- 1. The Dirac equation implies that the g-factor for the electron is  $g_e = 2$ . This result can also be obtained from the nonrelativistic limit of the Dirac Hamiltonian. For a charge e coupled to an electromagnetic vector potential  $\vec{A}(x)$  this nonrelativistic hamiltonian for the two-component wave function is  $H = [\vec{\sigma} \cdot (\vec{p} \frac{e}{c}\vec{A})]^2/2m$ . Show that this implies  $g_e = 2$ .
- 2. In section 14.1.3 Schwabl computes the Zeeman effect in hydrogen for an arbitrary magnetic field using the basis  $\{|n, j, m_j, l\rangle\}$  in which the spin-orbit term is diagonal but the Zeeman term is not. Redo the analysis using the basis  $\{|n, l, m_l, m_s\rangle\}$  in which conversely the Zeeman term is diagonal but the spin-orbit term is not, and check that you arrive at the same result for the energy levels.
- 3. The deuteron is the unique bound state of a neutron and a proton, both spin-1/2 particles. Since the strong interaction is short ranged, there must be a large S-wave component in the deuteron wave function, and in fact this wave function is a superposition  $a^{3}S_{1} + b^{3}D_{1}$ . Determine the fraction  $|b|^{2}$  of the *D*-wave component by calculating the magnetic moment and comparing with the observational value  $\mu = 0.85735\mu_{N}$  (where  $\mu_{N}$  is the nuclear magneton). The magnetic moment operator for the deuteron (neglecting the neutron-proton mass difference) is

$$\boldsymbol{\mu} = \frac{\mu_N}{\hbar} (0.5\mathbf{L} + 2g_p \mathbf{S}_p + 2g_n \mathbf{S}_n) \tag{1}$$

where  $g_p = 2.79275$  and  $g_n = -1.91315$ . The factor 0.5 in front of L occurs because the neutron does not contribute to the orbital part of the magnetic moment. The magnetic moment *value* is the expectation value of  $\mu_z$  in the state with maximal angular momentum in the z-direction. To evaluate the expectation value in the  $D_1$ -state, you can use the Clebsch-Gordan coefficients to write this state in terms of products of L and S eigenstates. Alternatively, you can use the *projection theorem* for vector operators. [I suggest that for practice you do it both ways, but that's not required.] (If you use the projection theorem method, I think you'll want to show as a lemma that  $\mathbf{S}_p$  and  $\mathbf{S}_n$  have equal expectation values in an eigenstate of  $\mathbf{S}^2$ , which can be shown using the projection theorem for vector operators with respect to the total spin.) Answer:  $|b|^2 \approx 0.04$ .

- 4. The structure of nuclei can be approximately described using the so-called *shell model*. Interacting nucleons (protons and neutrons) produce a self-consistent, spherically symmetric field of the nuclear force. The energy levels of a nucleon in this field can be classified by the values of the orbital angular momentum, the radial quantum number, and the total angular momentum of the nucleon. This is somewhat similar to the classification of electron energy levels in an atom. The neutrons and protons are fermions and therefore each obey the Pauli exclusion principle. The nuclei with completely filled energy shells—"(doubly) magic nuclei"—are particularly stable, similarly to the noble elements with completely filled electron shells. In addition to the common nuclear potential, each nucleon has a spin-orbit coupling  $-2a\mathbf{L} \cdot \mathbf{S}$ , where *a* is a positive constant.
  - (a) How many nucleons can be placed in the lowest 1S shell of a nucleus? What is the name of this particle?
  - (b) The first two levels of the nuclear potential are 1S, 1P (where "1" stands for the radial quantum number.) What is the second magic nucleus? Explain how the counting goes.

(c) Use the shell model with spin-orbit coupling to predict the spin and parity of the nuclei

$$_{1}H^{2}, _{1}H^{3}, _{3}Li^{7}, _{5}B^{11}, _{7}N^{15}$$

(where the subscript denotes the number of protons and the superscript the number of nucleons). Check your prediction by looking up the answer. For  $_1H^2$  there is more than one possibility, so list them all, and then select one by using the fact that, as a result of the "tensor force", the neutron and proton form a state that is antisymmetric under n-p interchange (i.e. an "isospin singlet" state). For  $_3Li^7$  there is also more than one possibility, so list them all, and then select that the neutrons in the outer shell form a J = 0 pair.

5. Calculate the magnetic moment (in nuclear magnetons  $\mu_N$ ) of the  ${}^{15}N$  nucleus, for which one proton in a  $P_{1/2}$  state is missing from a closed shell. (The *measured value* is -0.28.)

For the purposes of this calculation you can treat the hole state as a single spin-1/2 particle state. [The relevant general result justifying this is discussed in point 8 of the Tensor Operators supplement. I think in the present case you could argue that the 1-hole state can be combined with a single spin-1/2 particle state to form a singlet, so the 1-hole state must "behave as" a single spin-1/2 particle state. But this argument seems a bit too facile, because there is a potential sign, or phase factor that could enter. Eqn. (15) in the Supplement indicates that only for odd integer values of k is the sign in the particle-hole equivalence for the expectation values equal to 1. For a vector operator, like the magnetic moment, k = 1, which is indeed an odd integer. Please let me know if you can think of a simpler, direct argument justifying the + sign in the present case.]