$$n^a = \frac{1}{N}(t^a - N^a)$$

$$K_{ab} = \frac{1}{2}\mathcal{L}_n h_{ab} = \frac{1}{2}\mathcal{L}_{\frac{1}{N}(t^a - N^a)} h_{ab} = \frac{1}{2N}\mathcal{L}_{t^a - N^a} h_{ab} = \frac{1}{2N}\mathcal{L}_t h_{ab} - \frac{1}{2N}(D_a N_b - D_b N_a),$$

where D_a is the spatial covariant derivative operator determined by h_{ab}.

So: $$\boxed{K_{ab} = \frac{1}{2N}\dot{h}_{ab} - \frac{1}{N}D_{(a}N_{b)}}$$

$$\Pi_{ab} = \frac{\delta L}{\delta \dot{h}_{ab}} = \sqrt{h}\,(K^{ab} - K h^{ab})$$

constraints: $$\begin{cases} \Pi_N = 0 \\ \Pi_{N_a} = 0 \end{cases}$$

therefore: $$\mathcal{H} = \Pi^{ab}\dot{h}_{ab} - \mathcal{L} = N\mathcal{H}_\perp + N_a\mathcal{H}^a,$$ where

- $\mathcal{H}_\perp = \sqrt{h}\left[\frac{1}{h}\left(\Pi^{ab}\Pi_{ab} - \frac{1}{2}\Pi^2\right) - {}^{(3)}R\right]$
- $\mathcal{H}_a = -D_a\Pi^{ab}$

$\mathcal{H}_\perp$ is the Hamiltonian constraint

$\mathcal{H}_a$ is the momentum constraint

↳ compares with $\partial_a E^a = 0 \to -\partial_a \Pi^a = 0$

conj. mom. to A_a

compares to $(p^2 - m^2)$ in relativistic particle plus a potential term