler cites two possible explanations: 1) novice students may be more effective at using more informal, intuitive methods, 2) the act of prompting the diagram may cue epistemological resources that treat drawing and mathematical modeling as separate tasks (Hammer & Elby, 2003). In either case, prompting for the representation more often led to disconnected representational and conceptual sense-making.

Two cases studies by Lehrer, Schauble, Carpenter, and Penner (2000) also study the (dis)connectedness between drawing representations and conceptual or mathematical sensemaking. However, they draw slightly different conclusions explaining the source of this disconnectedness. In the two cases, elementary-aged children worked on a unit exploring physical features that affect the speed at which an object rolls down an inclined plane. In their inquiry, the students came upon the problem of ensuring that all ramps were equally steep. The students had identified three physical features of inclined planes that defined steepness: height, length, and "pushed-outness." One group was asked to invent a drawing that captured all three features. The other group was given a representational form (triangle) and asked to reason about the same issue (how the three features correspond to steepness). While the first group was successful in the task, the latter group's representations were lacking in their ability to show how the three physical features affected the steepness of the ramp. Lehrer et al. suggest that this group's sketches were "less representational, because we inadvertently began the discussion by providing the children with a solution to a problem that they had not yet accepted as problematic," (Lehrer, Schauble, Carpenter, and Penner, 2000).

In comparing to Heckler's work, two important differences arise. First, Lehrer et al. show the importance of giving students more choice in how they represent. For example, the first group chose something that was only partially abstracted because it included a crate propping up the ramp. However, this inclusion allowed the manipulation of the height through changing the number of crates. From this, the students could show how the height affected the steepness of the ramp. It also suggests that there is some issue of timing in their sensemaking. The representation should arise at the time and in a way to address something problematic in sensemaking. It seems naive to assume that this need arises at the same time for all students (i.e. in step 1 of a problem solving algorithm) or that it should be solved in the same way for each student, such as with a formal free-body diagram (Heckler, 2010). Often, nonstandard representations may serve local needs better than standard representations (Hall, 1996).

How do we approach creating opportunities for flexible representation use? Designing Disruptions to Representational Infrastructure

Based on personal experiences with curricular materials and textbooks (Griffiths, 2016; Serway, Moses, & Moyer, 2004; Liboff, 2003; Singh, 2008; Zhu &

Singh, 2012, Tutorials in Physics[12]: Quantum Mechanics, QuILTs[13], Intuitive Quantum Physics[14]), most standard instructional materials require the use of representations but don't explicitly scaffold or focus on flexibility. This is problematic given that research has shown that routine prompting of representations can lead to disconnected sense-making (Heckler, 2010; Lehrer, Schauble, Carpenter, & Penner, 2000; Kuo, Hallinen, & Conlin, 2017). Recent work on modeling instruction (McPadden & Brewe, 2017) focuses on having students use a variety of representations. However, none of these works focus on whether and how students use representations flexibly.

In this section, we explore work from socio-cultural studies in mathematics education to look for some answers to the problem of designing for flexible representation use. In doing so, we encourage instructors to take a broad view of the representations they ask students to produce in order to understand what opportunities are available to students. Then we turn to literature on 'disrupting' representational practices as a means of thinking about how to disrupt more routine representational practices to create opportunities for students to use representations more flexibly.

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[12] https://depts.washington.edu/uwpeg/tutorials-QM
[13] https://www.physport.org/examples/quilts/index.cfm
[14] http://umaine.edu/per/projects/iqp/

What is Representational Infrastructure?

In instruction, students may be asked to generate, sense-make around, or communicate about different types of representation (i.e. wavefunctions in quantum mechanics). Instructional histories, routines, and norms of practice around those types of representations then support that generation, communication, or sense-making by the students. This collection of histories, routines, and norms of practice are considered a representational (Hall, Stevens, & Torralba, 2002) or informational infrastructure (Star & Ruhleder, 1996).

Stable representational infrastructures have a certain scope in that they can, and should, be used beyond a single instance. In doing so, the infrastructure should support activity that follows the interests and norms of the community. The infrastructure should support performing heavily standardized tasks efficiently, but also have enough flexibility to be used in more customizable ways. For example, representational infrastructure used for supporting the generation of wavefunctions should be able to support students in quickly generating known wavefunctions for familiar systems, but also in reasoning about the wavefunctions of unknown systems.

Use of a representational infrastructure can often rely on a high degree of intersubjectivity between participants (Hutchins & Klausen, 2000). Those deeply embedded within the community of practice may take this intersubjectivity for granted. This can lead to situations with apparent intersubjectivity is reached, but without actual deep understanding. Differences in understanding may simply go unnoticed or unmarked. This raises the risk of misunderstanding of student actions by instructors. It may also mean that aspects of culturally sanctioned representations and their infrastructures are not readily apparent to students.

From this work on representational infrastructure emerged a line of inquiry of designing learning environments to disrupt more routine representational practices towards enabling students to use representations in new, flexible ways. The collection of work I review below suggests that designing disruptions to representational infrastructure might guide instructors in creating opportunities for flexible representation use.

Disruptions to Representational Infrastructure

Disruptions to representational infrastructure are bids for rejection, replacement, or challenges (Hall, Stevens, Torralba, 2002) that require participants to reorganize work practices to develop a new, or restructure existing, representational infrastructure. With this section, I aim to make two main points:

1. Disruptions often lead to adaptation and invention, requiring some cognitive and interactional work by participants to reorganize their work. The context of the representational infrastructure and the specifics of the disruption help shape the consequences of the disruption. The space may allow little innovation and creativity or let innovation go unbounded, sometimes at the expense of developing a stable infrastructure.

2. Learning opportunities and access to representational practices are tied to the representational infrastructure. Disrupting the representational infrastructure can shift or redistribute opportunities for learning or maintain marginalized access to the infrastructure.

Disruptions to Representational Infrastructure can serve to rebuke or better align with community standards and norms

In looking at two case studies, Hall, Stevens, & Torralba (2002) focus on how talk across disciplines helps shape disruptions to representational infrastructure and how the participants then develop new representational infrastructures. The context of the work being done in the two case studies puts some constraint or freedom on possible courses of action. The two case studies examine an entomology group and an architectural group.

In the "Bughouse," they developed a routine of practices for gathering, collecting, analyzing and representing data to make claims about the chemical profiles of different termite species and colonies. The problem is that, while the group is quite adept at looking across various representations to make these claims, the process too lengthy in publication. And so the group invites a statistician to help disrupt their routine, but also opaque, representational practices and develop ones that are easier to present in publications. Disciplinary differences between the entomologists and statisticians helps the BugHouse move away from using the awkward measures familiar to the entomologists and towards a more compact, computational method.

The architecture group is tasked with remodeling a library that has been coded as potentially unsafe through a representational infrastructure generated by the city. Within the unsafe code, there are multiple paths forward in retrofitting the library so that the building may be recoded as “safe.” The architecture group spends a lot time of time discussing various pros and cons of different retrofitting options. All options work to shift how the representational infrastructure would code the building. Instead of working to fit the building to a new code, the group’s historian suggests a total rejection of the coding scheme altogether. His suggestion is the representational infrastructure is based on “arbitrary margins of safety,” and that a more realistic infrastructure for coding buildings would classify the library as safe.

In comparing across the disruptions in the two groups, work within the given contexts places different constraints on how the representational can be repurposed or replaced. In both cases, the representational infrastructure embodies community standards and norms. In the Bughouse group, the entomologists cannot reject the need for concise, presentable data required of work in their professional community. Not adhering to or working towards this disciplinary expectation would jeopardize their very existence as members of that community. And so the representational infrastructure for classifying termites is adapted to better serve those standards and norms. In contrast, the architecture group is able to argue against community standards and norms. This is, in part, based on the historian's experience seeing similar "battles" fought in nearby cities.

Context of the representational infrastructure shapes opportunities for adaptation and learning

In this section, I review other disruptions literature to discuss how learning opportunities are tied to the representational infrastructure in routine and innovative uses. Disrupting the representational infrastructure to develop more innovative representational practice may help shift and redistribute learning opportunities. The context of the disruption plays a strong role in determining how this redistribution happens, if at all. In particular, a hierarchy of roles and differential access to aspects of practice effect opportunities for both learning and taking agency towards the innovation of the representational infrastructure.

Hutchins (1995) describes in great detail power loss in a Navy helicopter-carrier coming into port. The power loss disrupts both the ability to slow the ship down immediately and various technologies in the representational infrastructure for coordinating the ship's current position and in mapping what direction it is heading in. The situation is dire. The team of navigators must quickly repurpose aspects of the infrastructure (tools, technologies, forms of mutual monitoring) in order to find a place for the ship to drop an emergency anchor. Loss of military and civilian life will be unavoidable if the crew is unable to adapt the infrastructure to accomplish this end. Three main changes in work characterize the crew's adaptation to this disruption:

1. Routine practices for finding the ship's orientation are 'stretched' in the absence of quick feedback from downed technologies. The rudder angle is turned more sharply than it normally would be in the case where the rudder has electric power. With electric power, the rudder turns more quickly, providing faster feedback to the navigators.
2. Practiced back-up mechanisms for accomplishing the task of navigation are put in place. Instead of electric power turning the ship's rudder, two crew members turn large cranks to manually manipulate the rudder. There is no time to find inventive solutions.
3. More experienced navigators take control of various posts. Under routine practice, less experienced crew members usually take these positions, with the experienced crew members providing feedback to help newer members learn.

And so, the life-threatening context of the distribution leads the crew members to rely on well-established hierarchies of experience, backup technologies, and adaptation of existing practices to safely anchor the ship. In doing so, they essentially cut-off access to aspects of the navigational practice of various less-experienced crew members. Learning is of no concern when lives are on the line.

Like Hutchins, the case study on the Bughouse group by Hall, Lehrer, Lucas, and Schauble (2004) shows how a hierarchy of roles tied to the representational infrastructure shapes access to aspects of practice, both in routine and innovative uses of the infrastructure. As described above, the representational infrastructure used by the group coordinates various work practices and methods for analyzing and representing data to describe the behavior of (or classify) various termite species and colonies. Under routine use of the infrastructure, juniors coordinate field work while the seniors are in charge of the research more broadly. The need to disrupt and innovate the existing representational infrastructure arises from the need for continued funding. Even in the process of innovation, the differential access in determining the direction of the research is maintained. The seniors take charge in the innovation and in doing so, intentionally create learning opportunities for the juniors to understand the physical meaning behind the innovation. The seniors conceive of a way to adapt existing work practices to make claims about when certain of species of termites forage. The seniors help the juniors understand how these adaptations to their work practices will help answer this question.

Like the example described by Hutchins, the group's 'livelihood' is at stake, but to a much lesser degree. The Bughouse group's new line of inquiry needs to be disciplined in the sense that they must, in a somewhat timely manner, devise of an infrastructure that allows them to reliably address their new, fundable line of inquiry. This is maybe, in part, why the seniors take the lead. However, the situation is not so dire that loss of life is imminent if a working infrastructure is not immediately adapted. And so, seniors can take the time to make sure that the juniors are learning through the innovation.

Unlike the above two examples, instructional design can help engineer more distributed access to opportunities for learning and innovation (Ma, 2016; Hall, Lehrer, Lucas, & Schauble, 2004). Instructional environments that require or support innovative uses of infrastructure can shift the processes and content of student learning. However, overabundance of opportunities for innovation can come at the expense of opportunities for learning.

Ma worked with high school instructors to design Walking Scale Geometry; a spatial disruption to typical geometry classrooms. Instead of constructing geometrical shapes on a sheet of paper, students must invent tools and strategies for constructing shapes that are the size of a classroom. The scale of the disrupted work requires careful coordination between students and so learning becomes a joint activity. The group of students collectively has agency towards developing new, innovative means for solving previously mundane, individual problems. As the processes of learning shift through development of these strategies, the content of what students learn also shifts (Ma, 2016). For example, Walking Scale Geometry provides a particular experiential context for developing an understanding of adjacent-side relationships and congruency because students are able to see and manipulate these constructs through relative orientation with other students.

In Ma's classroom, the need or desire to innovate representational practices was not student-generated. The need was designed into the curriculum so that the students would be forced to contend with it. In contrast, a second case study Hall et al. (2004) documents a classroom where students have a different level of agency towards innovation. The case study examines a 6th grade classroom seeking to develop routine and innovative uses of a representational infrastructure of a system of "pond jars," used to answer questions about local pond ecology. The representational infrastructure includes the "pond jars" themselves, processes of making measurements, norms of representing finding to peers, etc.

