Now about $\frac{1}{2}\mathcal{L}_n h_{ab}$:

3) $\frac{1}{2}\mathcal{L}_n h_{ab} = \frac{1}{2}\mathcal{L}_n (g_{ab} + n_a n_b) = \frac{1}{2}\Big[(\nabla_a n_b + \nabla_b n_a) + n^c \nabla_c (n_a n_b) +$

$$+ \underbrace{(\nabla_a n^c)}_{0} n_c n_b + \underbrace{(\nabla_b n^c)}_{0} n_a n_c \Big] =$$

$$= \frac{1}{2}\Big[\nabla_a n_b + \nabla_b n_a + \cancel{(n^c \nabla_c n_a) n_b} + \cancel{n^c n_a (\nabla_c n_b)}\Big] =$$

$$= \frac{1}{2}\left[K_{ab} + K_{ba}\right] = K_{ab}.$$

$$\delta_a{}^c \nabla_c n_b = h_a{}^c \nabla_c n_b - \cancel{n_a n^c \nabla_c n_b}$$

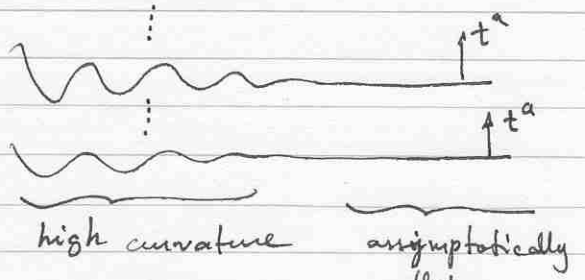
$$\delta_b{}^c \nabla_c n_a = h_b{}^c \nabla_c n_a - \cancel{n_b n^c \nabla_c n_a}$$

Boundary of space

In general a space time has no ~~the~~ time like killing vector so it's hard to define energy. But if spacetime is assymptotically flat at infinity, poincaré group acts as a symmetry group at infinity

So we can ~~define~~ relate: Energy $\longleftrightarrow$ generator of assymptotically time translations

If we choose t^a to be the assymptotic time translation symmetry, than <u>Hamiltonian must be the energy</u>



$$\begin{cases} h_{ab} \longrightarrow \delta_{ab} + \mathcal{O}(\frac{1}{r}) \\ \pi^{ab} \longrightarrow \mathcal{O}(\frac{1}{r^2}) \end{cases}$$

from Hamilton's eqn:

$$\dot{\pi}^{ab} = \{\pi^{ab}, H\} = \left\{\pi^{ab}, \int -N_\perp {}^3R\sqrt{h}\, d^3x\right\}$$

integrating by parts and comparing to $\dot{\pi}^{ab} = -\frac{\delta H}{\delta h_{ab}}$, we see we need to add boundary terms:

$$H = \int (\underbrace{N_\perp \mathcal{H}_\perp}_{0} + \underbrace{N_a \mathcal{H}^a}_{0})\, d^3x + \oint_{\lim R\to\infty} (\underbrace{h_{ij,i} - h_{ii,j}}_{\text{cartesian coords}})\, d^2S^j \equiv M_{ADM}$$