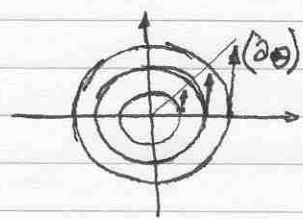
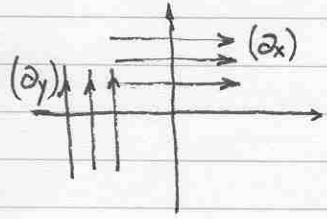


Examples:

2-d euclidian space (E^2): $ds^2 = dx^2 + dy^2 = dr^2 + r^2 d\theta^2$

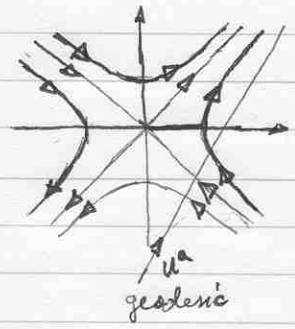


$$g_{ab}(\partial a)^a (\partial b)^b = g_{\theta\theta} = r^2 |\partial_\theta|^2$$

$$\rightarrow \|\partial_\theta\| = r$$

2-d minkowsky space (M^2): $ds^2 = -dt^2 + dx^2 = -\lambda^2 d\eta^2 + d\lambda^2$

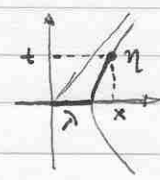
polar, with $\theta \rightarrow i\theta$



$\xi = \partial_\eta = x \partial_t + t \partial_x$ boost Killing in (t,x) coords. vector.

$u^a \xi_a$ is a constant

(see that it can't be called properly momentum or energy)



$$\begin{cases} t = \lambda \sinh \eta \\ x = \lambda \cosh \eta \end{cases}$$

$$ds^2 = -dt^2 + dx^2 = -(d\lambda \sinh \eta + \lambda \cosh \eta d\eta)^2 + (d\lambda \cosh \eta + \lambda \sinh \eta d\eta)^2 =$$

$$= -(d\lambda^2 \sinh^2 \eta + \lambda^2 \cosh^2 \eta d\eta^2 + 2d\lambda d\eta \sinh \eta \cosh \eta) +$$

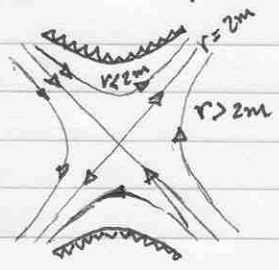
$$+ (d\lambda^2 \cosh^2 \eta + \lambda^2 \sinh^2 \eta d\eta^2 + 2d\lambda d\eta \sinh \eta \cosh \eta) =$$

$$= -d\lambda^2 \sinh^2 \eta + d\lambda^2 \cosh^2 \eta - \lambda^2 d\eta^2 (\cosh^2 \eta - \sinh^2 \eta) + 4d\lambda d\eta \sinh \eta \cosh \eta$$

$$= d\lambda^2 - \lambda^2 d\eta^2$$

Schwarzschild BH:

$$ds^2 = -(1 - \frac{2m}{r}) dt^2 + (1 - \frac{2m}{r})^{-1} dr^2 + r^2 d\Omega^2$$



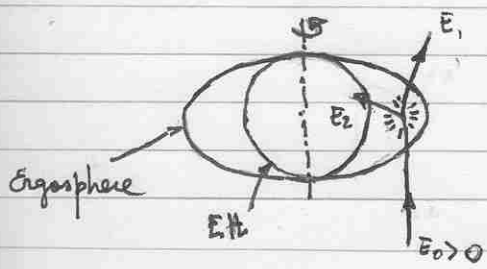
∂_t is a K.V.

$$\|\partial_t\|^2 = g_{tt} = -(1 - \frac{2m}{r})$$

In G.M. $\|\partial_t\|^2 > 0$ (spacelike) allows states with negative energy $\rightarrow$ Hawking radiation

Penrose Process:

rotating blackholes $\rightarrow$ classical negative energy.



(in Boyer-Lindquist coords)

∂_t is spacelike in the ergoregion

$$E = \xi^a u_a$$

Extract rotational energy from B.H. using ergoregion.

Arrange for E_2 to have a negative value so $E_0 = E_1 + E_2$, $E_2 < 0 \rightarrow E_1 > E_0$