obs:

~~in general~~ for conformally invariant theories we have $T^a{}_a = 0$.

$$S = \int d^n x \underbrace{\sqrt{g}\,\mathcal{L}}_{\mathcal{S}} \qquad g_{ab} \to \Omega^2 g_{ab} \quad \text{(conformal transformation)}$$

$$\sqrt{g} \to \sqrt{g}\,\Omega^{n}$$

if $\mathcal{L} \to \Omega^{-n}\mathcal{L}$ we have $S \to S$, therefore

Varying S with respect to the metric: $\delta_\Omega S = 0$.

$$\delta S = \int d^n x \frac{\delta \mathcal{S}}{\delta g_{ab}} \delta g_{ab} = \int d^n x\, T^{ab} \delta g_{ab} = \int d^n x\, T^a{}_a\, \delta(\Omega^2) = 0 \quad \forall\, \delta(\Omega^2)$$

for conf. transfo: $\delta g_{ab} = (\delta\Omega^2) g_{ab}$ therefore $T^a{}_a = 0$

In the Maxwell's example: $F_{\mu\nu} = \partial_{[\mu} A_{\nu]}$, $\mathcal{L} = g^{ar} g^{bs} F_{ab} F_{rs}$

~~then~~ for $g_{ab} \to \Omega^2 g_{ab}$,

$$\sqrt{g} \to \sqrt{g}\,\Omega^{4}, \qquad \mathcal{L} \to \underbrace{\Omega^{-2}\Omega^{-2}}_{\Omega^{-4}} g^{ar} g^{bs} F_{ab} F_{rs}$$

$$S = \int \sqrt{g}\, g^{ar} g^{bs} F_{ab} F_{rs}\, d^4 x = \int d^4 x \sqrt{g}\, \Omega^4 \Omega^{-4} F_{ab} F^{ab} = S.$$

for $n \neq 4$, EM is not conformal inv.

- Cosmological constant: $T_{\alpha\beta} \propto g_{\alpha\beta} f(x)$

conservation of energy: $\nabla^a T_{ab} = \nabla^a (f g_{ab}) = \nabla^a f\, g_{ab} = \nabla^b f = 0 \to$ <u>f is a constant</u>.

so: $\boxed{8\pi T_{ab} = -\Lambda g_{ab}}$

going to Einstein's eqn: $R_{ab} - \frac{1}{2} R g_{ab} = 8\pi T_{ab} = -\Lambda g_{ab}$

$$\boxed{R_{ab} - \tfrac{1}{2} R g_{ab} + \Lambda g_{ab} = 0}$$

it's a perfect fluid with $\boxed{P = -\rho_E}$ and $T_{ab} u^a u^b = \frac{\Lambda}{8\pi} = \rho_E$

$$T^a{}_a = -\frac{\Lambda}{2\pi} = -4\rho_E$$

therefore: $\boxed{\rho_{eff} = 2\rho_E + T = -2\rho_E}$ has a negative effective mass density

$$\rho_{eff} = -\frac{\Lambda}{4\pi}$$