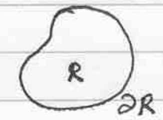
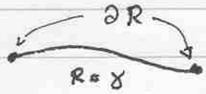
Stockes theorem

$$\int_{\partial R} \alpha_p = \int_R d\alpha$$

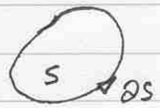


1. dim (0-forms) :



$$\int_\gamma df = \int_{\partial R} f = f(b) - f(a)$$

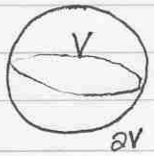
2. dim (1-forms) :



$$\oint_{\partial S} A = \int_S dA$$, in euclidian space ($\mathbb{R}^3$) :

$$\oint_{\partial S} \vec{A} \cdot ds = \int_S (\nabla \times \vec{A}) \cdot d\vec{a} .$$

3. dim (2-form) :



$$\int_{\partial V} B = \int_V dB$$, in E^3 :

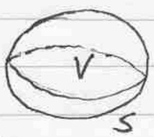
$$\int_{\partial V} \vec{B} \cdot d\vec{a} = \int_V \nabla \cdot B \, dV$$ (gauss' law) .

Magnectic charges.

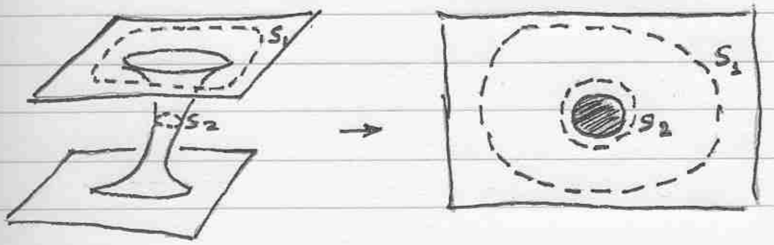
$$\Phi_B = \int_S \vec{B} \cdot d\vec{a} = 4\pi q_{mag} .$$

In manifolds with $\mathbb{R}^n$ topology, we have:

$$\Phi_B = \int_S F = \int_V dF \equiv 0$$



But if we have a "hole" on $\mathbb{R}^n$



we have : $\Phi_B = \int_{S_1} F = \int_V dF - \int_{S_2} F = -\int_{S_2} F$

where V is the volume enclosed by S_1 and S_2
$\rightarrow \partial V = S_1 \cup S_2$.

there's no reason to have 0 magnectic flux, what seems to the observers outside S_1 as a net magnectic charge.

4-vector potential : 1-form A

$$F = dA$$

if u is a unit vector tangent to observers world lines:

$$\boxed{E = u \lrcorner F} = i_u F = u^a F_{ab} .$$

the electric field.

$$dA = d(A_t dt + A_i dx^i) =$$
$$= \partial_x A_t \, dx \wedge dt + \partial_x A_y \, dx \wedge dy + \cdots$$

the spacial part gives:

$$\rightarrow \underbrace{(\partial_x A_y - \partial_y A_x)}_{(\nabla \times A)_z = B_z} dx \wedge dy + \cdots$$

---

if $\tilde{F}_{ab}$ is the orthogonal part of F with respect to u^a, $\tilde{F}_{ab} = h_a{}^c h_b{}^d F_{cd}$
$\tilde{F}$ is purelly spacial,
so $\vec{B}$ is the dual of $\tilde{F}$:

$$B^a = \varepsilon^{abc} \tilde{F}_{bc} .$$

So, if S is a purelly spacial slice,

$$\int_S \vec{B} \cdot d\vec{a} = \int_S \tilde{F} = \int_S F$$