The instructional set-up gave small groups of students agency over their own inquiries. Every group was able to innovate with their own jars, or even repurpose the entire class's collection of jars to identify broader patterns during class-wide "research meetings." In some cases, unbounded innovation with the representational infrastructure prevented disciplined inquiry. Students were unable to develop stable enough infrastructures to answer their inquiry questions. In other cases, interaction with the rest of the class and the instructional team helped students develop coherent, non-confounding infrastructures to answer their inquiry questions.

Takeaways from literature on Disruptions to Representational Infrastructure

1. Disruptions can lead to innovation and invention
2. The context of the disruption and differential access to aspects of practice can effect opportunities for learning
3. Careful instructional design and interaction in instructional spaces can help support innovation, invention, and disciplined inquiry

Designing for disruptions to representational infrastructure for generating Wavefunctions

Disruptions to encourage flexibility in wavefunction representational infrastructure

Wavefunctions are important in studying quantum systems because wavefunctions contain all knowable information about the system. In this work, my focus is the on the representational infrastructure associated with developing wavefunctions for elementary particles. I am not interested in disruptions that completely replace or reject representational infrastructure. Instead, I am interested in disruptions that encourage more of a 'stretching' of the representational infrastructure to include new contexts.

In the rest of this chapter, I will briefly describe the design of 'disruptive' interview tasks. After discussing analytical methods for studying student thinking on these tasks, I'll recount two episodes with Chad. At the time of these interviews, Chad was a senior physics major. Following these episodes with Chad, I will highlight the characteristics and flexible representation use in the data. Though I do mention instructional implications in this chapter, more can be found in the concluding chapter of this dissertation (Chapter 5).

Data Context

My data were selected from a collection of 14 hour-long interviews with a mix of physics and engineering students. Most students were of junior or senior standing. In the interviews, the students were given tutorial-style problem sets and were asked to think aloud with the interviewer as they went through the problem sets. If there were moments of silence interviewer would check in. Questions directed to students were meant to make the students' thinking more explicit.

The interviews were taken in two sets, with similar interview protocols for each set. On the first set of interviews, the protocol included a problem on an infinite well with a slanted problem, which I will refer to as the “slanted well” problem (described in more detail below) (McKagan et al., 2010, Cataloglu & Robinett, 2002). When viewing these interviews, I noticed a relatively consistent pattern of students finding ways to adapt the representation of the infinite square well to reason about the shape of the slanted well wavefunction. I then designed an interview protocol explicitly around creating opportunities for this type of sense-making. In the design of the second interview protocol, students first encountered problems on the particle in a box; these included drawing energy eigenstates, describing the energies, and thinking about the speed of the particle. The next problem is on a classical analog of the particle in a box, the “classical well” problem (described in detail below). Following problems on the protocol include explaining quantum systems to peers, the slanted well problem, and several problems on tunneling. Evidence of students adapting representations occurred on the classical and slanted well problems. However, not all students were able to get to these questions in the protocol. Across both interview sets, I had a total of 17 clips or instances of students reasoning about the slanted and classical well problems. The interview protocols can be found in the appendices.

Classical well problem

Problem statement: Suppose you had a classical particle in a physical situation analogous to the quantum particle in the box. Consider a bead on a string, and the string is knotted at x=0 and x=L so that the bead is confined between 0 to L, and can move smoothly and freely between these bounds. The bead has some energy E, and can bounce elastically at the knotted ends. Sketch the wavefunction of this classical particle.

Opportunities for flexible representation use: Traditionally, wavefunctions are used to describe the quantum state of a system. The wavefunction can be used to describe possible outcomes of different types of measurements, such a position of the particle or energy of the particle. In this problem, students are asked to apply this quantum formalism to a fundamentally different type of system, a bead-on-a-string. Because the physical behavior are so different between quantum and classical entities, students will likely have to adapt their quantum practices to this classical context.

Solution: A flat wavefunction showing equal probability at every location.

Slanted well problem

Problem statement: Consider a quantum system with $V(x) = \infty$ for $x=(-\infty, 0)$ and (L, ∞), and $V(x)= Ax$ for $x=(0, L)$. Sketch the wavefunction for the first allowed state, or ground state, of the particle.

Opportunities for flexible representation use: I anticipated this problem may encourage students to adapt because finding the wavefunction directly from the Schrodinger Equation is not easy. Doing so would require students to recognize the type of differential equation emerging from the Schrodinger equation, which I think is not widely familiar to undergraduate students. Also, the slanted well problem is not a heavily routinized example (like the infinite well). For these reasons, I suspected that the slanted well problem would encourage to adapt what they've learned.

Solution: The canonical solution for the ground state is a first-order Airy function.

Analytical Flow

Identifying relevant cases

I aim to model the reflexive development of conceptual and representational meaning in student sense-making. My orienting research questions are then:

- How does the student's sensemaking shape the inscriptional space?
- How does the inscriptional space shape the student's sensemaking?

And so, my first pass at the data was to find relevant cases to further study flexible representation use, using reflexiveness a broad-scale structure to look for. In doing so, I looked for episodes of sense-making where there appeared to be several turns of back-and-forth between conceptual reasoning and inscriptional development. I settled on two episodes with Chad because of his talkativeness in these episodes. My hope is that with these two episodes, I can begin to highlight key aspects of flexible representation use.

Analytical Methods[15]

Orientation: Looking for coordination among a system composed of people and media

I tend direct my analytical focus 'from the outside, in'. I take a systems-level perspective first, then model how different aspects of the systems are coordinated to generate the coherences exhibited by the system. i.e. What shared meaning is being generated? How does interaction between the participants and their drawings/representations help generate that meaning?

I take a systems-level perspective as primary because I see the student's social and material environment as playing a strong role in supporting student thinking. So much so that the cognitive work required of the student can be significantly different than the "cognitive work" accomplished by the entire system. An external representation is not only a form of external memory, it can be a computational medium (Hutchins 1995, Kirsh 2010). Take the example of the naval nomograph (fig. 4.1):

[15] The analytical methods are adapted from the previous chapter: Learning with and about toy models in QM

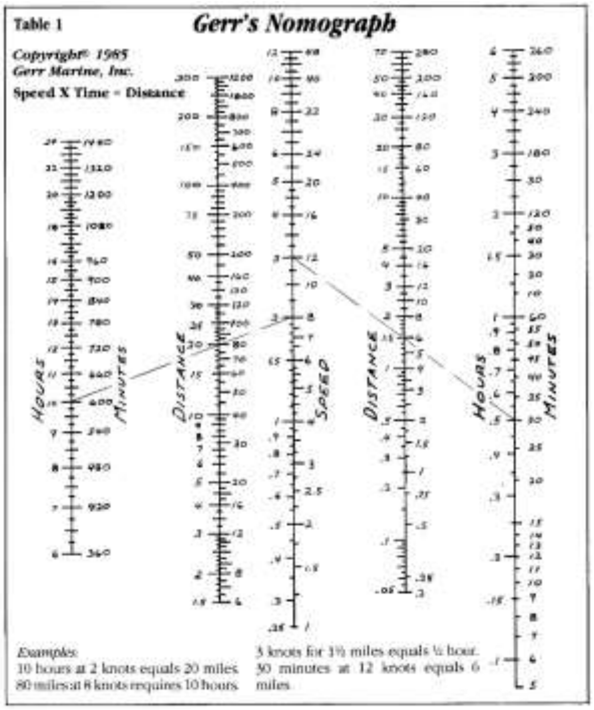

FIG. 4.1 Naval nomograph

The nomograph consists of logarithmic line numbers for coordinating any two of a ship's speed, distance traveled over an amount of time, and the amount of time to find the third quantity. For example, a navigator may take a straight-edge to the nomograph, lining up the edge with a known travel time and speed to find the distance the ship has traveled in that time. At a systems-level perspective, some "cognitive" work has been done to accomplish this computation of speed. However, the cognitive work that the navigator does is not computational, but more so focused on the manipulation of the tool and perceptual pattern matching (Hutchins & Klausen, 2000; Rumelhart, Smolensky, McClelland, & Hinton, 1986).

And so I start with a systems-level perspective to understand how integral parts of the system -- people and drawings they create -- are working together in the process of meaning-making. I only provide conjectures about what's going on in heads of participants when obvious mental transformation is functional towards the meaning being made. This avoids the issue of overattributing cognitive process to reasoning (Hutchins, 1995), either due to failing to confront the fact that cognition is embedded in socio-material settings or that our own cultural embeddedness can affect the meaning we (as instructors or researchers) attach to students' reasoning (Star and Ruhleder, 1996). The most appropriate unit of analysis is then studying talk-in-interaction[16] to understand what shared meaning the participants are developing through coordination with each other and material structure such as drawings (Hutchins, 1995).

The analytical question becomes: how do I study talk-in-interaction to model meaning being generated?

1) look to the contents of interaction and speech to determine conceptual or representational constructs

2) look at the structure and organization of the conversation provides a means for understanding the *functional* meaning of those conceptual constructs

[16] This is contrast to taking internal cognitive processes as a unit of analysis.

Conversations can be organized and structured at various grain-sizes. For example, single words or sounds may help meaningfully connect different clauses in a single utterance. Or, turns in a conversation may be oriented towards the broader conversational goal. Below, I provide a list of conversational structures/means for organization that I attend to in my analysis.

- [17]Progression; on a large scale, there is a sense that the conversation should move towards accomplishing the mutually determined purpose
- Turn-taking; individual turns at conversation
- Repair; attempts to alleviate conversational trouble or breakdowns in mutual understanding
- Turn construction; conversational turns are structurally comprised of turn-construction units, which may be single words, clauses, questions, etc.
- Adjacency pairs (Sidnell, 2010; Sacks and Schegloff, 1973); distributed conversational sequence of two utterances, where the first-pair part mutually constrains second-pair part
- Discourse markers (Schiffrin, 1988; Bolden, 2009); words such as “so”, “oh”, “well”, “okay”

Example Analysis

[17] This list is adapted from the previous chapters.

Here, I provide a short snippet of data to show how I analyze the content and structure of the interaction to understand meaning being generated. In the clip, "Paul" is reasoning about the slanted well problem. The left column contains the transcript, the middle column contains the inscription he is drawing or referencing in the associated transcript. The third column highlights things I am noticing in the data. In particular, I highlight; 1) how material structure becomes attached with conceptual meaning and 2) the *reflexive* generation of meaning.

A more detailed example of analysis can be found in Chapter 3.

Transcript[18]	**Inscriptions**	**Noticing**
Paul: Ok. So it would be more likely to be found over here ((gestures to left side)), so that means you would get something kind of like....		Gesture to part of the inscription 'attached' conceptual meaning to the left side. "So that means" shows some causal connection between the area of least likelihood and the wavefunction. Context

[18] Transcript conventions can be found in the appendices.

Something kinda like... tha:t? ((draws wavefunction))		leads us to infer what he is drawing is the wavefunction.
Interviewer: So, can you explain the different parts that you've drawn?		Interviewer positions the wavefunction Paul has drawn has having different "parts."
Paul: So, it has to be zero on the edge ((points to left edge)). And then, I guess, and then it would have to asymptotically approach zero over here because the potential gets higher. Right here ((points to peak)), it's kind of we:ird.		Paul takes up the piece-wise treatment of the wavefunction in his explanation, explaining three different parts. New conceptual and representational meaning is emerging in his explanation; asymptotic behavior on the right side is due to the raising potential. Paul highlights the peak as "weird," and will go on to sense-make about it in the rest of the episode (not shown). The representation he has drawn is shaping his conceptual sense-making in that it necessarily contains a feature that he does not understand. And so

		the representation has introduced a new feature that he must sense-make about.

Table 4.1 Example analysis of episode in interview with Paul

Now, I move to discuss my focal episodes from the interview with Chad. In particular, I aim to demonstrate the reflexiveness in Chad's sense-making on the two different prompts. In showing the ways in which Chad's sense-making shapes, and is shaped by, his representation(s) I highlight important features/processes in his sense-making that support reflexiveness in his sense-making.

The first episode analyzes Chad's sense-making on the classical well problem. The second episode analyzes Chad's sense-making on the slanted well problem. Each episode is split into several subsection. The subsections try to show at least one feedback loop in reflexiveness. Each loop shows how Chad's understanding becomes inscribed and how that inscription, or process of inscribing, comes to play an integral in Chad uncovering new meaning about the system.

Chad on the Classical Well Problem

Chad's modeling of the bead-on-a-string focuses on the knotted ends

After reading the prompt, Chad moves from sense-making about the knots constraining the bead to reasoning about what these constraints physically mean for the bead's wavefunction.

Chad: Let's start drawing.

((Draws shading in probability representation, box lines, string))

So this is L.

((labels L in probability representation))

We have, knots on the, edges, right?

