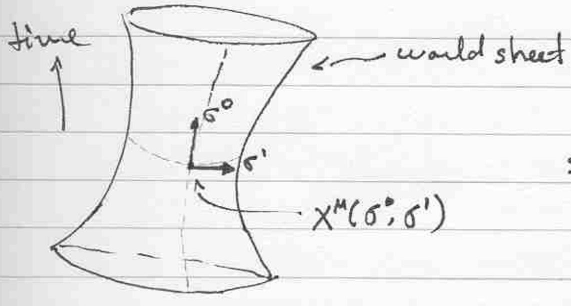
Relativistic String

Particle:

$$s = m\int d\tau = m\int\sqrt{\dot X^\mu \dot X_\mu}\,d\lambda \longrightarrow S[e,X] = \frac{m}{2}\int\left[e^{-1}\dot X^\mu \dot X^\nu g_{\mu\nu} + e\right]d\lambda.$$

omit e for massless case



$S = T\int d^2A$: Nambu - Goto action (nothing more than the worldsheet area)

string tension

index: $a,b,\ldots = 1,2$

$\mu,\nu\ldots = 1,2,3,4$ $\qquad \partial_a X^\mu \longrightarrow$ tangent vector to the worldsheet

in analogy to the particle, we set:

$$S[X^\mu, h^{ab}] = \frac{T}{2}\int\sqrt{h}\,h^{ab}\underbrace{\partial_a X^\mu \partial_b X^\nu g_{\mu\nu}(x)}_{\text{call it } H_{ab}}\,d^2\sigma \quad \longleftarrow \text{Polyakov action}$$

(~~[illegible]~~)

(non-linear σ model)

if replace $h_{ab} \to \Omega^2 h_{ab}$ (a Weyl or conformal transformation)

so, $\det h \longrightarrow \Omega^4 \det h$, $\quad h^{ab} \longrightarrow \Omega^{-2}h^{ab}$

$\sqrt{h} \longrightarrow \Omega^2\sqrt{h}$ $\quad \boxed{\sqrt{h}\,h^{ab} \longrightarrow \sqrt{h}\,h^{ab}}$ invariant

Symmetries:

- Reparametrization invariance (on the worldsheet: $\sigma^i \to \sigma'^i(\sigma^j)$)
- Weyl ~~[illegible]~~ invariance

try to eliminate h_{ab}: $\quad \dfrac{\delta S}{\delta h_{ab}} = 0 \Longrightarrow \boxed{H_{ab} - \frac{1}{2}h_{ab}\left(h^{cd}H_{cd}\right) = 0}$

so: $h_{ab} = \dfrac{2}{\mathrm{tr}(H_{ab})}H_{ab}$, $\quad |h| = \dfrac{4\,|H|}{(\mathrm{tr}\,H_{ab})^2}$

Like in GR:

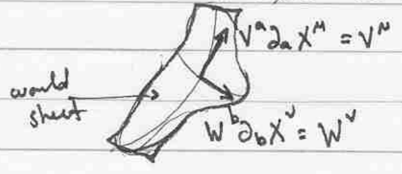
$$\frac{\delta}{\delta g_{\mu\nu}}\int d^4x\sqrt{-g}\,g^{\mu\nu}R_{\mu\nu} = (R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R)$$

$$S = \frac{T}{2}\int\sqrt{h}\,\mathrm{tr}\,H_{ab} = \frac{T}{2}\int\frac{2\sqrt{H}}{(\mathrm{tr}\,H)}\,\mathrm{tr}\,H = \frac{T\,2}{2}\int\sqrt{H}\,d^2\sigma =$$

$$= T\int\sqrt{H}\,d^2\sigma \text{, Nambu-Goto action.}$$

Hab is the induced metric:

- the restriction of $g_{\mu\nu}$ to the worldsheet



$$g_{\mu\nu}V^\mu W^\nu = g_{\mu\nu}V^a\partial_a X^\mu\, W^b\partial_b X^\nu = (g_{\mu\nu}\partial_a X^\mu \partial_b X^\nu)V^a W^b =$$

$$= H_{ab}V^a W^b.$$