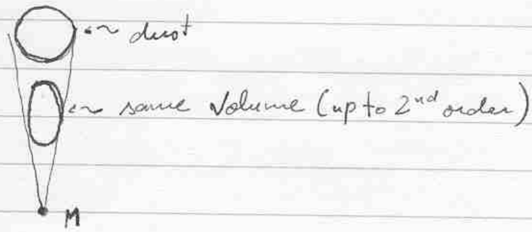
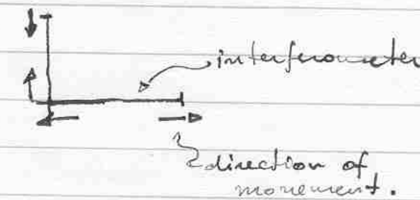
$\frac{\ddot V}{V} = \mathrm{tr}^N \Phi$. Volume is preserved in empty space ($\Phi^i_i = 0$). [Tidal forces still exist but volume don't contract].

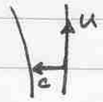
falling matter :



gravitational waves :

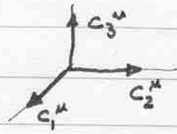


GR analogue :



$$\Phi^\mu{}_\nu = -R_{\alpha\nu\beta}{}^\mu u^\alpha u^\beta \ , \quad \Phi^\mu{}_\nu c^\nu = \frac{D^2 c^\mu}{d\lambda^2}$$

volume :



$$V = \varepsilon_{0\mu\nu\rho}\, c^\mu_{(1)} c^\nu_{(2)} c^\rho_{(3)} \ , \quad \frac{D^2 c^\mu_{(i)}}{d\lambda^2} = \Phi^\mu{}_\nu c^\nu_{(i)} = \text{"}\ddot c^\mu\text{"}$$

$$\frac{\ddot V}{V} = \varepsilon_{0\mu\nu\rho} \frac{\left(\ddot c_1^\mu c_2^\nu c_3^\rho + c_1^\mu \ddot c_2^\nu c_3^\rho + c_1^\mu c_2^\nu \ddot c_3^\rho\right)}{\varepsilon_{0\alpha\beta\gamma}\, c_1^\alpha c_2^\beta c_3^\gamma} \ , \quad \text{choosing} \quad \begin{array}{l} c_1 = \partial_1 \\ c_2 = \partial_2 \\ c_3 = \partial_3 \end{array} \ , \text{ and } \Phi^\mu{}_\nu \text{ diag.}$$

$$= \cancel{\frac{\ddot c_{(1)}}{c_{(1)}} + \frac{\ddot c_{(2)}}{c_{(2)}} + \frac{\ddot c_{(3)}}{c_{(3)}}} = \left(\Phi^1{}_1 + \Phi^2{}_2 + \Phi^3{}_2\right) = \mathrm{tr}\,\Phi .$$

To what extent does Einstein's eqn determine the whole $R_{abc}{}^d$?

$$\left(R_{ab} - \tfrac{1}{2} R g_{ab}\right) = 8\pi T_{ab}$$

- G_{ab} has 10 indep. components in 4d $\Leftrightarrow \frac{d(d+1)}{2}$
- $R_{abc}{}^d$ has 20 indep. comp. in 4d $\Leftrightarrow \frac{d^2(d^2-1)}{12}$

$d=1$, R has 0 degrees of freedom ($R \equiv 0$, no $T \equiv 0$).

$d=2$, R has 1 d.o.f ($T^\mu{}_\mu = 0$

in $d=3$: R_{ab} determines $R_{abc}{}^d$

$\rightarrow$ no independent gravitational d.o.f.

$\rightarrow$ flat in vacuum