((traces prompt))

Interviewer: Mhm.

Chad: Does it say that? The thing is knotted, at, so yeah, right.

((traces lines of prompt and draws knots))

So it can bounce at the knots, and it has some energy E. Bounce elastically, so it keeps all of its energy. So that means.... it's bouncing, ela:stically, but yeah it has to be accelerated, so it does slow down still. Um, yeah. So, that means, the wavefunction is the thing that squared would be the probability.... It has to look like this.

((draws wavefunction above axis in probability representation)).

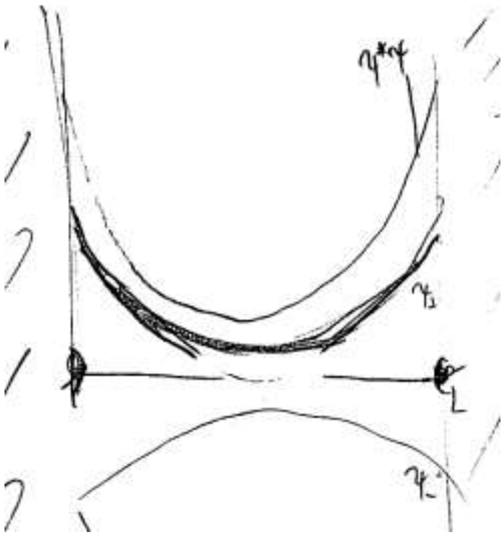

FIG. 4.2 Chad’s Probability Representation

Chad moves from referencing the problem set-up to sensemaking about what the setup means physically for the bead. In doing, Chad focused on marking the knots at the edges and what the bounce means physically. He reasons that the elastic bounce means that the bead keeps all of its energy. The bead also “has to be accelerated, so it does slow down still.” For Chad, this implies that the wavefunction resembles what is shown in Fig. 4.2.

It’s not yet clear how the bead’s acceleration and speed imply a wavefunction or a probability density for Chad. Or how a constant energy may help shape the wavefunction. In any case, it’s apparent from Chad’s drawing and reasoning that the knots on the string are so far an important focus in his sensemaking. It may also seem strange that Chad clearly marks the knots on a plot for the wavefunction. However, this is reminiscent of the cultural norm of plotting a quantum particle's potential in same space as its wavefunction. Work in that type of blended space may allow Chad to visualize how the ways in which the bead is confined or constrained gives rise to physical/wavefunction behavior.

Construction of the Position Representation of the Bead-on-a-string initiates a change in the Wavefunction Representation

Following his drawing of the wavefunction, the interviewer asks for an explanation of where the wavefunction came from. This sets of a new chain of reasoning. Chad introduces a representation of the bead's position over time to explain his wavefunction to the interviewer. In doing so, he comes to realize that his original wavefunction is not quite right. New representational (and associated conceptual meaning) are then generated through explaining his position representation.

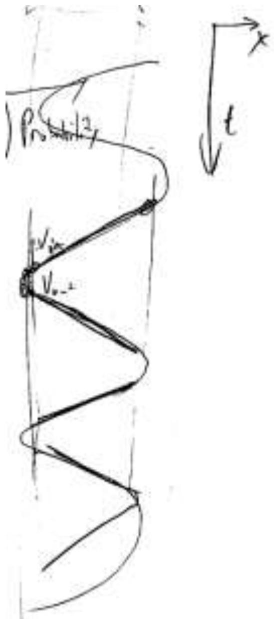

FIG. 4.3 Chad’s Position Representation

Interviewer: So, can you explain where that comes from?

Chad: Well, um, so the wavefunction, always has to be such that psi star psi is the probability distribution.

((writes on paper psi star psi equal box probability))

Interviewer: Ok.

Chad: Right?

Interviewer: Mhm.

Chad: So, if you start with the probability distribution, that, it's going to be spending the majority of its time at the edges

((gestures to edges in probability representation)) because, uh, it has to be accelerated at the edges.

So, it's essentially doing….. thi:s

((draws position representation))

if this is time, and this is x

((labels x and t above position representation)).

It's doing that. Which means that the majority of it is here,

((draws partitions in two position representation by drawing two loops encompassing the edges)),

the minority of it is here.

((gestures to center of position representation))

So... it's not really, doesn't go to zero there

((erases center of the wavefunction))

Higher

((redraws center of wavefunction))

Chad: Alright, um. So if you have that, actually, technically, it could be also be this

((Draws lower wavefunction in probability representation))

But it could never cross, because then it would go to zero.

((gestures over center of wavefunction))

So yeah. It has to be either this or this, ((points to wavefunctions in probability representation on “this” and “this”))

because psi star psi would be the probability distribution

((points to psi star psi equals probability))

which has to look like this.

((points to upper wavefunction in probability representation))

So psi is plus or minus the square root of the probability

((write psi equals +- probability))

Interviewer: Ummm.

Chad: Almost.

((continues writing))

Interviewer: Oh, I see. Ok. So you're saying the probability looks, like, uh

((E points to upper psi in probability representation))

Chad: ((Draws thin line of probability above wavefunction in probability representation))

Interviewer: The probability...

((points to line Chad just drew))

ok. And then the square root, is like, either that one or that one

((pointers touch both wavefunctions in 1)).

Chad: Yes, this would be psi plus this would be psi minus, and this one is psi star psi.

((labels)) ((labels)) ((labels))

Chad: So, essentially deconstructing it from what the probability distribution is

((gestures to probability distribution in probability representation))

Interviewer: Ok, that makes sense.

Chad: Yeah.

Chad's sensemaking is now attending to new areas of the bead's motion, in addition to the bounce. This attention helps generate new meaning (nonzero probability) through close coordination of Chad's two representations. Interestingly, it seems that his focus on the bounce region naturally led him to point out the complementary region (the center region), leading to new conceptual and representational meaning being attached to the center.

Chad sets up his explanation for the wavefunction through the introduction of the probability distribution. He partitions the position representation to highlight the edge ("bounce") region where the bead is accelerated and therefore spending the "majority" it's time. Then, he moves his attention to the center region of his position representation, highlighting the region where the bead spends the "minority" of its time. With his attention on this center region, Chad comes to the sudden realization that something is amiss in the associated center region of his wavefunction representation. The wavefunction should not go to zero in the center, it should be somewhat higher. Chad moves to make the adjustment.

Turning back to his explanation initiates another change in the wavefunction representation. Chad realizes that another, negative wavefunction is also possible and so adds it to the representation. Chad's explanation comes full circle in reiterating the relationship between the probability distribution and wavefunction. In doing so, he codes the upper wavefunction as the probability. This leads to some ambiguity as it seems like Chad has now referred to the upper wavefunction in his representation as both probability and the wavefunction. An implicit need for explanation arises as the interviewer stumbles over the "probability," leading to another representational development. Chad adds another curve showing the probability distribution.

Chad's Explanation of his Position Representation leads to a deeper understanding of the bead's energy and a refinement of the position Representation

The interviewer again asks for an explanation. This time, of Chad's position representation. This again sets off a new chain of reasoning in which Chad sense-makes around new areas and comes to a deeper understanding of the bead's energy and motion.

Interviewer: Ok. Cool. Can I ask you, did, where this, did this come from somewhere? Or is it.

Chad: Well, uhhh...

Interviewer: Besides your brain?

Chad: Yes, it did. Uh, because, it is, the:..

((pencil hovers over position representation))

O:h actually, it's not perfectly like that.

((pencil hovers over position representation))

Because this is the wavefunction for if, for if the relation is, the, the distance away from it is

((writes diff equation x equals negative double-dot x))

equal to the negative acceleration of it.

Interviewer: Oh, ok. That's your diff equation for the--?

Chad: Yeah, that's wave, essentially. Cus x dot dot plus x equals zero.

((writes diff equation x plus double-dot x equals zero))

That would give you a wave. But this isn't exactly that because it's bouncing off the edges

((points to differential equation))

But I assumed, fully elastic, it would have to have some sort of, uhhh some sort of squishing element to it.

Interviewer: Uh huh, yeah.

Chad: To take in energy, because you can't just go

((gestures bead bouncing over table, makes noise at the gestured bounce))

Interviewer: Yeah.

Chad: That

((holds hand in ball form in area where it hand bounced before in gesture))

Yeah.

Interviewer: So it kinda has to squish, and then reform.

Chad: Yeah. So it would, it has some energy E, which is normally in 1/2mv^2.

((Writes E = 1/2mv^2)

But then it'll be so:me, let's make it E elastic kx involved.

((Adds +kx to energy equation))

Interviewer: Ok, cool.

Chad: Just for the edge

((darkens the knot on the right side in probability representation)).

Because the knot, since it's not, not, we're talking about classical situations it’s got some associated size, and bounceable-ness because it's elastic. So yeah, it has to, in, so this region would be flatter than how I drew it before. So, it'd be like that.

((straightens wiggly lines inside of the vertical lines he drew in position representation)).

Interviewer: Ok.

In looking at his position representation, poised for an explanation, Chad again realizes that his representation is not quite right. Chad’s continued sensemaking around the bounce at the edge provides a means of justifying why the position representation does not accurately describe the bead-on-a-string through the generation of a new understanding of the bead’s energy.

He first says that his (incorrect) position representation is described by his written differential equation but cites the bounce at the edge as a reason to reject the differential equation. In doing so, he implies his position representation needs further development. That he cites the bounce, mentions acceleration in his description of the differential equation, and then goes on to adjust the inside region of the position representation may indicate that he’s realizing that the differential equation only applies to the bounce region. We might infer this because Chad has mentioned the acceleration at the edge, first somewhat implicitly and then more explicitly in mentioning the “negative acceleration of it”.

In any case, he continues his algebraic modeling for the bead with the bounce region coordinating sense-making. His mention of the bouncing at the edges as disqualifying the differential equation is followed by some consideration of how energy is flowing at the bounce. Specifically, the energy takes kinetic form in the center region with some additional elastic energy in the bounce regions. In doing so, Chad writes a force instead of an energy. However, his intentions are clear and the misstep seems inconsequential for his developing model of the situation.

Chad’s understanding of the bead’s energy is significantly developed from his first modeling of the bead-on-a-string. In his first modeling, Chad simply reasoned that bouncing elastically meant that it kept all of its energy. Here, Chad has deepened his consideration of the energy, both in terms of what form the energy takes (elastic and kinetic) and how those forms may differ in different regions of the bead’s motion, through his sense-making which continuously coordinated by the bounce region. From this, Chad flattens the lines in the center region of his position representation (where the bead spends the “minority” of its time). This flattening follows conceptually from his new modeling of kinetic energy in the center region where the velocity is constant.

New meaning in the Position Representation leads to a new understanding of the Probability and Wavefunction of the bead

Chad’s new understanding of the energy has a cascade of consequences through his representational system. It leads to new conceptual and representational understandings of the bead’s position, wavefunction, and probability.

Chad: And then this bit is where

((Traces vertical line in position representation))

Oh yeah, so that's not quite right, is it?

((gestures to probability representation))

Interviewer: What made you say that?

Chad: Cus, the entire inside bit has equal probability.

((traces partitioning lines in position representation))

Because it spends the same amount of time here as it does here

((gestures to two points in position representation, unclear which ones))

Interviewer: Mhm.

Chad: Because this is following the same velocity

((points to energy equation))

because, I'm saying the elastic collision only happens at the end bits

((traces area outside of partitions in position representation))

And that's where it's accelerated, enough to make it go back to the same velocity at this point

((marks points in position representation))

Interviewer: Ok.

Chad: So that's, this vin and this is vout

((labels vin and vout in position representation)).

Interviewer: Ok. Sorry, so you're saying the velocity is the same along those straight lines.

((pointer traces lines in position representation))

And then, so can you relate that to probability?

Chad: Ye:s. It would make it, so that, it looks more like

((draws frame of revised probability representation))

So it would be a flat line until the knot region, in which case it curls up,

((draws wavefunction in revised probability representation))

because where it's stopping is where it's spending the most time.

((dots left knot in probability representation))

So it's the highest probability at that point. And this point

((marks left peak of revised probability representation))

((marks right peak in revised probability representation))

So, this is probability distribution.

((draws probability in revised probability representation))

So that makes, the psi plus and minus into.... flat li:ne ((draws top wavefunction in revised probability representation))

Just, essentially a smaller version of it, and a negative version of it.

((points to probability in revised probability representation))

((draws lower wavefunction in revised probability representation))

Interviewer: Ok, that makes sense. Cool.

Chad: Cool.

Interviewer: Yeah.

