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1. Compute the lifetime of the 21 cm hyperfine transition in hydrogen. (a) Find the decay rate for each of the three values of m_F. (b) If the atom has equal probability of starting with any m_F, what is the average lifetime?

2. Consider the matrix elements of the form
$$\langle \omega' J' M_J' L' S' | \hat{T} | \omega J M_J L S \rangle$$
where the states describe an atom and the operator $\hat{T}$ is either the electric dipole, orbital magnetic dipole, spin magnetic dipole, or electric quadrupole transition operator. Determine in each of these cases the selection rules for parity, J, M_J, and L. Make a table displaying the selection rules, and explain very briefly your reasoning.

3. This problem concerns spontaneous transitions of a hydrogen atom.

   (a) Make an argument using dimensional analysis and basic physics to determine how the total rate Γ for an electric dipole transition of hydrogen depends upon e, c, $\hbar$, the frequency ω of the emitted photon, and the dipole matrix element $\mathbf{d} = \langle f|\mathbf{r}|i\rangle$. **Explain your logic**. Do *not* just copy from your answer in part 3c.

   (b) Use your result from part 3a to make an order of magnitude estimate of the rate for the transition $2p \to 1s$. Next, compare this with the rate for the transitions $2p_{3/2} \to 2s_{1/2}$ and $2s_{1/2} \to 2p_{1/2}$. (The $2p_{3/2}$—$2s_{1/2}$ fine structure energy splitting is of order α^2 Rydbergs, and the $2s_{1/2}$—$2p_{1/2}$ Lamb shift is about ten times smaller than that.)

   (c) Using Fermi's golden rule, write down an explicit formula for the transition rate for an electric dipole transition between atomic states $|i\rangle$ and $|f\rangle$. Evaluate any integrals or sums except those implicit in the dipole matrix element. Verify that your result agrees with your result for part 3a.

   (d) Suppose an atom makes an electric dipole transition from a $2p,\ m_l = 1$ state to the $1s$ state. What is the polarization of the emitted photon if the photon emerges in the (i) $\hat{\mathbf{z}}$ direction, (ii) $\hat{\mathbf{x}}$-direction? Do this *both* by physical reasoning and by explicit computation.

   (e) Verify eqn. (2) below.

Possibly useful information:
$\alpha = e^2/\hbar c \simeq 1/137$
$\hbar \simeq 2/3$ eV-fs (1 fs = 10^{-15} s)
1 Rydberg = 13.6 eV

$$A^j(\mathbf{r},t) = \sum_{\mathbf{k},\lambda} \left(\frac{2\pi\hbar c}{Vk}\right)^{1/2} \left(a_{\mathbf{k},\lambda}\, \epsilon_\lambda^j\, e^{i\mathbf{k}\cdot\mathbf{r} - i\omega_\mathbf{k} t} + a^\dagger_{\mathbf{k},\lambda}\, \epsilon_\lambda^{j\,*}\, e^{-i\mathbf{k}\cdot\mathbf{r} + i\omega_\mathbf{k} t} \right) \tag{1}$$

where periodic boundary conditions in a box of volume V are assumed, and ϵ_λ, $\lambda = 1, 2$ are the two polarization vectors corresponding to a given wavevector $\mathbf{k}$. The combined integral over angles and polarization sum is

$$\int d^2\hat{\mathbf{k}} \sum_\lambda \epsilon_\lambda^j \epsilon_\lambda^{l\,*} = \frac{8\pi}{3}\delta^{jl}. \tag{2}$$