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$$H = \int (N_\perp \mathcal{H}_\perp + N_a \mathcal{H}^a)\, d^3x$$

$$\mathcal{H}_\perp = \sqrt{h}\left[h^{-1}\left(\pi^{ab}\pi_{ab} - \tfrac{1}{2}(\pi^a{}_a)^2 - {}^3R\right)\right]$$

$$\mathcal{H}^a = -D_b \pi^{ab}$$

So $\mathcal{H}_\perp$ and $\mathcal{H}^a$ should have weight 1

Densities:

$\int_R \sqrt{h}\, d^3x$: geometric volume (invariant)

$\int_R f(x)\sqrt{h}\, d^3x$: invariant.

$\sqrt{h}$ — scalar density of weight 1

1) π^{ab} is a tensor density of wt. 1 :

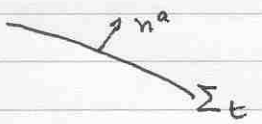
$$\pi^{ab} = \frac{\delta L}{\delta h_{ab}} = \underset{\text{wt. 1}}{\sqrt{h}}\,\underbrace{(K^{ab} - K h^{ab})}_{\text{tensor of wt. 0}}$$

$\longrightarrow$ But just by the definition of the variation of a scalar:

$$\delta L = \int \underbrace{\frac{\delta L}{\delta h_{ab}} \delta h_{ab}\, d^3x}_{}$$

must be scalar density of wt. 1 $\Longrightarrow$ $\frac{\delta L}{\delta h_{ab}}$ has wt. 1

2)



$h_{ab} = g_{ab} + n_a n_b$, purelly spatial tensor. In the Hamiltonian formalism, we restrict g_{ab} to Σ_t to obtain a 3-D Riemannian metric $(+++)$.

3) Why "$\bullet$" $= \mathcal{L}_t$?

$$\begin{cases} \dot{h}_{ab} = \mathcal{L}_t h_{ab} = \dfrac{\delta H}{\delta \pi^{ab}} \\ \dot{\pi}^{ab} = \mathcal{L}_t \pi^{ab} = -\dfrac{\delta H}{\delta h_{ab}} \end{cases}$$

In a coord (t, x^i),

$$\mathcal{L}_{\partial_t} = \partial_t = \text{"}\bullet\text{"}$$

See that, $t^c \nabla_c h_{ab}$ depends on the dynamical variables $(\nabla = \nabla(g))$ whereas $\mathcal{L}_t$ does not.

also, $$\delta H = \int \underset{\text{wt. 0}}{\frac{\delta H}{\delta \pi}}\, \underset{\text{wt 1}}{\delta\pi}\, d^3x$$

$\mathcal{H}^a$ - spatial diffeomorphism constraint.

$$\int N_a \mathcal{H}^a d^3x = \int -N_a D_b \pi^{ab} d^3x = \int \Big[(D_a N_b)\pi^{ab} - \underbrace{D_a(\underbrace{N_b \pi^{ab}}_{\text{vector density.}})}_{\text{surface term}}\Big] d^3x =$$

$$= \tfrac{1}{2}\int \mathcal{L}_N h_{ab}\, \pi^{ab}\, d^3x .$$