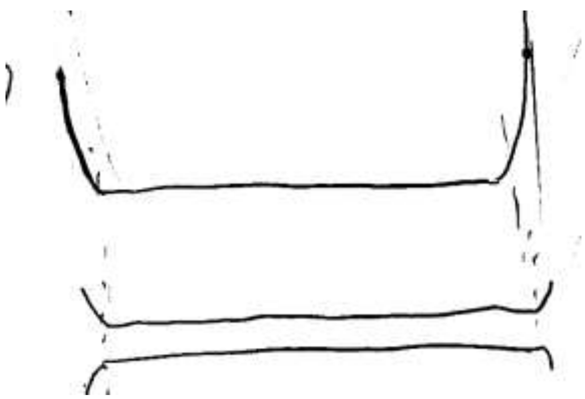

FIG. 4.4 Chad’s Revised Probability Representation

Chad’s explanation of his position representation quickly leads to new conceptual and representational meaning in both his position and wavefunction representation.

Chad is explaining the consequences of his new understanding of the bead’s energy when, mid-explanation, when Chad’s attention is on the boundary line between the bounce and center regions, Chad realizes that his wavefunction is not right. In the position representation, Chad explains the form of the probability in the inside region by reference to two arbitrary two points, reasoning that the points have equal probability because the velocity is constant in that region because the "elastic collision" and acceleration only happens in the bounce region. From this, Chad draws another set of wavefunctions.

Chad's explanation has now come full circle. His original wavefunction was reasoned through attention only to the bounce region; the acceleration at the bounce meant the bead spent more time there, yielding a higher probability in that region. This conceptual and representational understanding remains unchanged. However, Chad has further fleshed out the conceptual and representational consequences of the acceleration at the bounce through reasoning more carefully about the center region. Here, the acceleration at the bounce puts the bead back to a constant velocity and therefore constant probability in the center region.

Discussion of Chad on the Classical Problem

In this section, I illustrate key features of flexible representation use. In particular I show the reflexiveness of between Chad's developing conceptual understanding and his representational system. Representations not only came to reflect Chad's sensemaking but also helped generate new lines of reasoning. In turn, new lines of reasoning become embodied in his representational system. In this back and forth, Chad's sensemaking was continually coordinated by a focus on the bounce area. Highlighting and sensemaking around the bounce in his various representations helped drive sensemaking in the regions complementary to the bounce, the center region and the boundary.

Chad's position representation served multiple roles. Sometimes the representation was a tool for explaining and justifying his thinking. At other times, the same representation became generative towards his sensemaking. I recount how the close conceptual and perceptual coordination between the representations Chad has drawn and his attention to particular areas in those representations help coordinate his sensemaking, leading to new conceptual and representational meaning.

1. Construction of the Position Representation of the Bead-on-a-string initiates a change in the Wavefunction Representation:

Explaining his wavefunction representation led to the generation of the position representation and then the partitioning of the representation to highlight the edge regions. Introducing a different representation of the bead helps Chad 'see' and reason about regions of the bead's motion that he had previously not considered. Highlighting the complementary center region led to the realization that the wavefunction should be non-zero in the center. Chad doesn't fully flesh out why the wavefunction can't go to zero in the center. It seems that when he makes this realization, it's the first time in the episode that he is coordinating his visual perception with the center region in the representation, along with the conceptual notion of where the bead is spending its time. It seems likely that in looking across his plot of the bead's position over time, he can easily read-out that every point is being occupied by the bead at some point in time. Coordinating his notion of 'more-time yields a higher probability' may allow him to quickly infer that the probability is non-zero in the center.

2. Chad's Explanation of his Position Representation leads to a deeper understanding of the bead's energy and a refinement of the Position Representation:

When the interviewer asks about the source of the position graph, Chad has to reason more directly about the representation. In doing so, Chad describes his deepening understanding of the bead's energy to alleviate issues that became apparent to him when he was poised to explain the position graph to the interviewer. The consequences of this modeling lead him to the new conclusion that the velocity is constant in the center of bead's motion and therefore the position representation should be flatter in the center region. This point shows how the using velocity representation reflexively shaped Chad's sense-making, leading to new conceptual meaning about the bead's motion.

3. New meaning in the Position Representation leads to a new understanding of the Probability and Wavefunction of the bead:

Chad's new understanding of the center region in the position representation becomes quickly generative towards developing new representational and conceptual meaning in the wavefunction representation of the bead. The flatness of the position representation implies a constant probability in the wavefunction representation. The quick flow of consequences from new meaning coordinated in the position representation to the wavefunction representation shows the close coupling of the representational system Chad has developed to help coordinate his sensemaking.

Chad on the Slanted Well Problem

Chad recruits the Finite Well representation to reason about how potential walls affect the wavefunction

After reading the prompt, Chad mentions that the slanted well was a problem on one of his final exams, although it was not something they went over in class. Although the situation is familiar to Chad, we see him constructing aspects of his sense-making as the interaction unfolds. In his sense-making, Chad first draws on the finite well representation to sense-make about how potential walls affect the wavefunction.

Chad: ((Reads prompt))

So another particle in a box. V=Ax oh. Oh. Ok, it's particle in a box.

((draws axes and box in slanted well representation))....

Interviewer: Ummmm sooo I think it's maybe not quite--

Chad: Oh Ax. Sorry, I did not quite read that right. You're right.

((slanted well representation-- changes bottom to slant, erases flat bottom))

It is. Alright, this was one of my final questions.

Interviewer: Was it really?

Chad: Yeah.

Interviewer: No wa:y.

Chad: Draw the states allowed.

Interviewer: Is this something you guys did in class?

Chad: No. It was only on the final. We talk about how the potential, uh walls affect it

((traces vertical wall on slanted well representation)).

And how it would be uhh...

((re-traces vertical wall on slanted well representation))

Yeah 'cus we talked, yeah if you talk about, uhh finite regions,

((draws well in finite well representation)) you have the wavefunction in here

((begins to draw wavefunction in finite well representation)),

it doesn't go to zero here

((crosses boundary in finite well representation))

it goes to the points that it does, then exponentially decays in it

((draws decaying wavefunction in left, then right region))

Interviewer: I see.

Chad: And then... if they're tall enough you get

((adjusts potential walls in finite well representation to go down to V=0))

Tunneling!

((extends wavefunction in finite well representation))

Interviewer: Yay.

Chad: But yeah for this one it would just be, uhh it starts off like it and then it decays

((ground state in finite well representation with matching speech, draws n=2 with no comment))

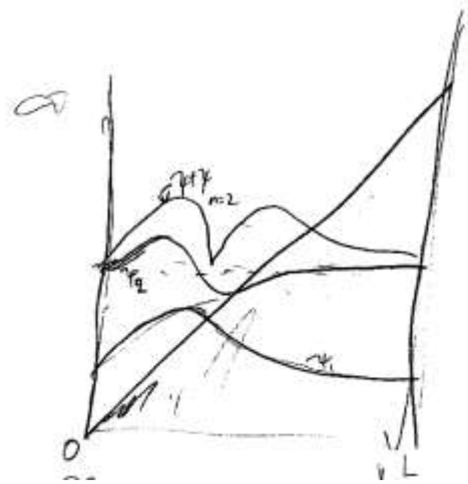

FIG. 4.5 Chad’s Slanted Well Representation

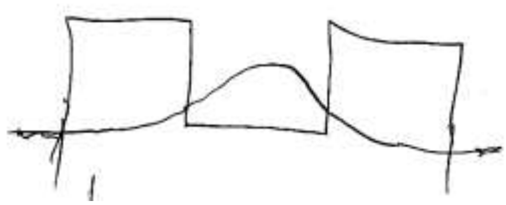

FIG. 4.6 Chad’s Finite Well Representation

At the beginning of his turn, Chad has a platform ready (the slanted well representation) to hold his representation of the “states allowed.” Chad begins a chain of causal reasoning in stating that the “potential walls affect it.” In doing so, he highlights and re-highlights the vertical, right line in his slanted well representation, bringing forth one aspect of the inscription as particularly relevant to the shared visual field. The overlapping speech and tracing indicate that Chad is referring to the vertical line as a “potential wall.” As the episode progresses, we see continued evidence that the regions and boundaries created by walls in his inscriptions structure and coordinate his reasoning.

After an abandoned start, “and how it would be…,” Chad moves vertically down his page and begins to draw the finite well representation, a standard representation of a finite potential well. The finite well representation unfolds with his speech; he notes that the wavefunction “exponentially decays within it.” The use of the preposition “within” and his concurrent drawing of the wavefunctions in the regions of high potential connects the idea of “decay” to regions bounded by potential walls in the inscription.

In this short piece of speech, the physical structure of the potential walls feeds into his sense-making in several ways. Looking at his utterance, once he begins to expand on his causal reasoning of how the “potential walls affect it,” his speech is naturally punctuated at the potential wall boundaries. The clauses of his speech either refer to a region of the inscription (“uhh finite regions,” “you have the wavefunction in here,” and “then exponentially decays in it”) or refer to the values at boundaries in the inscription (“it doesn't go to zero here” and “it goes to the points that it does”). His reasoning about the wavefunction is compartmentalized into reasoning about its properties in regions and at boundaries. I would therefore infer that the representation is playing some role in ‘bounding’ Chad’s reasoning, perhaps by supporting a causal or descriptive account he is remembering, or partially generating on the fly, about wavefunction behavior.

Chad takes the outermost walls of the inscription and extends them further down. When he adds to the wavefunction, he draws the piece of the wavefunction that extends horizontally beyond the boundary walls that he had just previously extended. In this brief turn, Chad recounts a causal story about the emergence of tunneling, which can occur if these pieces of the wavefunction are “tall enough;” a condition which does not have empirical conceptual underpinnings, but is more readily a condition met by the physical form of the inscription. The first form of the inscription (without extended walls and wavefunction), provides the setup conditions necessary for the tunneling to occur. To be clear, the causal story of tunneling is told more through the manipulation of the physical form of the inscription than readily apparent conceptual ideas about the particle or probability. As before, the region-based structure of the inscription structures Chad’s speech in his implicit treatment of the wavefunction as piece-wise, through his coding of these pieces of the wavefunction as “they”, where “they” end at the boundaries of potential walls.

Chad models the Slanted Well wavefunction as two competing influences: the Particle in a Box wavefunction and the effect of the rising potential

The interviewer looks at the wavefunctions Chad and drawn and asks for an explanation of the shapes. Chad draws two representations, the infinite-well wavefunction and a slanted potential below. In doing so, Chad shows how the effect of the rising potential can give rise to the slanted well wavefunction.

FIG 4.7 Chad’s Infinite well representation above the slanted potential representation: Chad draws diagonal line (slanted potential representation) directly below his infinite well representation. Lettering below the slanted well representation occurred later in the episode and so should be ignored.

Interviewer: So can you tell me like how you kno:w like, kinda of the shapes of those guys?

((points to wavefunctions in slanted well representation))

Chad: So it's, you can kind of take it as perturbation on the particle in a box

((infinite well representation))

So it's going to be essentially particle in a box

((draws n=1, 2 in infinite well representation)) but then, uhh what's happening is as the potential increases

((potential slant representation))

it uhh, reduces the probability of being in that region.

Which means that if you still normalize it,

((traces n=2 in infinite well representation))

it would have to follow, it would have to follow the same energy, stepping, where it's going by nodes added, but it will reduce the probability of this region, linearly.

((circles right-hand side of n=2 in potential slant representation))

Interviewer: Uhh, this region? Is that...

((E points to right-hand side of n=2 in slanted well representation))

Chad: Yeah, this is the potential.

((shades under slant in slanted well representation))

The new conceptual ideas emerging are that the problem is essentially a perturbation on the usual infinite well, and the potential and probability are inversely related. Again, it’s important to note that the relationship between potential and probability is coordinated through reference to given “regions,” now referring to the higher-potential vs. lower-potential parts of the interior of the “box.” The potential-probability relationship he posited is also continuous with, and in some ways a refinement of, his previous conclusion that the potential walls have an effect on the wavefunction. Chad’s verbal coordination of these two conceptual ideas leads to the creation and manipulation of new inscriptions. The representational system grows to include a common form of the infinite well representation with the potential slant representation positioned directly below. His reasoning appears to structure and organize the construction of the inscriptions through this vertical alignment: Chad can more easily read-out the locations of areas of high potential, and therefore high effect on probability. Chad’s speech initially sets-up the mediating relationship between two conceptual ideas. However, it’s the strong visual coordination between inscriptions the infinite well representation and the potential slant representation that provides the platform for reasoning and showing that the right side of the wavefunction in the infinite well representation will see reduced “probability of being in that region.” He circles the region of interest in the infinite well representation, the interviewer then confirms that he is implicitly also reasoning about the corresponding region in the slanted well representation, to which he agrees. Ultimately, through this interaction, Chad manipulates a standard, culturally-sanctioned form of infinite well representation, through the mediating effect of the potential slant representation, in order to draw conclusions about reduced probability in the right side of the ground state in the slanted infinite well representation.

