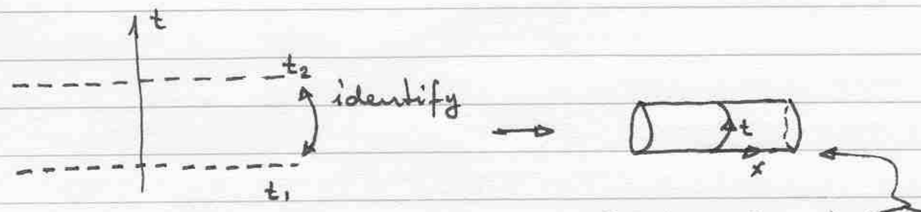
→ Closed timelike curve problem:

ex: Minkowsky:



(to get Mink.) cut and unwrap it
↳ (covering space)

So, to avoid closed timelike curves, use the covering space, just assume θ goes from $(-\infty$ to $\infty)$.

Symmetry

All these spaces have the symmetry of the embedding space:

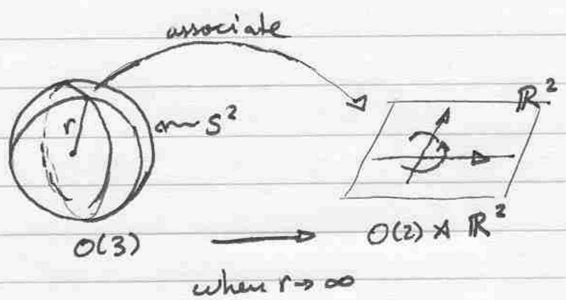
$\Lambda > 0$, deSitter (dS) : $O(4,1)$ → $\frac{5 \cdot 4}{2} = 10$

$\Lambda < 0$, anti-dS (AdS) : $O(3,2)$ → $\frac{5 \cdot 4}{2} = 10$

$\Lambda = 0$, M : $O(3,1) \rtimes \mathbb{R}^4$ ⁓ translations → $\frac{4 \cdot 3}{2} + 4 = 10$

↑ semidirect product

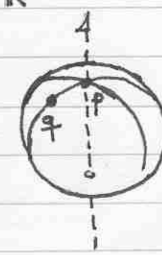
in $\mathbb{R}^3$



Suppose S^2 imbedded in $\mathbb{R}^3$

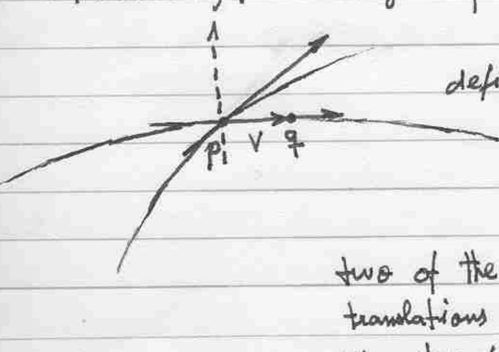
$\mathbb{R}^3 \to O(3)$

consider $p, q \in \mathbb{R}^3$
$g \in O(3)$



$q = g p = e^{\theta^\mu J_\mu} p$

Infinitesimally (on the tangent space)



define: $V = q - p = (e^{\delta\theta^\mu J_\mu} - 1) p = (\delta\theta^\mu J_\mu) p$, assuming $p = (0, 0, R)$ (cartesian)

$= (\underbrace{\delta\theta^x R, \delta\theta^y R}, R)$ that is a vector on the subspace $\mathbb{R}^2$ tangent to S^2 at p

two of the $O(3)$ rotations are translations on the $\mathbb{R}^2$ subspace. The remaining rotation forms the $O(2)$ symmetry of $\mathbb{R}^2$