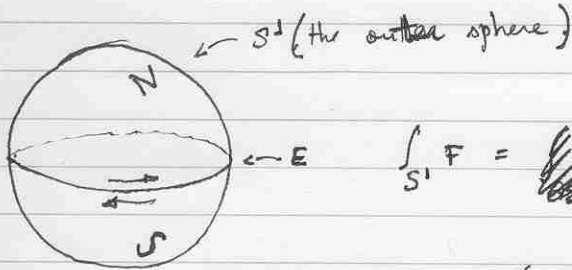
But even though for black-holes (or topologically no trivial spaces) we could have $\Phi_B \neq 0$, there are some constraints:



$$\int_{S^1} F = \int_{S^1} dA = \int_{\partial S^1} A = 0$$, but also:

$$= \int_N F + \int_S F = \int_N dA + \int_S dA = \int_{\partial N} A_N + \int_{\partial S} A_S =$$

$\partial N = E$

$\partial S = -E$

$$= \int_E A_N - \int_E A_S = \int_E (A_N - A_S) = 0$$

$\underline{\underline{\text{if } A_N = A_S}}$.

Even though the fact that $dF = 0$ in a blackHole space would allow $\Phi_B \neq 0$, the fact that $F = dA$ seems to impose $\Phi_B = 0$,

Unless we make A a discontinous function at the equator E. (using a gauge transformation).

$$A_N = A_S + d\Lambda \quad , \text{ so: } \int_{S^1} F = \int_E (A_N - A_S) = \int_E d\Lambda = \Lambda(2\pi) - \Lambda(0) = \Phi_B .$$

so we make Λ a discontinous function at E.

But quantum mechanics impose a new constraint. ψ, the wave function, must be continuous at E:

$$\psi \rightarrow \psi \, e^{i\frac{q\Lambda}{\hbar c}} \Longrightarrow \frac{q(\Lambda(2\pi) - \Lambda(0))}{\hbar c} = 2\pi n$$

$$\frac{q}{\hbar c} 4\pi q_{mag} = 2\pi n \longrightarrow \boxed{q \cdot q_{mag} = \frac{\hbar c}{2} n}$$

Dirac quantisation condition.