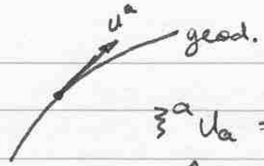
2/20/03

Killing vectors: $\xi^a \leftrightarrow \partial_{\hat\alpha}$



$$\xi^a u_a = \text{const. along geod.}$$
$$\Updownarrow$$
$$g_{\mu\hat\alpha}\dot{x}^\mu = \text{const}.$$

the same as in Lagrange's eqn:

for $L = \frac{1}{2} g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu$, $\quad P_{\hat\alpha} = \frac{\partial L}{\partial \dot{x}^{\hat\alpha}}$, $\quad \frac{d}{d\lambda} P_{\hat\alpha} = \frac{\partial L}{\partial x^{\hat\alpha}} = 0$ if $x^{\hat\alpha}$ is ignorable.

then used this to derive: $\nabla_a \xi_b + \nabla_b \xi_a = 0$.

But has an integrability condition: $\nabla_c \nabla_b \xi_c = -R_{bca}{}^d \xi_d$

$\Longrightarrow$ ξ is determined by ξ^a and $\nabla_a \xi_b$ <u>at one point</u>

<u>Maxmally symmetric spacetime has maximal number of Killing Vectors.</u>

Relation of $\nabla_{(a}\xi_{b)} = 0$ and the fact that metric doesn't depend on x^1, $\xi^a = (\partial_1)^a$.

$$\nabla_a \xi_b + \nabla_b \xi_a = g_{c(b}\nabla_{a)}\xi^c = g_{\beta\gamma}\underbrace{\partial_\alpha \xi^\gamma}_{=0,\text{ for } \xi^\alpha = (\partial_1)^\alpha} + \underbrace{g_{\beta\gamma}\Gamma^\gamma_{\alpha\delta}\xi^\delta}_{\frac{1}{2}(g_{\beta\alpha,\delta} + g_{\beta\delta,\alpha} - g_{\alpha\delta,\beta})\xi^\delta} + (\alpha \leftrightarrow \beta) =$$

$$= g_{\alpha\beta,\delta}\xi^\delta = \partial_1 g_{\alpha\beta} = 0.$$

so $g_{\alpha\beta}$ doesn't depend on x^1.

<u>Symmetry and conserved quantities.</u>

- momentum along geodesics
- conserved current: $\nabla_a T^{ab} = 0$ (a local statement)

but if have a killing vector: $\boxed{j^a_\xi = T^{ab}\xi_b}$, conserved current.

$$\nabla_a j^a_\xi = \nabla_a(T^{ab}\xi_b) = \underbrace{\nabla_a(T^{ab})}_{0}\xi_b + T^{ab}\underbrace{\nabla_a \xi_b}_{=0} = 0$$

In flat space (t, x^i): $\nabla_a j^a = \partial_t \rho + \partial_i j^i = 0 \leftarrow$ continuity eqn.