Prof. Ted Jacobson
PSC 3151, (301)405-6020
jacobson@physics.umd.edu

1. The scattering amplitude for neutrons of energy E incident on a certain species of heavy nuclear target is given to a good approximation by $f(\theta) = A + B\cos\theta$.

(a) For approximately what range of energies E could this be true?

(b) What is the s-wave phase shift?

(c) If the incident beam has a number flux I, how many neutrons per unit time are back-scattered into a small solid angle $\Delta\Omega$ about the backward direction $\theta = \pi$?

2. Consider scattering of a particle of energy E and mass m from the potential $V(r) = V_0\theta(a - r)$, where V_0 can have either sign.

(a) Give an "observational" definition of the differential scattering cross section $d\sigma/d\Omega$.

(b) Express the differential cross section in terms of the scattering amplitude.

(c) State two different conditions on (E, m, V_0, a) which are independently sufficient for validity of the Born approximation. One condition should depend on E and the other should not.

(d) State the conditions on (E, m, V_0, a) for s-wave scattering to dominate the partial wave expansion. What is the angular dependence of the scattering amplitude in this case?

(e) Find an exact transcendental equation for the s-wave phase shift δ_0. Write the scattering amplitude in the s-wave approximation in terms of δ_0.

(f) Find the scattering amplitude using the Born approximation in the common domain of validity of the Born and s-wave approximations.

(g) Show that in the common domain of validity of the Born and s-wave approximations, your results for parts 2f and 2e agree. (*Hint*: You may wish to use the expansion $\tan x = x + x^3/3 + \cdots$.)

3. You showed in HW#10 that the first Born approximation gives a finite, nonzero cross section for a delta-function potential, while the second Born approximation is infinite. (a) Now use the partial wave expansion to show that the exact cross section is *zero* (!) for the delta-function limit of the spherical square potential $V(r) = V_0\theta(a - r)$ (i.e. the limit $a \to 0$, $V_0 \to \infty$, with $\int d^3x V(\vec{x})$ held fixed). (b) Show that if, instead of the delta-function limit, V_0 times the *surface area* of the potential is held fixed, then the limit is finite and *nonzero*.