

1. Problem 11.1, *Schwabl*. (Use units with $\hbar = m = \omega = 1$, and use Schwabl's hint.)
2. Problem 11.3, *Schwabl*. (Use the x and y ladder operators. N.B. also $\omega = 1$.)
3. Consider the Hamiltonian $H = H_0 + V$ of a three state system, with

$$H_0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad V = \begin{pmatrix} 0 & 0 & v \\ 0 & 0 & v \\ v & v & 0 \end{pmatrix}. \quad (1)$$

[2+1+5+2=10 *points*]

- (a) Find the exact eigenvalues and eigenvectors of H .
 - (b) Expand the exact eigenvalues and eigenvectors to order v^2 .
 - (c) Use degenerate perturbation theory (with V as the perturbation) to find the first and second order energy shifts $E^{(1)}$ and $E^{(2)}$, and check that they agree with part 3b.
 - (d) Find the eigenvectors of the second order secular equation and compare with the $v \rightarrow 0$ limit of the exact eigenvectors. They should agree.
4. Consider a two-state quantum system described by the Hamiltonian

$$H = \begin{pmatrix} E + U & \Delta e^{i\phi} \\ \Delta e^{-i\phi} & E - U \end{pmatrix}, \quad (2)$$

with E , U , Δ , and ϕ all real. This is the most general hermitian 2×2 matrix.
[2+1+1+3+3=10 *points*]

- (a) Find the exact eigenvalues and eigenvectors of H . (*Tip*: Expand H in Pauli matrices and use what you know about them.)
- (b) Sketch the eigenvalues as functions of U when U changes from $U \ll -\Delta$ to $U \gg \Delta$. Notice that the energy levels “repel” in the region $U \approx 0$ where they would cross if Δ were zero.
- (c) Expand the exact eigenvalues to lowest nonvanishing order in U/Δ when $U \ll \Delta$.
- (d) Considering the Δ terms of the Hamiltonian (2) as a perturbation, compute the first and second order energy level shifts using non-degenerate perturbation theory (assume $U \neq 0$.)
- (e) The approximate eigenvalues of parts (4c) and (4d) do not agree when $0 < U \ll \Delta$. Explain why non-degenerate perturbation theory does not give good results even though the unperturbed eigenvalues are non-degenerate when $U \neq 0$.

5. In most calculations of atomic energy levels the nucleus is taken as a positive point charge Ze . Actually, the nuclear charge is more accurately represented by a uniform charge distribution reaching to a radius of about $Z^{1/3}$ Fermi. (1 Fermi = 10^{-13} cm = 2×10^{-5} Bohr radius.) [6+2+1+1=10 *points*]
- (a) Calculate the correction to the energy of a 1s electron due to this nuclear size effect. How does this correction depend on the nuclear charge Z ? (*Note:* You can simplify the calculation by noting that the nucleus is much smaller than the Bohr radius, so the wave function is approximately constant inside the nucleus.)
 - (b) What is the ratio of the nuclear size effect to the hyperfine splitting of the 1s level as a function of Z and the nuclear mass M_N ? (See (12.40) of Schwabl.) What is the ratio for hydrogen? Assuming $M_N \sim 2Zm_{\text{proton}}$, $I = 1/2$, and $g_N \sim 1$, at roughly what value of Z do these effects become comparable?
 - (c) What is the ratio of the nuclear size effect to that of the Darwin term? (See (12.22) of Schwabl.)
 - (d) Why is the shift for the 2p state is negligible compared to that for the 1s state?