HW#7—Phys623—Spring 2016

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1. Consider the Hamiltonian $H = H_0 + \alpha(t)V$ where $\alpha(t) = 0$ if $|t| > \tau$.

(a) Write out the formal solution for $|\psi(t)\rangle$ using first order perturbation theory.

(b) Solve for $|\psi(t)\rangle$ *exactly* in the limit $\tau \to 0$ with $\gamma \equiv \int dt\, \alpha(t)$ fixed. (*Hint*: Make sure the evolution is unitary. You can't integrate the Schrodinger equation from $t = -\varepsilon$ to $t = \varepsilon$, because $|\psi(t)\rangle$ is discontinuous at $t = 0$.)

(c) Under what conditions does the first order perturbative result give a good approximation to the exact result for an impulsive perturbation?

2. Consider a perturbation $V(t) = e^{\eta t}V_0$. Show that if η is small enough the first order time-dependent shift of an eigenstate of a hamiltonian H_0 with discrete spectrum agrees with the first order shift of the state in time-independent perturbation theory. How small is "small enough"? (As $\eta \to 0$, the exact solution must coincide with an exact eigenstate of the hamiltonian $H_0 + V_0$. This is an instance of the *adiabatic theorem*.)

3. Schwabl, Problem 16.1

4. Schwabl, Problem 16.5. This asks for the exact solution. Make sure the probabilities for all n add up to 1. Add parts: (b) Compute the transition probabilities in first order time-dependent perturbation theory. (c) Under what conditions is the result from part (b) accurate? (d) Discuss and explain the behavior of the exact and approximate transition probabilities as a function ofΩ and $\Delta t \equiv t_2 - t_1$, paying attention to the interesting limits. (*Further hints for (a)*: The transition amplitude you're looking at for part (a) is $\langle n|U|0\rangle$, where $|n\rangle$ is the nth level of the unforced oscillator, and U is the evolution operator from t_1 to t_2 in the presence of the external force. You can solve exactly the equation of motion for the Heisenberg annihilation operator $a_H = U^\dagger a_S U$, and use the solution to express the transition amplitude in terms of $f(t)$ and $\langle 0|U|0\rangle$. Unitarity of U then allows you solve for $|\langle 0|U|0\rangle|$.)

5. A particle of mass m moves in a one-dimensional attractive potential $U(x) = -\lambda\delta(x)$, where $\delta(x)$ is the Dirac delta-function, and $\lambda > 0$. Use periodic boundary conditions with $x = \pm L$ identified, with $L \to \infty$.

(a) Find the wave function and the energy E_0 of the bound state. What is the parity of the wave function with respect to the operation $x \to -x$?

(b) Find the wave functions and the energies of the unbound states. Choose the wave functions to be symmetric or antisymmetric with respect to the parity operation $x \to -x$.

At time $t < 0$, the particle is in the ground state of the potential. At time $t > 0$, a small oscillating potential

$$V(t) = Ax^2 \sin(\omega t) \tag{1}$$

of frequency $\omega > |E_0|/\hbar$ is turned on.

(c) Which matrix elements of the perturbation (1) between the ground state and the symmetric or antisymmetric unbound states vanish because of the parity selection rule? Calculate the nonvanishing matrix elements.

(d) Using Fermi's golden rule, calculate the probability of transition of the particle from the ground state to an unbound state per unit time. Make sure the dimensionality of your final result is 1/time. (*Caution*: How should you compute the density of final states when the unbound states are not free particle states?)

(e) Discuss and interpret the behavior of the ejection rate as ω approaches ∞, and as ω approaches $|E_0|$.

(f) How would the result change if for the final states you used free particle states rather than the exact unbound states? How does the difference behave in the limit $\omega \to \infty$? Interpret your result.