Modeling the Slanted Well wavefunction as two competing influences generates a new conceptual and representational feature: the turning-point between the two influences

Chad continues to flesh out his modeling of the slanted well representation. As he goes back and forth between conceptual and representational sense-making, new representational features emerge as his sense-making develops.

Chad: But, uhh so it will follow essentially it

((traces part of ground state of the infinite well in slanted well representation))

I think I made it too big for my waves to look right. But it will go into it generally like that, ((points to n=slanted well representation in infinite well representation)).

but it will also decay after it enters the region.

((traces remainder of wavefunction in slanted well representation, leaving little line showing where two wavefunctions deviate))

The probability function looks a bit weird on this because it goes past zero and comes out to it slowly.

((draws dotted line in slanted well representation for n=2))

Chad: Looks like the....

((adds presumably psi squared for n=2 in slanted well representation))

Interviewer: So is that psi squared for the... n=2?

((points to n=2 in slanted well representation))

Chad: Yeah. That's psi star psi for... This is ...

((labels wavefunctions))

He traces the wavefunction as he attends to the deviation from the unperturbed infinite well, leaving a little line that the normal ground state might follow, where the slanted ground state begins to decay away.

In this concluding bit of speech, Chad finalizes his conceptual coordination of the unperturbed infinite well and the mediating effect of the potential increasing. He turns to the slanted infinite well, and shows how the state takes on characteristics of the unperturbed infinite well and then begins to decay towards the right side of the inscription. Although interpretations are possible whereby the various representations merely express Chad's thinking, we argue for a more reflexive relation between the inscriptions and Chad's thinking whereby the inscriptions influence and help shape his thinking: the decaying wavefunctions inscribed in the forbidden regions of the finite well representation, with forbidden corresponding visually to where the potential is higher than the wavefunction, combined with the visual coordination of the infinite well representation and the potential slant representation as discussed above, contributes to Chad's in-the-moment drawing/thinking about the lowest wavefunction in the slanted well representation. The particular shape of the wavefunction—unperturbed then decaying—as well as the conceptual insight that the probability is lower where the potential is higher are constructed through the generation, manipulation, and reasoning with the inscriptions in the finite well representation, the infinite well representation, and the potential slant representation.

Discussion of Chad on the Slanted Well Problem

In this brief episode, Chad's sense-making and inscription use are deeply entwined, occasionally unfolding together and other times feeding back into each other reflexively. In some moments, his talk was structured in terms of the properties of regions of the inscription. In some places, Chad's judicious use of inscriptions may provide additional structure to the conceptual ideas he brings up. For example, his speech explicitly relays that the problem is like the infinite well, but that there is some mediating effect of the potential increasing. However, the cognitive work to find the region affected in the slanted well representation, where the wavefunction is "below" the potential, is not accomplished in his speech, but through the coordination of other inscriptions.

For Chad, different inscriptions do different work, and he generates new inscriptions to serve specific, emergent purposes; even the culturally-sanctioned representations (infinite and finite well representations) he draws do not simply serve read-out purposes. Chad does work on these inscriptions to coordinate different conceptual ideas, giving additional meaning to the non-standard form he constructs, the slanted well representation. Not only do different inscriptions serve different purposes for Chad, different parts of individual inscriptions play different roles. For example, the right side of the slanted well representation initiates the causal reasoning about how the rising potential affects the probability, with the regions of wavefunction decay in the finite well representation playing a role in that reasoning.

Comparing the two episodes

What's similar? Features of Flexible Representation Use

In looking across the two episodes of Chad's sense-making, we see similar patterns of flexible representation use emerge:

1. Chad's sense-making was highly reflexive in both cases. His representations 'grew' with his conceptual understandings of the two situations. In turn, drawing and attending to the representations he had drawn helped develop his conceptual understanding.
2. Chad generated highly coherent, coupled systems of representations. These representations served different purposes, depending on emergent sense-making or communication needs. Sense-making within one representation often led to conceptual and representational consequences in another representation.
3. Attention to new features or areas in his representations became generative towards Chad's sense-making, often through coordination across coupled representations. Not only was noticing new features important for Chad to

develop a deeper understanding, but acting on those features became generative.

a. On the classical well problem; attention to the middle region in his position representation, and straightening the middle region, led to a deeper understanding of the probability distribution of the bead.
b. On the slanted well problem; a new representational feature (representational turning-point) in the slanted well wavefunction emerged through the coordination of his infinite well representation and the potential slant.

Overall, the above three points led to a reflexive form of sense-making in which Chad repeatedly circled over his representations. As he circled over, Chad was able to generate new meaning through the system he created and attending to new areas within and across that system.

What's dissimilar? The kinds of representational stepping stones Chad introduces

On the two problems, Chad introduced various representations to serve as 'stepping stones' to reason towards his final products (wavefunctions). In the classical well problem, Chad generated a 'non-standard' representation to reason with (plot of position versus time). I say it's non-standard because it appears to be something Chad constructs in the moment, based on the problem set-up. In any case, the representation is some abstraction of the physical behavior of the bead. In other cases of students reasoning on this problem, students use other representations of the bead to reason about the bead's behavior and its wavefunction. For example[19], these may be drawings of the bead-on-a-string, over which the student can simulate the bead's motion, then infer the wavefunction from the simulated behavior.

In contrast, on the slanted well problem, Chad recruited two iconic representations of different quantum toy model situations. This pattern of using quantum toy models as stepping stones in reasoning about the slanted well also holds. Often, it is the infinite well. Others, like Chad, utilize the finite well too. This is likely, in part, due to the conceptual and physical continuity between the infinite well and the slanted well situations. However, it raises an important issue in supporting flexible representation use in quantum. The issue is mainly a question of what types of representations are *available* for students to sense-make with in quantum. It's unclear what a less-abstracted representation of the particle in the slanted well would look like or whether it would be physically correct enough to be an appropriate tool to reason with.

[19] Quinn and Oliver in the previous chapter.

Arguably, the most common quantum toy models (infinite well, finite well, harmonic oscillator) are meant to be powerful examples of the application of the Schrodinger Equation in different potentials. But clearly, they can also serve as stepping stones towards understanding other situations. And so, this highlights the importance of instruction providing students with opportunities to reason about quantum behavior is a variety of situations.

Conclusions

In this chapter, I began with Greeno & Hall's invocation for teachers to create opportunities for students to use representations flexibly. Towards this purpose, I described some design principles used in problem development and recounted two episodes of student reasoning in the context of the designed problems that illustrates flexible, reflexive use of representations. My goal is to expand upon the notion of reflexive use of representation through my case studies of Chad, showing the coupled, mutually influencing nature of Chad's sense-making and his developing inscriptional system. Through this case study, I hoped to begin challenging what counts as proficiency in using representations. Reflexive use describes a pattern of sense-making that may involve a back-and-forth between the traditionally characterized as proficient actions of (re)-generating a representation to express one's thinking and appropriately reading-out from a representation. However, as I've have demonstrated, inscriptions can provide more than a place to read-out information, but can be a site for action.

Designing disruptions to representational infrastructure became a lens to critically examine the opportunities for innovation and learning available to students. Thinking along these lines also provided a way to start thinking about what supports are available to students in developing ‘disciplined inquiry’ when innovating and inventing with representations. I found that the prompts encouraged innovation, with interaction with the interviewer helping generate some coherence in Chad’s reasoning. Bids for explanation sometimes treated Chad’s representations as piece-wise, which may have encouraged Chad to sense-make around different features of his representations. These bids often meant that Chad had to go back over representations he had already drawn. Doing so helped initiate more ‘feed-back’ loops of reflexive sense-making and more coherent conceptual and representational modeling of the two situations.

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Chapter 5: Conclusions

The three studies in this dissertation focused mainly on studying student thinking along various dimensions and providing insights to instructors based on those studies. In this concluding chapter, I aim to expand on the 'lessons learned' from those studies. In doing so, I discuss the chapters individually.

Chapter 2: Tension in collaborative group-work

This chapter sought to model one way in interactional **tension** can function in students' conceptual reasoning. I analyzed three cases of students working in collaborative to show tension in the groups became a driving mechanism in the group's taking an 'escape hatch.' In these escape hatches, students found creative means to close a conversational topic while leaving some conceptual query unresolved. In doing so, I showed the entanglement of various analytical dimensions of interaction: social, epistemological, conceptual, and affective dimensions. The entanglement of these dimensions of interaction, and that resolution multi-dimensional tension through an escape hatch, leads to various implications for research and instruction.

Implications for researchers

The three cases discussed all shows various ways in which cognitive, social, emotional, epistemological dynamics of group interaction can feed into one another.

Conflict within one layer often leads to conflict in another, occasionally resulting in an escape hatch to relieve the multidimensional conflict and tension. This raises question of whether there exists well-designed learning environments or interactional patterns in which these dimensions are not so tightly coupled. I think it's likely that such design, or developed classroom norms, will never be able to fully decouple emotion from conceptual sense-making. But there may be ways to make such coupling less severe.

The issue of entanglement should provide some methodological insights to researchers who may be interested in different types of conflict, as opposed to the coupling of different types. For example, a researcher may be interested in epistemological conflict. And so that researcher seeks out moments where students are negotiating 'what counts' as knowledge in a particular context. Understanding how and why students go about this negotiation should necessarily involve mapping out tension and different types of conflict (conceptual, social, and epistemological) to be able to fully contextualize the epistemological conflict and its resolution.

Instructional implications

In this work, we showed how the conversational closing in an escape hatch can come in the forms of epistemological humor, exploiting tutorial wording, or "agreeing to disagree," (Lampert, Rittenhouse, & Crumbaugh 1996). These are just particular instantiations of the broader interactional phenomenon of taking an escape hatch. Of course, they may come in other forms. The point of the work then is not to direct instructor attention to these particular closings, to help instructors develop practices of noticing that enable to see both; 1) entanglement in these different analytical dimensions of interaction, 2) conversational closings that enable tension and conflict resolution.

Understanding dimensions of interaction to be entangled has important consequences for facilitating small group work. It shows that what facilitators are responding to is not just in-the-moment sense-making, but the interactional history of the group, as well. For example, a facilitator walking by a group may observe some epistemological conflict being negotiated in a group. Maybe half the group is leaning towards conceptual sense-making whereas the other half is arguing for more quantitative approaches. Intervening immediately to help the group through their negotiation may be unproductive, if the root cause of the tension experienced by the group is not understood. For this reason, it becomes clear that some amount of 'sampling' needs to be done of group dynamics to better understand what exactly, as a facilitator, you are actually responding to.

Students and emotions

A final point we advocated for is helping students notice conflict and emotions in their interactions. This includes helping students see how interactional tension and emotion function in their reasoning, more loading. The goal here being to reconceptualize classrooms as a place where students can deal with emotionally-charged disagreements and find appropriate ways to find a resolution in that disagreement. Resolution may be working towards a point of more clarity or being explicit about why the discussion should be left and taken-up later on. Depending on the context, both options are perfectly valid ways of dealing with disagreements. Indeed, depending on the nature of the disagreement, the discussion may need to be tabled many times over before the participants are able to find points of more clarity. The hope is that this instructional orientation, towards understanding and managing disagreement, would help students acquire tools to better engage with difficult issues, both in and out of the classroom.

Chapters 3 and 4: flexible representation use

Both chapters 3 and 4 studied certain aspects of flexible representation use by students. This line of work originated in seeing evidence of common student difficulties in interview and focus group data. My noticing here was likely influenced by the plethora of research on student difficulties in quantum. When I was able to look past this, I began to notice the creativity and inventiveness of student's reasoning. And so this work sought to understand different ways that students use and generate representations to develop new understanding and model new situations.

The works are quite complementary. However, I had a very specific reason for separating, as I wanted to use chapter 2 to make a particular point to instructors. Mainly, I try to encourage instructors to give students opportunities to sense-make about more complex systems, even if they feel these students aren't ready. Below, I'll talk more about the inspiration for this chapter and then implications I have drawn from this line of work.

Inspiration for Chapter 2: allowing students to move on to real-world applications

Two summers ago, I was attending the annual Physics Education Research Conference. The conference often has focused sessions and I was listening-in on several researchers talk about their work in studying the teaching and learning of quantum mechanics. Near the end of the session, when the panelists were taking questions from the audience, a student made a request of the researchers (and educators). He understood that a lot of new, fundamental knowledge is needed to understand and learn quantum mechanics. But he was missing opportunities for learning about real-world applications. In his words, “why the **** are these courses not helping me understand how Pokemon-go works on my phone?”

