Conservation of energy

→ conservation along flow of X_H : $X_H^i \partial_i H = X_H^n \omega_{ij} X_H^j \equiv 0$

or

$$i_{X_H} dH = i_{X_H} i_{X_H} \omega = \omega(X_H, X_H) \equiv 0$$

→ change of the sympleatic form (along flow of X_H):

$$\mathcal{L}_{X_H} \omega = \underbrace{i_{X_H} d\omega}_{d\omega \equiv 0} + d\underbrace{(i_{X_H}\omega)}_{dH} = 0$$

→ flow is a symplectic transformation = canonical transformation

so canonical transformation are the ones that preserve ω.

Liouville's theorem :

2n-form generated by ω : $\omega^n = \underbrace{\omega \wedge \omega \wedge \dots \wedge \omega}_{n\text{-times}}$

so $\mathcal{L}_{X_H} \omega^n = (\underbrace{i_X \omega}_{0}) \wedge \omega^{n-1} + \dots = 0$

in particular, if n is the phase space dimention we have

$\omega^n = dq_1 \wedge dp_1 \wedge \dots \wedge dq_n \wedge dp_n$, the volume element.

→ so flow in phase space preserves the volume.