3/10/03

last time: Hamiltonian formulation of

- relativistic particle: $H = \frac{e}{2m}\left(p^\mu p_\mu - m^2\right)$, $\{x^\mu, p_\nu\} = \delta^\mu{}_\nu$

- EM field: $H = \int d^3x \left\{\frac{1}{2}(\Pi^2 + B^2) + A_0 \partial_i \Pi^i\right\}$, $\{A_i(x), \Pi^j(y)\} = \delta_i{}^j\, \delta^3(x-y)$

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Constraints $\longleftrightarrow$ Symmetries : arise from and generate them

Hamilton's eqn: $\dot q = \{q, H\}$

$\dot p = \{p, H\}$ , $\{f, g\} = \frac{\partial f}{\partial q}\frac{\partial g}{\partial p} - \frac{\partial f}{\partial p}\frac{\partial g}{\partial q}$

EM: $A_0$ – lagrange multiplier, variation $\longrightarrow \partial_i \Pi^i = 0$ (gauss' law), $\vec E = -\vec \Pi$.

$$\dot A_i(x) = \{A_i(x), H\} = \Pi^i + \left\{A_i(x), -\int \partial_j A_0(y)\, \Pi^j(y)\, d^3y\right\} = \Pi_i - \boxed{\partial_i A_0(x)}$$

gauge transformation of $A_i$.

so $H_{constr} = \int A_0 \partial_i \Pi^i d^3x$ generates the symmetry transf.

$$\{A_i, H_{constr}\} = -\partial_i A_0$$

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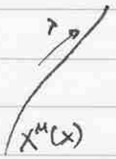
Relativistic particle: 


$H_{constr} = H = \frac{e(\lambda)}{2m}(p^2 - m^2) \Longrightarrow$ for relat. particle, evolution corresponds to a "gauge transformation", i.e.: reparametrization !

$$\dot X^\mu(\lambda) = \left\{X^\mu, \frac{e}{2m}\left(p^\nu p_\nu - m^2\right)\right\} = \frac{e}{m} p^\mu \ , \quad p^\mu = \frac{m}{e}\frac{dX^\mu}{d\lambda}$$

if using proper time: $p^\mu = m\frac{dX^\mu}{d\tau} \xrightarrow[\text{general}]{\text{in}} p^\mu = m\underbrace{\left(\frac{d\lambda}{d\tau}\right)}_{e^{-1}}\frac{dX^\mu}{d\lambda}$

$e^{-1} \longleftarrow$ arbitrary function of $\lambda$.