Having a student push for seeing the application of what they’re learning is really wonderful. It’s what instructors should hope for in their students. However, the response from a senior researcher and educator was quite the opposite of wonderful; “well, how far have you gotten [in quantum]?”

And so, this work aims to make the case to researchers and educators in quantum mechanics, that students should be given opportunities to move on to more complex applications. In doing so, I hope to elevate this one student’s voice and potentially others who are not privileged enough to be able to put themselves in spaces to speak up for themselves.

Balancing different intuitions as an instructor

As mentioned above, this work is mainly responding to a pervasive instructional notion that fundamentals should be mastered first. This notion of fundamentals-first came up recently in a conversation with a well-respected physics instructor who has written several textbooks, including one of quantum mechanics. He expressed some tension emerging from differences in his instructor-intuition and his intuition abstracted from being a learner. His instructor-intuition would encourage him to make sure no student is left behind in understanding these fundamental examples. But his intuition as a learner of physics leads him to realize that learning itself is a non-linear process, and so mastering fundamentals continues to happen as one learns about more complex systems. And so careful instruction should be a balance of giving into these different intuitions, making sure not to prioritize one consistently over the other.

Reconceptualizing toy models in QM: students should learn with and about toy models

In showing different cases of students reasoning with toy models and their iconic representations in new situations, I hoped to make a case for instructors to reconceptualize what these toy models are for students. Instead of only being strong examples of the application of the Schrodinger Equation in simple potentials, they can also be tools for learning with in making sense of new situations.

In this chapter, I illustrated three cases of students ‘learning with’ toy models in order to show how the adaptation of the toy model could lead to a deeper understanding of the toy model itself, as well as the new situation being modeled. In particular, continuities that students saw between the two situations, and fleshing out differences, helped direct student sense-making to particular areas in the toy models and new situations. Most importantly, misunderstandings that students had about the toy models, or sometimes their faltering at finding the words to describe the toy model, did not prevent the students from using the toy model productively in generating new meaning.

The main point for instructors here is that they should provide students’ opportunities to invent and adapt even if they think students aren’t ready. The process of invention and adaptation may provide students opportunities to better learn ‘fundamentals’ that concern instructors. This perspective is complimented by literature suggesting that even if early adaptation leads to incorrect results, i.e. the process does end up providing the student with new meaning, early adaptation is still valuable (Schwartz & Bransford, 1998; Schwartz & Martin, 2004). These studies show that early invention can eventually lead to greater gains in student learning later on.

Chapter 4: Flexible Representation Use

I reproduce part of the discussion points to remind the reader of important features of flexible representation use. Chad's sense-making was highly reflexive in both episodes I discussed. His representations 'grew' with his conceptual understandings of the two situations. In turn, drawing and attending to the representations he had drawn helped further develop his conceptual understanding.

1. **Stepping Stone Representations**: Chad generated highly coherent, coupled systems of representations. These representations served different purposes, depending on emergent sense-making or communication needs. Sense-making within one representation often led to conceptual and representational consequences in another representation.
2. **Figural features**: Attention to new features or areas in his representations became generative towards Chad's sense-making, often through coordination across coupled representations. Not only was noticing new features important for Chad to develop a deeper understanding, but acting on those features became generative.

a. On the classical well problem; attention to the middle region in his position representation, and straightening the middle region, led to a deeper understanding of the probability distribution of the bead.

b. On the slanted well problem; a new representational feature (representational turning-point) in the slanted well wavefunction emerged through the coordination of his infinite well representation and the potential slant.

Standard and non-standard Stepping Stone Representations

From analyses of the episodes with Chad and others not described in this dissertation, it's clear that 'stepping-stone' representations are often necessary for student sense-making towards the final, ultimate representation that is typically the goal in those instances. However, this raises two important instructional issues, 1) what types of 'stepping stones' are available to students and 2) how do instructors provide space for students to utilize stepping stones.

Comparing the two episodes with Chad, we see that different stepping stones are available to his sense-making because of the physical context in which he is reasoning. On the classical well problem, he is able to draw a graph of the bead's position versus time. Other students, such as Oliver and Quinn, drew pictures of the bead-on-a-string and/or the velocity of the bead as representations to sense-make with. In contrast, stepping stone representation on quantum problems tended to be iconic representations associated with toy models. This thread, along with the work in chapter 2, highlights the importance of toy models and their representations in students' reasoning.

Highlighting Figural Features

In both chapters 3 and 4, I showed that student attention to and sense-making around different figural features became integral in the generation of new meaning. With the case of Chad in chapter 3, his sense-making was consistently coordinated around particular figural features; the knots in the classical well problem and potential walls on the slanted well problem. Focusing on these regions helped coordinate Chad in developing a deeper understanding of regions outside of the focal features.

In the episodes with Quinn and Oliver, a similar pattern was present; their sense-making focused around particular features or areas. In these cases, the infinite well toy model helped direct their reasoning to the 'same' areas in the toy model and bead-on-a-string. Unlike with Chad, these interviews proceed with the interviewer highlighting new regions. Coordinating these new regions in their sense-making provided a mechanism for Quinn and Oliver to develop deeper understandings of the infinite well representation and the bead-on-a-string.

The differences in how students come to reason about new regions in their representing and represented world's raises important instructional considerations. For Chad, his reasoning about the bead-on-a-string naturally led him to consider additional regions as his sense-making developed. However, it was important that there was space in the classical well prompt for him to coordinate his reasoning through a focus on the knots to begin with. I can imagine two scenarios that could impede this focus in his reasoning. The first being a preceding question to this prompt, as in on a tutorial, that positions all positions on the axis with equal likelihood by asking something about what a measurement on the bead's position would yield. Another way to impede this sense-making may be to overly simplify the situation by suggesting the bounce at the ends happens instantaneously. And so, it seems as if the conceptual space left in the prompt may help provide space for students to sense-make in ways that are heavily coordinated by particular areas or features.

While Chad's sense-making led him to consider additional regions in his representations as he came to a better understanding of both situations, it took highlighting by the interviewer to get Quinn and Oliver to consider new regions. This highlighting intervention became routinized across the collection of interviewers as many students failed to consider off-center regions in their first modeling of the bead-on-a-string. And so, it leads me to believe that this type of intervention could be incorporated into a follow-up prompt.

Providing Opportunities for flexible representation use: thinking about students' access and opportunities

I turned to literature on designing disruptions to representational infrastructure to think about designing for flexible representation use. In responding to disruptions students need to be inventive and creative because they do not have to tools to efficiently solve the problem at hand.

Thinking about disrupting representational infrastructure should lead instructors to consider of more than just the conceptual content that students may develop in response to a disruption. Disrupting representational infrastructure illuminates issues of access of opportunities for learning through considering what opportunities are available to students through routines being developed in the course. Getting a good grip on this can then help instructors think about ways to disrupt any problematic norms. For example, students should have the opportunity to invent in new situations because it can lead to the generation of new meaning and because it may serve student desire to see applications of what they're learning. Students should also have access to instructional support, either through an individual and/or well-designed curricular materials that support the students in adapting and inventing in a disciplined way.

Takeaways from interviews

Some careful consideration is needed in taking these results, which come from clinical contexts, and translating them into implications for instruction. Below, I'll briefly discuss a few of those differences that I project may be consequential for such translation of findings:

1) While I was able to get to know students some over the course of several interviews and/or focus groups, instructors have the benefit of being able to get to know their students over a semester, or even longer. When considering interactional norms and escape hatches, this knowledge of the student can be a huge asset. Instructors can develop a sense of what students work well together, what norms are developing in the classroom, or what students might need more

interventions than others. Such patterns can then help inform instructors' responsiveness to group-work.

2) Similar to the first point, instructors have the opportunity in classrooms to spend time developing interactional and classroom norms. These can involve shaping what types of thinking and interactions are accepted in the classroom.

3) I believe it's very likely that spaces, like classrooms, where students are assessed, and where norms suggest that a student display canonical knowledge, come at the expense of a learning space that supports creativity and trying out non-canonical ideas. This means there may be some tension associated with asking students to reason 'beyond' norms that they're used to. In interviews and focus-groups, I tried to create spaces where students were encouraged to express their thinking but finding the right answer (or not) was less valued. This may have freed students to be more creative and flexible in their thinking, because there was less risk of assessment.

4) Students themselves may see the interview or focus-group spaces as very different from what's accepted or normal in a classroom. This may mean that the sense-making that emerged in the interview spaces is not as accessible in the classroom. As mentioned above, this might be remedied by working to set norms and expectations for what's expected in the class.

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Appendix 1

Transcript Conventions

The transcripts use the following protocols (Sacks, Schegloff, & Jefferson, 1974; Jefferson, 2004). :

:: Elongated words or vowels

CAP Emphasized words are capitalized

[Start of overlapping speech of first speaker is shown with open bracket

// Start of overlapping speech of second speaker

-- Turns that are cut off by other speakers or end abruptly are marked with a hyphen

… Speaker turns that trail off are marked with an ellipsis

(()) actions other than speech, including gestures, are represented in italics and surrounded by double parentheses

(?) Pieces of speech that are difficult to discern are preceded or replaced

(#) Length of a pause

Jefferson, G. (2004). Glossary of transcript symbols with an introduction. *Pragmatics and Beyond New Series*, *125*, 13-34.

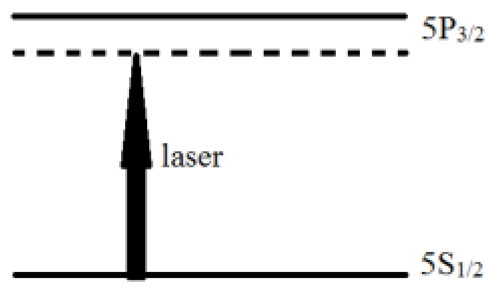
Sacks, H., Schegloff, E. A., & Jefferson, G. (1974). A simplest systematics for the organization of turn-taking for conversation. *language*, 696-735.

Appendix 2

Tutorial on Doppler Cooling

Tutorial 8:

Doppler cooling



How does an ambulance sound speeding towards you, as compared to sitting still or moving away?

Suppose we have an atom that can move along the horizontal dimension, with laser light coming in from the right and from the left.

If the atom moves to the right, how do the wavelengths of each laser change, as seen by the atom?

What about if the atom moves to the left?

Suppose the atom in #2 has an energy level structure, as shown to the right. The arrow shows the energy of the photons produced by the lasers. This laser is “red-detuned" from the atomic transition because photons from the laser have a lower frequency and energy, hence a longer or more-red wavelength, than the atomic transition.

Draw an energy level diagram of the atom when it is moving to the right, showing the photon energy of both lasers, as seen by the atom.

Does motion along the axis change the likelihood of excitation of the atom by photons from one of the lasers? If so, which laser?

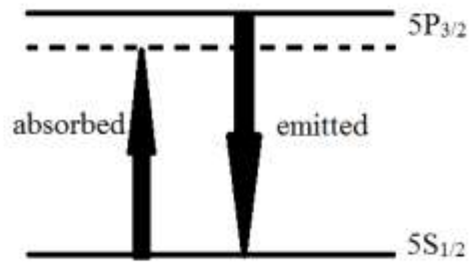
✯ *Consult an instructor before you proceed.*

If the atom is moving to the right and absorbs a photon, how does the atom's momentum change, if at all? (*Hint:* A photon carries momentum.)

After absorbing the photon, the atom will then emit a photon, with an energy equal to the

$5P_{3/2} \rightarrow 5S_{1/2}$ transition, in a random direction. Describe how the momentum of the atom changes after many, many cycles of absorption from the right laser and random emission.

The diagram at right shows the energy absorbed and then emitted by the atom in one cycle *as seen by a motionless observer*. So in each cycle, the atom emits more energy than it absorbs. How can you reconcile this with the conservation of energy?



Let's pull things together. Describe what happens to an atom that is moving in one dimension between two red-detuned laser beams that are shining in opposite directions (as shown in the diagram in question #2).

Now let's try to generalize to three dimensions. Instead of having an atom that is confined to move in one dimension, it is now allowed to move in all three. How could additional lasers be arranged and tuned so that the atom loses speed no matter which way it's moving?

If you had a large number of atoms in the system you described in #9, how does the temperature of the atoms change as they undergo many cycles of absorption and emission?

Atoms in the system described in #9 are called an optical molasses. Why do you think physicists chose that name?

