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1. Write the $2p_{1/2}(F = 0)$ state of hydrogen in terms of the product kets $|2p, m_l\rangle|m_s\rangle|m_I\rangle$. (Here $F = J + I$ is the total angular momentum of the atom, where I is the nuclear spin and J is the total angular momentum of the electron.)

2. Consider the product $1 \otimes 1$ of two spin-1 representations of the rotation group.

(a) Find the state $|0\rangle$ with $J = 0$ in terms of the states $|mm'\rangle$. Do this from first principles, by imposing $J_z|0\rangle = 0$ and $J_\pm|0\rangle = 0$.

(b) Compute from scratch the Clebsch-Gordan coefficients for all of the irreps (irreducible representations) in $1\otimes 1$. Check your results against a table of CG coefficients or Mathematica.

3. A deuteron is the unique bound state of a neutron and a proton, both spin-1/2 particles.

(a) Using only the additional fact that the deuteron has total angular momentum $J = 1$ ("spin 1"), what would be the possible values of ${}^{2S+1}L_J$ in the deuteron?

(b) Which combinations of the ${}^{2S+1}L_J$ found in part 3a could occur in the deuteron, given that parity is a symmetry of the nuclear hamiltonian? Justify your answer.

4. The ground state of the hydrogen atom is split into two hyperfine states separated by 1.42 GHz. What is the hyperfine splitting in the deuterium atom? The respective magnetic moments are $\mu = 2.8\mu_N$ and $\mu_d = 0.86\mu_N$, where μ_N is the nuclear magneton.

5. For a particle of zero mass, the Dirac equation falls apart into two independent equations,

$$i\partial_t\chi = \pm\vec{\sigma}\cdot\vec{p}\,\chi,$$

where $\vec{\sigma}$ is the vector of Pauli matrices, χ is a two-component spinor, and $\vec{p}$ is the usual quantum mechanical momentum operator. This is called the Weyl equation. It is the time evolution equation for a quantum system with Hamiltonian $H = \pm\vec{\sigma}\cdot\vec{p}$. The two possible signs correspond to the "chirality" of the massless spin-1/2 particle, called right-handed (+) and left-handed (−).

(a) Use dimensional analysis to restore the factors of $\hbar$ and c in the Weyl equation (but after this part set $\hbar = c = 1$).

(b) Establish the following:

i. The Hamiltonian is translation invariant, and a momentum eigenstate is an eigenstate with energy E satisfying $E^2 = |\vec{p}|^2$.

ii. The Hamiltonian is invariant under time reversal, but not under parity (space inversion).

iii. The "helicity" $\frac{\hbar}{2}\vec{\sigma}\cdot\hat{p}$ (spin along the direction of momentum) of a momentum eigenstate with positive energy is $\pm\hbar/2$, where the sign corresponds to the chirality defined above.

iv. The Heisenberg velocity operator $d\vec{x}/dt$ is $\pm\vec{\sigma}$.