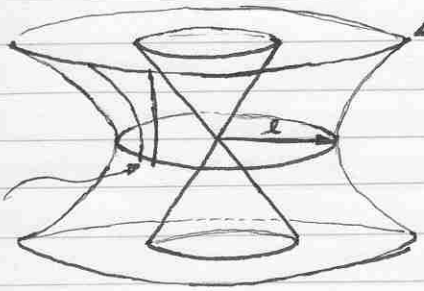
de Sitter spacetime : hyperboloid in $d+1$ dimensional Minkowsky spacetime

eg: 2-d de Sitter ⟶ 2+1 Mink. metric: $ds^2 = -dt^2 + dx^2 + dy^2$

hyperboloid : $-t^2 + x^2 + y^2 = \ell^2$

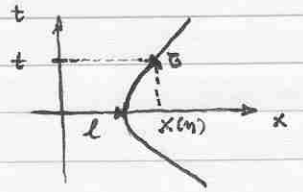


two points initially at rest

for 4-d de Sitter : $ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 + dw^2$

$-t^2 + x^2 + y^2 + z^2 + w^2 = \ell^2$

hyperbolic trigonometry :

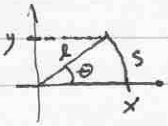


$\sigma = \eta \ell$ (definition of hyperbolic angle η)

$t = \ell \sinh \eta$

$x = \ell \cosh \eta$

Analogous to



$s = \theta \ell$

$t = \ell \sin \theta$

$x = \ell \cos \theta$

because of spherical symmetry $\dot{V} = \frac{4}{3}\pi 3R^2 \dot{R}$, $\ddot{V} = \frac{6\dot{R}^2}{R^2} + \frac{3\ddot{R}}{R}$

but setting $\dot{R}\big|_{t=0} = 0$, we have $\ddot{V}\big|_{t=0} = \frac{3\ddot{R}}{R}$

so:

$$\frac{\ddot{V}}{V} = \frac{3\ddot{R}}{R} = \frac{3}{R}\frac{d}{d\sigma}\left(\frac{d\eta}{d\sigma}\frac{dR}{d\eta}\right) = \frac{3}{R}\left(\frac{d\eta}{d\sigma}\right)^2 \frac{d^2R}{d\eta^2}$$

, but $\frac{d^2R}{d\eta^2} = \frac{d(\ell \cosh\eta)}{d\eta^2} = \ell \cosh\eta = R$

$$= \frac{3}{R}\left(\frac{d\eta}{d\sigma}\right)^2 R = 3(\ell)^{-2} = \frac{3}{\ell^2}$$

because $\frac{\ddot{V}}{V} = -4\pi f_{eff} = \Lambda$, we get $\boxed{\Lambda = \frac{3}{\ell^2}}$