Instructor Guide: Tutorial on Doppler Cooling

Instructor Guide 8:

Doppler cooling

This tutorial is designed to introduce students to the concept of laser cooling, specifically Doppler cooling. By the end of the tutorial, the students should recognize that counter-propagating, red-detuned laser beams may be used to create an optical molasses, or a cloud of cool atoms. This tutorial can be used in conjunction with Tutorial 9 (Zeeman Effect), where students see that a magnetic field can then be used to provide spatial confinement of the cooled atoms. Taken together, the laser and magnetic-field configurations described in Tutorials 8 and 9 form a Magneto-Optical Trap (MOT). Doppler cooling can be used to cool atoms for atomic clocks. In creating Bose-Einstein Condensates, scientists usually create a cloud of cooled atoms with a MOT.

In the tutorial, some of the big ideas students will focus on are the following:

- Doppler shift of sound and light
- Atomic energy levels and transitions between levels
- Laser-matter interactions

Doppler Shift

1) Students should recognize that the sound waves from the ambulance are shifted to a higher frequency when coming towards the observer, and a lower frequency when moving away.

2) This is the same question, but on the Doppler shift for light. The lasers are blue-shifted when the atom is moving towards them, and red-shifted when moving away.

Doppler Shifted Energy levels

3) There are many way to represent the energy levels, but many students choose to draw diagrams similar to the one shown. Students should see that one laser (right) gets shifted up in energy and the other laser gets shifted down.

4) Students should see that the laser that the atom is moving towards is shifted closer to the atomic resonance.

Momentum Considerations

5) The photon's momentum is absorbed by the atom. The atom gets a kick in the direction of the laser's propagation, essentially slowing the atom down *in the direction* antiparallel to the laser's propagation. The change in velocity that the atom

receives is called the recoil velocity and can be calculated using the momentum of the photon and mass of the atom. The recoil velocity is related to the minimum temperature that is achievable through Doppler cooling.

6) Each cycle, the atom is slowed in its direction of motion through absorption and then receives a kick in a random direction through emission. After many cycles, there is a large slowing effect in the direction of motion, while the kicks that the atom gets through reemission should average out to no net change.

Energy Considerations

7) One important consideration is that energy and energy conservation are frame-dependent! This can be a difficult concept for students, as energy conservation is one of the tools used most frequently in physics. In the lab frame, the atom does absorb a photon of lower frequency than what it re-emits, leaving some small amount of energy unaccounted for. This energy comes from the atom itself; kinetic energy is being taken from the atom and given to the second photon.

8) The atom is slowed in its direction of motion, in both directions.

9) Three orthogonal pairs of counter-propagating, red-detuned beams.

10) The atoms are losing kinetic energy, so the temperature decreases.

11) Answers may vary. A molasses is something that moves slowly.

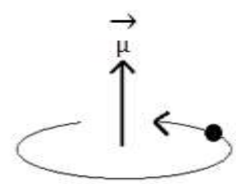
Appendix 3

<u>Tutorial on Zeeman Effect</u>

Tutorial 9:

Zeeman effect

In this tutorial, we will investigate how a magnetic field and a pair of lasers might be used to confine atoms in a small region of space.



1) Moving charges experience a force in a magnetic field. The potential energy of the interaction is $U = -\boldsymbol{\mu}\cdot\mathbf{B}$, where $\boldsymbol{\mu}$ is the magnetic moment of the charge. It is proportional to the charge and points in the direction perpendicular to the plane of motion.

    a. If we have a charge that is forced to move in a circle, what happens to the charge if we turn on a magnetic field that points in, for example, the positive z-direction?

    b. Does your answer above depend on what direction the magnetic moment points in? Explain why or why not.

2) Atoms have angular momentum, which we can investigate by modeling the atom as an electron orbiting a nucleus. If we apply a magnetic field in the positive z-direction, what orientation of the atom has the lowest potential energy? (I.e., in what direction

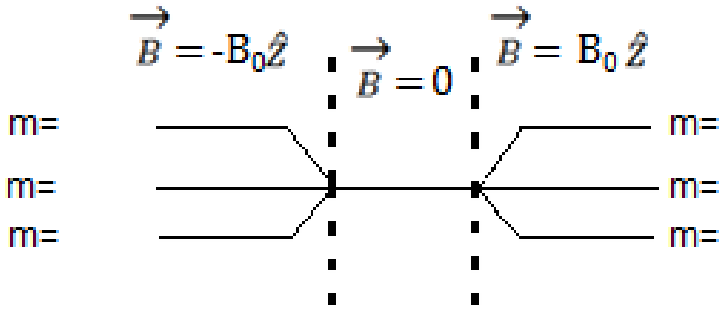
should the magnetic moment point?)

3) Thinking of the atom as a charge orbiting the nucleus is a classical model for a quantum system. How accurately do you think this model describes the physical situation?

4) Is there any value in using the classical model, even if we see that it breaks down when we consider that an accelerated charge would radiate away energy? Explain your reasoning.

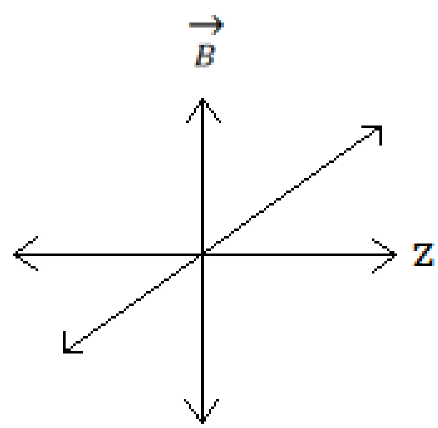
5) For angular momentum of l =1, the magnetic quantum number m can take on values of -1, 0 and 1. (The magnetic quantum number tells us the projection of the angular momentum vector in a given direction, say z.) Which of these states (m = -1, 0, 1) gives maximum, zero and minimum potential energy, given a magnetic field that points in the positive z-direction?

6) In the absence of a magnetic field, the three levels would have the same energy. Fill in the energy level diagram of the three magnetic levels splitting in the two cases shown. On the left, the magnetic field is in the -z direction and on the right it is in the +z direction. B_0 is a constant.



7) Suppose we have the linear magnetic field shown in the graph. Describe how the energy levels change as you move away from the origin. Is it the same going in the +z direction as the -z direction?

8) Time for a quick detour into polarization. (Polarization tells us which direction the electric field of an electromagnetic wave points in.) Light can have many polarizations, such as horizontal, vertical or circular. With circular polarization, the E-field traces out a circle as the wave propagates. This rotation can be clockwise or counterclockwise. We'll call these σ^+ (or σ^-), respectively. When an atom absorbs or emits a σ^+ photon, the electronic transition must satisfy $\Delta m = +1$. Similarly, absorption or emission of a σ^- photon allows a transition with $\Delta m = -1$.

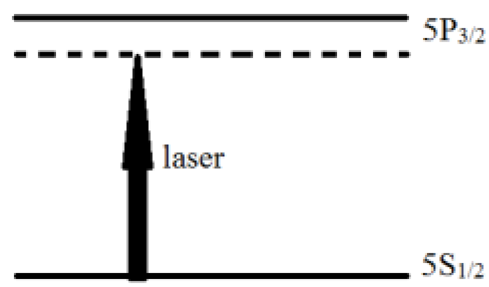


Which types of polarization (σ^+, σ^-, or none) can enable the following transitions?

a) $l=0$, $m=0 \rightarrow l=1$, $m=+1$

b) $l=0$, $m=0 \rightarrow l=1$, $m=0$

c) $l=0$, $m=0 \rightarrow l=1$, $m=-1$



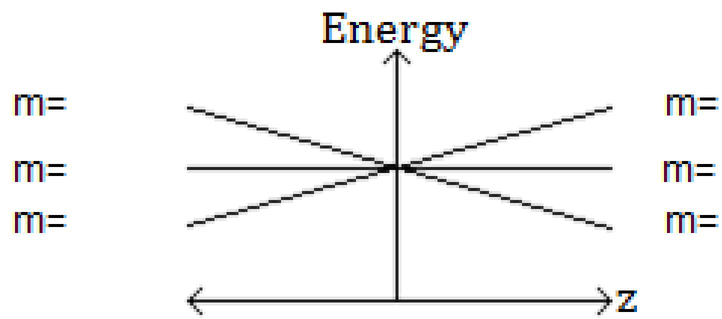
d) l=1, m=+1 → l=0, m=0

e) l=1, m=0 → l=0, m=0

f) l=1, m=-1 → l=0, m=0

9) Imagine we have the magnetic field described in question 7, one that varies linearly with z-position. We can adapt our energy level diagram from problem 6 to the diagram shown below. Label the magnetic *m* levels on the diagram.

10) Suppose we have a laser with an energy level that is shown by the dotted line on the diagram. The atom will absorb a photon from the laser if the energy of the photon is close to the energy of an appropriate transition of the atom.



If we have an atom in the ground state that is to the right of the origin, what kind of polarization (σ^+ or σ^-) is it more likely to absorb?

What about a ground state atom to the left of the origin?

☆ *Consult an instructor before you proceed.*

11) A photon carries a momentum of $\hbar k$, where $k = 2\pi\lambda$. If a stationary atom absorbs a photon, where does the photon's momentum go? Explain your reasoning.

12) If the photon in the previous question is coming in from the right, what is the velocity of the atom (magnitude and direction) after the collision? This is called a *recoil velocity*.

13) Now suppose we have the same linear magnetic field described in question 7, and two lasers coming in from the right and left. The one from the left is σ^+ while the one from the right is σ^-, and both carry photons with the energy depicted in the diagram with question 10.

    Describe what happens to ground state atoms at different positions along the z-axis. (Note: in this situation, the atoms can only move along the z-axis.)

14) When the atom absorbs a photon, it quickly re-emits another photon in a random direction.

a) Imagine the atom is to the right of the origin. What is the effect of many, many cycles of absorption and reemission?

b) What happens to an atom that is to the left of the origin after many cycles?

15) It's possible that an atom that originally started to the right of the origin ends up moving to the left past the origin after going through many cycles of reemission. Can this atom move back towards the origin? Explain your reasoning.

16) Our goal was to confine atoms to a region near the origin. Have we accomplished this? Why or why not?

17) Can we adapt this system to confine atoms in three dimensions? Explain your trapping setup.

Instructor Guide 9:

Zeeman Effect

This tutorial is designed to introduce students to the Zeeman Effect. The students will see that a magnetic field can then be used to provide spatial confinement of cooled atoms. This tutorial should be used in conjunction with Tutorial 8 (Doppler cooling). Taken together, the laser and magnetic-field configurations described in tutorials 8 and 9 form a Magneto-Optical Trap (MOT). Doppler cooling can be used to cool atoms for atomic clocks. In creating Bose-Einstein Condensates, scientists usually create a cloud of cooled atoms with a MOT.

In the tutorial, some of the big ideas students will focus on are the following:

- Motion of charged particles in magnetic fields
- Models of atomic orbits
- Quantized angular momentum
- Polarization
- Momentum transfer

Motion in magnetic fields

1a-b) This may be a bit of a trick question, as we don't say what plane the atom rotates in. The point of this question is that the students should see that this matters. If students do not reach this point for whatever reason, they can move on to the next question without the facilitator providing too much help, as 1b asks the students directly whether the relative orientation matters. In any case, the rotating charge will experience a torque, which will force the charge's magnetic moment to align with the magnetic field.

- Students may struggle with the concept that a single charge can be modeled as experiencing a torque. The facilitator may initiate the idea of an electron attached to the end of a string, with the other end fixed in place. The electron rotates quickly in a circle while in the presence of a weak magnetic field. Ask the students to imagine how the plane of rotation would change in response to this field; it should slowly tilt towards the the direction of the magnetic field.

2) The potential energy of the interaction is lowest when the magnetic moment and magnetic field are aligned. This means the magnetic moment should point in the positive z-direction.

- To check for comprehension, a facilitator might ask what this magnetic moment would imply for motion, e.g. asking the students what plane the rotation lies in and what direction it goes. A magnetic moment in the positive z-axis corresponds to counterclockwise motion in the x-y plane.

3) The students' answer to this question will not likely influence their work in the rest of tutorial. This question provides a point for students to discuss some of the interpretive issues in quantum mechanics. Some students may see the classical model as a good description. Students may also look ahead and choose an answer based on #4. The facilitator should choose whether they want to further discuss this point with students.

4) The facilitator should use their judgment on how to approach this problem with students. We would likely expect students to come up with the response that the model (like the Bohr model) is useful in some ways, but maybe not all.

Quantized Angular Momentum

5) The states m = -1, 0, 1 would correspond to maximum, zero and minimum energies, respectively. The state m=+1 is the state where the projection on the z-axis is positive, so the angular momentum vector is above the x-y plane. Students may lose track of the negative sign in the equation for potential energy, thinking that the parallel configuration yields a maximum energy. Have the students check to make sure their answers are consistent.

