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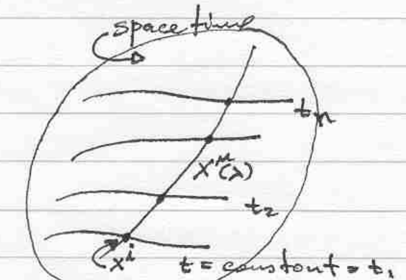
Hamiltonian description of GR

$\nabla^\mu G_{\mu\nu} = 0$, $G_{0\nu}$ has only 1st derivatives of X^0 : 4 initial value constraints from $G_{0\nu}$.

Arbitrary slicing of space-time



- easiest way: pick ~~an~~ coordinate system (t, x^i) and in general $x^a \to x'^a(x^i)$

- But can make a "less" coordinate dependent formulation, avoid choosing x^i.

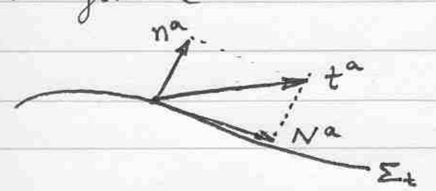
- choose t : time function. (t = constant, are spacelike surfaces)

- choose t^a : time flow vector field. (t^a is time-like)

such that $\boxed{t^a \nabla_a t = 1}$, t^a is the dual vector associated to the covector $\partial_a t$. In the same way that $\underline{\partial_t}$ and $\underline{dt}$ are dual : $\langle \partial_t, dt \rangle = 1$

In special coords: $\partial_t = t^a \partial_a \Rightarrow (1,0,0,0)$
$dt = \partial_a t \, dx^a \Rightarrow (1,0,0,0)$

In general:



n^a - unit time like normal to Σ_t (the time slice), N^a is arbitrary and $N^a n_a = 0$

$$\boxed{t^a = N n^a + N^a}$$

Relation between N, n^a and $\nabla_a t$:

t is scalar function, $\nabla_a t = \partial_a t$ is a 1-form field, the gradient of t. And $\partial_a t$ is normal to Σ_t. Therefore $n_a = c\, \partial_a t$.

* Define: $\boxed{n_a = -N \partial_a t}$, so:

$t^a \partial_a t = g^{ab}(-N^2 \partial_a t\, \partial_b t) = N^2 |\nabla_a t|^2 = 1$

$$\boxed{N = |\nabla_a t|^{-1}}$$

metric splitting:

$$\boxed{h_{ab} = g_{ab} + n_a n_b}$$

$h_{ab} V^a W^b = g_{ab} V^a W^b$ for V, W tangent to Σ_t

$h_{ab} n^b = (g_{ab} + n_a n_b) n^b =$
$= n_a - n_a = 0$.

In special (orthonormal) coords adapted to n^a : $h_{ij} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & & & \\ 0 & & g_{ij} & \\ 0 & & & \end{pmatrix}$

h_{ab} is called a spatial tensor.