$g_{ab} = h_{ab} - n_a n_b$, $n^a = \frac{t^a - N^a}{N}$. therefore :

$$\boxed{g^{ab} = h^{ab} - n^a n^b = h^{ab} - \frac{1}{N^2}(t^a - N^a)(t^b - N^b)}$$

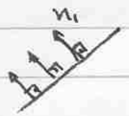
$g_{ab} \longleftrightarrow (h_{ab}, N, N_a)$, where $N_a \equiv N^b h_{ab}$ (like in Wald)

N - lapse function

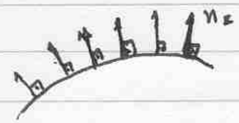
N_a - shift vector

G.R. Lagrangean: $\mathcal{L} = \sqrt{-g}\,{}^{(4)}R = \sqrt{h}\,N\left[{}^{(3)}R + K_{ab}K^{ab} - (K^a{}_a)^2\right]$

- ${}^3R$ - Ricci scalar of the 3-surface : ${}^3R = {}^3R(h)$
- K_{ab} - extrinsic curvature of the 3-surface embedded in 4D. caracterizes how a lower dimension manifold (submanifold) bends in embedding space.



$\nabla n_1 = 0$ ← no extrinsic curvature



$\nabla n_2 \neq 0$ ← has non zero extrinsic curvature

} embedding in flat 2D space

n^a - unit normal vector field.

$\boxed{K_{ab} = h_a{}^c \nabla_c n_b}$ derivatives on the direction of surface

depends only upon n_b on ~~the~~ the surface

$h_a{}^b$ - spatial projector.

One can ~~prove~~ show that

$\mathcal{L}_n h_{ab} = \mathcal{L}_n (g_{ab} + n_a n_b) =$

$= \nabla_a n_b + \nabla_b n_a = 2K_{ab}$.