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1. Consider scattering of a particle of mass m by a spherical square well

$$V(r) = -V_0 \theta(a - r). \tag{1}$$

    (a) For which energies is the Born approximation valid for a nucleon scattering from a nuclear potential for nucleon number A, approximated by a square well with with $a \sim A^{1/3}$ fm and $V_0 \sim 50$ MeV? (*Note:* $\hbar c \approx 200$ MeV-fm and $mc^2 \sim 1$ GeV for a nucleon.)

    (b) Find the differential and total cross sections for the potential (1) in the Born approximation.

    (c) Study and discuss fully the behavior in the limits $ka \ll 1$ and $ka \gg 1$ (within the validity of the approximation). Try to explain this behavior, and relate it to the general physical features expected for scattering from any potential. How does the cross section depend on angle and energy at low and high energies? At small angles? How about the total cross section?

2. (*a*) Show that the Born approximation is not valid for a delta-function potential $V(\mathbf{r}) = A\delta^3(\mathbf{r})$ in the sense that it is not valid for the regularized potential $V_a(r) = A\theta(a - r)\left((4/3)\pi a^3\right)^{-1}$ in the limit $a \to 0$. (*b*) Show that the second term in the Born series (iterative solution of the time-independent Schrodinger equation) diverges for a delta-function potential. (*c*) It is nevertheless sometimes accurate to model potentials by delta-functions for use with the first Born approximation. Characterize the potentials and energies for which this gives a good approximation.

3. A particle of mass m scatters from a spherically symmetric "Yukawa potential"

$$V(r) = \frac{A}{r}e^{-r/a}, \quad \text{with } A \text{ a constant.} \tag{2}$$

    (a) Find all conditions (on A, a, m, k) under which the Born approximation is valid.

    (b) Find the differential and total cross sections in the Born approximation.

    (c) Study and discuss fully the behavior in the limits $ka \ll 1$ and $ka \gg 1$ (within the validity of the approximation). Try to explain this behavior, and relate it to the general physical features expected for scattering from any potential. How does the cross section depend on angle and energy at low and high energies? At small angles? How about the total cross section?

4. The interaction energy between a neutron and an atom in a ferromagnet can be approximated for slow neutrons using the first Born approximation with

$$V(\mathbf{r}) = (A + B\hat{n} \cdot \vec{\sigma})\delta^3(\mathbf{r}) \tag{3}$$

where $\hat{n}$ is a unit vector in the direction of the magnetization of the atom.

    (a) Calculate the scattering cross-section for the two possible spin orientations of a beam of slow neutrons in the Born approximation.

    (b) Assume the incoming beam is unpolarized, and find the polarization $P = (I_+ - I_-)/(I_+ + I_-)$ of a beam of slow neutrons that has passed through a block of completely magnetized iron of thickness L and with N atoms per unit volume. Here I_+ and I_- refer to the intensities of the transmitted (i.e. unscattered) beams with spins parallel and antiparallel to the magnetization.