last time 3/6/3

Integration of differential forms.

Stoke's theorem : $\int_{\partial R} \omega = \int_R d\omega$

magnetic monopole :

$F = dA$, $\Phi = \oint_E (A_N - A_S) = \oint_E d\theta = 0$ unless θ is discontinuous on the equator (E)

in generic gauge theory :

$F = dA \longrightarrow F = dA + A\wedge A$, $DF = 0 \Rightarrow dF + [A,F] = 0$. So we don't have a

$d \longrightarrow D = d + [A,\cdot]$, contradiction but still need to have charge quantization.

Hamiltonian formulation of GR

plan : Relativistic particle $\longrightarrow$ EM field $\longrightarrow$ relativistic string $\longrightarrow$ GR

In general :

$S = \int L\, dt$, $p = \frac{\partial L}{\partial \dot q}$, so $S = \int (p\dot q - H)\, dt \xrightarrow[\text{p and q}]{\text{varying}} \begin{cases} \dot q = \frac{\partial H}{\partial p} \\ \dot p = -\frac{\partial H}{\partial q} \end{cases}$

exchange $(q,\dot q) \leftrightarrow (p,q)$. $H = p\dot q - L$

• Relativistic particle $(+---)$: Action is the proper length (time duration) of world-line :

$$S = m\int ds = m\int \sqrt{g_{\mu\nu}\dot x^\mu \dot x^\nu}\, d\lambda = m\int \sqrt{g_{\mu\nu}dx^\mu dx^\nu}$$ parametrization independent.

$L = m\sqrt{g_{\mu\nu}\dot x^\mu \dot x^\nu}$

$p_\mu = \frac{\partial L}{\partial \dot x^\mu} = \frac{m\, g_{\mu\nu}\dot x^\nu}{\sqrt{g_{\alpha\beta}\dot x^\alpha \dot x^\beta}}$, $\boxed{p^2 = g^{\mu\nu}p_\mu p_\nu = \frac{m^2 g_{\mu\nu}\dot x^\mu \dot x^\nu}{g_{\alpha\beta}\dot x^\alpha \dot x^\beta} = m^2 : \text{constraint on momenta}}$

it's related to the fact that $x^\mu(\lambda)$ as a soln. still has freedom of reparametrization.

⊕ Problems to find Hamiltonian : L is "homogeneous in first degree" $\left(L = \dot x^\mu \frac{\partial L}{\partial \dot x^\mu}\right)$ what implies $H \equiv 0$ (Goldstein, pp 358, also easy to check)

- could handle this (the constraint) by fixing $\lambda = \tau$ (gauge fixing of rep. inv.)
- could use a lagrange multiplier $\lambda(p^2 - m^2)$ on action

- Instead :