2/13/02

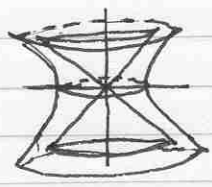
last time

* $R_{ab} = 8\pi(T_{ab} - \frac{1}{2}T g_{ab})$, $\rho = T_{ab}V^aV^b$, $V^2 = -1$
* $R_{ab}V^aV^b = 4\pi(2\rho + T) = 4\pi(\rho + 3P)$ for perfect fluid

* $T = 0 \longleftrightarrow$ conformal invariance ($g_{ab} \to \Omega^2 g_{ab}$)
* cosmological constant: $8\pi T_{ab} = -\Lambda g_{ab}$

$\Lambda > 0$, de Sitter spacetime : 4+1 Mink: $ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 + dw^2$

$-t^2 + x^2 + y^2 + z^2 + w^2 = \ell^2$ $\longrightarrow$



$\Lambda = \frac{3}{\ell^2}$

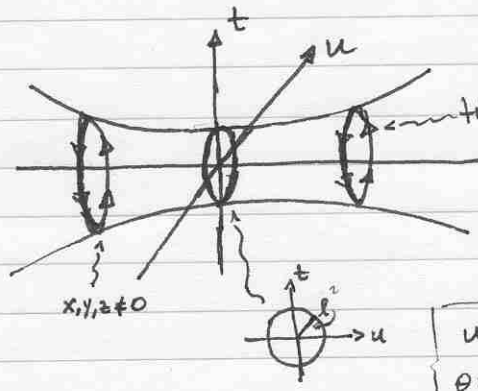
c.f.: Strominger, Volovich, (Maloney)

$\Lambda < 0$, anti-de Sitter spacetime : embeded in 3+2 (+++--) flat :

$ds^2 = \boxed{-du^2 - dt^2 + dx^2} + dy^2 + dz^2$

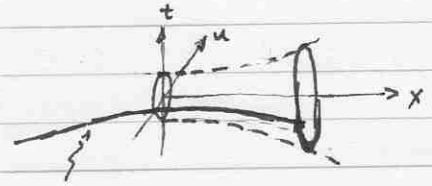
$\underbrace{-u^2 - t^2 + x^2 + y^2 + z^2 = -\ell^2}_{\text{"hyperboloid"}}$, if $x = y = z = 0$, have a circle $\boxed{u^2 + t^2 = \ell^2}$,

and it grows with x, y or z :



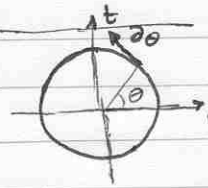
closed timelike curves

take $t = 0$,



$u^2 + t^2 = \ell^2$

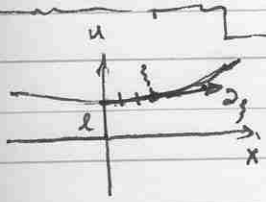
θ: $u = \ell\cos\theta$, $t = \ell\sin\theta$



$V = \partial_\theta$, $V^u = \partial_\theta u = -\ell\sin\theta$

$V^t = \partial_\theta t = \ell\cos\theta$

$V^2 = -(V^u)^2 - (V^t)^2 = -\ell^2$: it's timelike



$-u^2 + x^2 = -\ell^2$

ξ: $u = \ell\cosh\xi$, $x = \ell\sinh\xi$

$V^u = \partial_\xi u = \ell\sinh\xi$

$V^x = \partial_\xi x = \ell\cosh\xi$

$V^2 = -(V^u)^2 + (V^x)^2 = +\ell^2$: it's spacelike