6) The states m = -1, 0, 1 would be maximum, zero and minimum energies on the right side of the diagram. The states m = +1, 0, -1 would be maximum, zero and minimum energies on the left side of the diagram.

7) The states would be linear in energy with respect to the z-axis. The m=-1 state would have a positive slope, the m=+1 state would have a negative slope and the m=0 state would remain horizontal. By #7, students should have come to see that energy levels will shift in response to the magnetic field, but they may struggle with the idea that the energy now depends on position along the z-axis. However, students will need to fill a graph showing the linear tilts of the energy levels in #9. So if they are unable to get the linear tilt, they will be given it in #9. There may be some confusion here about what the axes of the graph represent; specifically, the vertical axis now represents the magnitude of the magnetic field (rather than energy as in #6).

Polarization

8) Students may struggle with circular polarization. With circular polarization, the electric field vector traces out a helix around the axis of propagation.

- a. l =0, m=0 → l=1, m=+1
  - Δm= +1, σ^+
- b. l=0, m=0 → l=1, m=0
  - Δm= 0, none
- c. l=0, m=0 → l=1, m=-1
  - Δm= -1, σ^-

- d. l=1, m=+1 → l=0, m=0
    - Δm= -1, σ^-
- e. l=1, m=0 → l=0, m=0
    - Δm= 0, none
- f. l=1, m=-1 → l=0, m=0
    - Δm= +1, σ^+

9) The m=-1 state has the positive slope, the m=+1 state has the negative slope and the m=0 state is horizontal.

10) The students should be looking for the energy levels that are shifted *down*, closer to the wavelength of the laser. On the right side of the origin, the atom's m=-1 energy level is shifted closer to the energy of the laser. An atom here is unable to absorb σ^-, as σ^- would can only drive a transition with Δm= -1, but m=-1 is the lowest magnetic state available. Hence, an atom is more likely to absorb a σ^+ photon and transition from m=-1 to m=0. To the left of the origin, an atom is likely to absorb a σ^-, and transition from m=+1 to m=0.

Momentum Transfer

11) The photon's momentum is transferred to the particle, by conservation of momentum. Throughout the tutorial, we draw both on the wave-like and particle-like characteristics of light. For some students, it may be difficult to discuss light as an electromagnetic wave with polarization and then consider what momentum a photon can carry.

12) The momentum of the atom after absorption would be equal to the momentum of the photon before. To find velocity, divide by mass, $v = \hbar km$. This velocity points in the direction the photon was moving.

13) When atoms are to the right of the origin, the m=+1 state is closer to the laser's energy, making it more likely that the atom can absorb a σ^- photon coming in from the right side. On the left side, the m=-1 state is closer to the laser's energy, which makes it more likely that the atom can absorb a σ^+ photon coming in from the left side.

Atomic Confinement

14a-b) If the atom is to the right (left) of the origin, the atom gets many kicks towards the origin. The kicks the atom receives from re-emission average out to zero.

15) Once the atom is to the left of the origin, it is closer in resonance to the laser coming in from the left, making it more likely that the atom will get kicked back towards the center.

16-17) In order to confine atoms in three dimension, we would have three orthogonal pairs of counter-propagating lasers. Once students reach this point in the tutorial, have them try to draw connections to the previous tutorial. A facilitator might ask whether the temperature of the atom cloud has changed or how the speed of individual atoms changes over time.

Appendix 4

Interview Protocol 1

1. Prompt:
    a. Thanks for agreeing to do the interview! We'll be doing the same kind of thing we've been doing this semester, testing out some curriculum materials that we might use in future quantum classes. So I just ask that you think and work out loud. I'll give you a few sheets of problems but I just ask that you don't look ahead because there's spoilers!
2. Quantitative/conceptual question: (Particle in a box) If you were to measure the position of the particle at some point in time, what position(s) would you expect to measure?
    a. Follow-ups:
        i. How did you get those values?
        ii. If you were to repeat the measurement, would you get the same value every time?
    b. Goal: Contextual priming of particle in a box.
3. How confident do you feel in your answer to the position measurement question?
    a. If high confidence: How did you know that your answer was correct?
        i. Do you generally feel confident in your answers?
        ii. What makes you feel this way?

iii. How do you know when your answer to a problem is correct or incorrect?

b. If low confidence: What made you feel that you didn’t have a correct answer?

i. What makes you feel confident, or not, in your answers?

ii. How do you know when you have a correct or incorrect answer?

c. Goal: This is hinting more at declarative knowledge, as we ask them how they know in general when they’re right. However, I would call this metacognitive priming, priming students to think about metacognition.

4. With the particle in a box, if you were to measure the speed of the particle at some point in time, what would you expect to measure and why?

   a. How did you come to your answer?

   b. Will you get the same measurement every time?

   c. Does your answer make physical sense?

   d. Goal: Contextual priming of particle in a box.

5. I gave this problem to a group of students and while they were solving, I heard the following discussion.

   a. Student 2: What would be the speed of the particle?

i. Student 1: Well it's obviously getting at momentum. So the expectation value of the momentum over the mass would give us the speed, right?

ii. Student 1: Would the speed change? It's just bouncing back and forth.

iii. Student 2: Well, the speed wouldn't change because the potential is constant. But the velocity would have to change direction at the walls.

b. As a student in the group, what might you say to your peers at this point?

   i. Goal: Awareness of the classical description of "just bouncing back and forth." This is assessing conditional knowledge, but in context.

c. The conversation followed with:

   i. Student 3: Well now it sounds like we're switching from quantum to classical explanations.

   ii. Student 1: You're right. So maybe we should stick to the expectation value of momentum.

d. Can you comment on the contributions by each of these students to the discussion?

   i. I will have them comment on each student, if they don't on their own.

ii. Are their contributions different?

iii. Are there ways in which they are similar?

iv. How does it relate to your pretend role in the group?

6. Consider the hydrogen atom in the ground state. If you were to measure the position of the electron, what might you measure?
   a. How did you find that position?
   b. Are there other positions where you might find the electron?
   c. Is it equally likely that you can find it everywhere?
7. I recently overheard two students discussing what the speed of an electron in a hydrogen atom might be.
   a. Student 1 said, "picture the electron-proton as the moon-earth system, we know that closer objects move faster. So a closer electron moves faster." To which the other student replied "that's classical reasoning for a quantum system." Student 1 responds with "but the model is classical mechanical and reproduces the correct energy levels, so it's okay to use."
   b. As a student in the group, how might you respond to your peers?
8. Here's a tough one! Consider the particle in a box, but now slant the bottom of the well. How might you go about finding the allowed energy eigenstates and energies? (Need a diagram here!)
   a. This problem is not solvable and will be used to assess student's use of planning and monitoring of possible solutions paths.

b. If the student jumps into solving:

   i. What are doing?

   ii. How does what you're doing get you to the solution?

c. If the student appears stumped:

   i. Can we take a few steps towards a solution?

9. I gave this problem to a group of students and heard the following:

   a. When guessing what a possible energy eigenstate for the slanted potential problem. A student makes the following statement. "Well, the wavelength of the particle comes from the de Broglie wavelength, which tells me that the wavelength is inversely proportional to difference between the particle's energy and the potential. This means that in regions of low potential, where the difference between E and V is large, the wavelength is small, and in regions of higher potential, where the difference between E and V is small, the wavelength is large. I can reason the amplitude of the wavefunction by picturing the particle as a ball rolling up and down the hill. The amplitude of the wavefunction is largest where the particle spends more time, this would be at the top of the hill because the particle's energy is all potential energy here. So the amplitude should be large at the top of the slant and small at the bottom.

   b. What do you think about this student's reasoning?

      i. Does it appear to valid reasoning?

ii. Is there anything that makes you think twice about using this type of reasoning?

c. This question is designed to test the conditional knowledge of classical reasoning in terms of velocity. Can maybe get at the correspondence principle as well.

Interview Protocol 2

This document contains the interview protocol for my next round of interviews. The goal of the protocol follows the lines of analysis that we have been talking about in the QM meetings recently, disruptions to representational infrastructure within distributed cognition. Through the interview, I would like to first prompt canonical or typical quantum inscriptions. Questions designed to elicit these representations are labeled SW or SV, for Standard Written or Standard Verbal. Subsequent questions, both verbal and written, are designed to disrupt the representational infrastructure surrounding and including the inscription. These are labeled as verbal or written questions (DV or DW).

Disruptions may change the physical form of the inscription or any of the layers of the infrastructure below the representation. These layers include, but are not limited to, tacit design choices, conventions of communication and conventions of use. While it is not possible to determine a priori whether an interviewer move will be taken up as a disruption by the student, it seems reasonable to attempt to introduce such disruptions through conceptually difficult questions or re-framing typical prompts. Both mechanisms for creating disruptions seem to fall within the conventions of use, extending the representational infrastructure in conceptual and epistemological dimensions.

My research questions seem to follow three interrelated threads: the canonical representations that students present, distribution and redistribution of cognition in the system, finding the mechanisms that initiated the redistribution.

1) What are the canonical representational systems that students are representing in the interviews?
2) How is cognition distributed and redistributed throughout the system? or How are changes in the representational infrastructure coupled to the emergent reasoning about the physics? (I want to say that the second question does treat the system as the unit of analysis, but sort of places an emphasis on the student's role.)
3) What causes the redistribution? or What causes disruptions to the representational infrastructure?

Answering this first question, through my standard question types (written and verbal), I will be able to show what representations students are bringing forth that are historically and socially propagated forms. However, pursuing the second and third threads will allow me to show that there's a lot more to the student's understanding and reasoning than we might first anticipate by simply looking at the canonical representations elicited through standard question. This would come in conjunction with my analysis showing that the cognition is truly distributed across the system, and to ask questions about the student's understanding and reasoning, we must also answer how the reasoning/understanding is coupled to representation and representational change.

PIAB

- SW: A quantum particle is in a region of potential defined as $V(x)=\infty$ for $x = (-\infty, 0)$ and (L,∞), and $V(x) = 0$ for $x = (0,L)$. Sketch the wavefunction for the first allowed state, or ground state, of the particle.
    - SV/DV: What are the values of the wavefunction at the edges?
    - SV/DV: What about the peak?
    - *If students jump into heavy algebra, ask them how they would finish to save time.*
- SW: For the particle described in the previous problem, what are the first few allowed energies and wavefunctions of the particle?

    - *If the participant stacks the wavefunctions as in standard representation:*
        - SV/DV: Could you say more about why you plotted the wavefunctions vertically?
        - Does the vertical stacking represent something physical?
        - SV/DV: What do the vertical and horizontal axes represent?
    - SV/DV: Is there more you can say using this drawing?
        - SV/DV: The system, the particle, energy, anything you want!
- SW: Suppose a particle in the potential described in problem 1 was excited to the n=2 state. If you were to measure the position of the particle, what might you measure?
    - DW: Suppose you performed an experiment to measure the position of the particle in its n=2 state. You get a positive result at a specified location. Where was the particle right before you measured it?
- DW: Suppose you had a classical particle in a physical situation analogous to the quantum particle in the box. Consider a bead on a string, and the string is knotted at x = 0 and x = L so that the bead is confined between 0 to L, but can move smoothly and freely between these bounds. The bead has some energy, E, and can bounce elastically at the knotted ends. What is the wavefunction of this classical particle?
    - *If they are unsure about applying the wavefunction idea to a classical particle*

        - DV: Is there anything about the wavefunction idea that you can bring in to understand the motion of the classical particle? Is there anything about that kind of thinking that you can bring in here?
    - DW: Compare this wavefunction to that of the quantum particle in the ground state and in a very highly excited state.
- DW: Suppose a friend from your quantum class missed the lecture on the particle in a box. How would you explain it to your friend?
    - *If unclear on "it"*
        - DV: Essentially what you put for problem 1
- DW: How might you explain your response to problem 1 to a friend who's an English major?

Tunneling through a barrier (if time, keep separate)

- SW: A free quantum particle with energy E coming in from the left encounters a potential barrier of the form, $V(x) = 0$ from $(-\infty, 0)$ and (a, ∞). From (0,a), the potential is $V(x)= V_0$. (consider $E<V_o$)
    - SW: Sketch what the wavefunction of the particle will look like.
    - SW: If you were to measure the position of the particle, where are you *most likely* to find the particle?
    - DV: Can you describe what the particle is doing?
    - DV: Can you explain the relative heights of the different pieces of the wavefunction?

    - DV: Can you describe the probability of finding the particle inside the barrier?
- DW: What would the wavefunction of the particle look like if it had an energy above that of the potential barrier?
    - DV: Can you describe what the particle is doing?

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