So we can impose Newton's gravity law to GR framework:

- Newton's law valid for observers with 4-velocity $V^a$ provided

$$\boxed{\left({}^{E}\Phi_V\right)^a{}_a = 4\pi\rho} \quad (G=1) \qquad , \quad g_{ab}V^aV^b = -1 .$$

but what is $\rho$ ? ~~[illegible]~~ scalar from $T_{\mu\nu}$ : $\rho \leftrightarrow T_{ab}V^aV^b$ (but only this gives a problem!)

$\leftrightarrow T_{ab}g^{ab}$ – the trace of $T_{ab}$.

so in general:

$$\boxed{\rho \leftrightarrow \left\{\alpha T_{ab} + (1-\alpha)(T g_{ab})\right\} V^aV^b}$$

therefore:

$$(\underbrace{-R_{abm}{}^{b}}_{R_{am}})V^aV^m = 4\pi\left\{\alpha T_{ab} + (1-\alpha)T g_{ab}\right\}V^aV^b ,$$

if this is true for all $V^a$, thus:

$$R_{ab} = -4\pi\left\{\alpha T_{ab} + (1-\alpha)T g_{ab}\right\}$$

$$\to R = -4\pi\left\{\alpha T + 4(1-\alpha)T\right\} = -4\pi(4-3\alpha)T .$$

See: $\nabla^a(R_{ab} - \frac{1}{2}Rg_{ab}) \equiv 0$
Contracted Bianchi identity

$$R_{ab} - \left(\frac{1-\alpha}{4-3\alpha}\right)Rg_{ab} = 4\pi\alpha T_{ab}$$

since $\nabla^a T_{ab} = 0$ (local conservation of $T_{\mu\nu}$)

we have: $\left(\frac{1-\alpha}{4-3\alpha}\right) = \frac{1}{2} \to \alpha = 2 .$

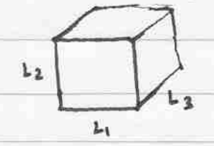
So finally:

$$\boxed{G_{ab} = R_{ab} - \frac{1}{2}Rg_{ab} = 8\pi T_{ab}} \quad , \text{ and } \rho = (2T_{ab} + Tg_{ab})V^aV^b .$$

Einstein's eqn.

---

Meaning: (Newton) free falling box full of particles:




$V = L_1L_2L_3$

initially

$$\dot{V}\Big|_{t=0} = 0$$

$$\ddot{V}\Big|_{t=0} = \left(\frac{\ddot{L}_1}{L_1} + \frac{\ddot{L}_2}{L_2} + \frac{\ddot{L}_3}{L_3}\right)V = tr({}^{N}\Phi)\cdot V ,$$

so, $V(t) = V(0) + \frac{1}{2}t^2\left({}^{N}\Phi^i{}_i\right)V(0) + \cdots$

$\vec{L}_1 = \hat{i},\ \vec{L}_2 = \hat{j},\ \vec{L}_3 = \hat{k}$,

and $\Phi^i{}_j$ is diagonal.

$$= \left(\frac{|\Phi^i{}_j L_1^j|}{|L_1|} + \frac{|\Phi^i{}_j L_2^j|}{|L_2|} + \frac{|\Phi^i{}_j L_3^j|}{|L_3|}\right) =$$

$$= \left(\Phi^1{}_1 + \Phi^2{}_2 + \Phi^3{}_3\right) = tr\,\Phi$$