Decomposition of Riemann tensor ($d \geq 3$)

$$R_{abcd} = \underbrace{C_{abcd}} + \frac{2}{d-2}\left(g_{a[c} R_{d]b} - g_{b[c} R_{d]a}\right) - \frac{2}{(d-1)(d-2)} R \, g_{a[c} R_{d]b}$$

Weyl tensor.

↓

it's traceless → in 4d - has 10 independent components

3d - 0 components

$C_{abc}{}^{d}$: invariant under conformal transformations

$$g_{ab} \longrightarrow \Omega^2(x) g_{ab}$$

$$C_{abc}{}^{d} \longrightarrow C_{abc}{}^{d}$$

Therefore, if $C_{abcd} = 0 \longleftrightarrow g_{ab} = \Omega^2(x) \eta_{ab}$