02/03/03

Recall Newtonian Gravity:

$$a^i = g^i = \varphi_{,i}$$

$$\ddot{C}^i = {}^N\Phi^i{}_j C^j \quad , \quad {}^N\Phi^i{}_j = -\varphi_{,ij} \quad , \quad {}^N\Phi^i{}_i = 4\pi G\rho$$

Newton $\longleftrightarrow$ Einstein

free fall $\longleftrightarrow$ geodesic motion

relative acceleration of free falling obs $\longleftrightarrow$ relative acceleration of geodesics $\Rightarrow$ curvature

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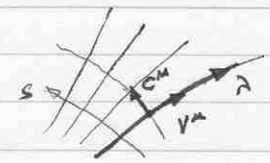
Congruence of Geodesics, $X^\mu(\lambda, s)$



$V^\mu = \dfrac{\partial X^\mu}{\partial \lambda} \longrightarrow V^a$ abstract index notation (by Geroch after Penrose ?)

$C^\mu = \dfrac{\partial X^\mu}{\partial s} \longrightarrow C^a$ Wald's notation, latin lower case.

(also Wald defines)

$$\boxed{\begin{aligned} [\nabla_a, \nabla_b] C^n &= (\nabla_a\nabla_b - \nabla_b\nabla_a) C^n = -R_{abm}{}^n C^m \\ [\nabla_a, \nabla_b] \omega_m &= (\nabla_a\nabla_b - \nabla_b\nabla_a)\omega_m = +R_{abm}{}^n \omega_n \end{aligned}}$$

$$-R_{abm}{}^n C^m = +g^{n\theta}(R_{ab\theta m} C^m)$$

geodesic deviation eqn:

$$(V^a\nabla_a)(V^b\nabla_b)C^n = -R_{abm}{}^n V^a V^m C^b$$

$\longleftrightarrow$ could have guessed.

1- need to have curvature: $R^{\cdot}{}_{\cdots}$

2- but R already contains 2∇'s, so $\dot{X}$ and C appears undifferentiated $\rightarrow R\dot{X}\dot{X}C$

3- by index symmetry it must be: $\rightarrow \alpha R_{abm}{}^n \dot{X}^a \dot{X}^m C^b$ (up to a constant α)

Analogously to Newton:

$$(V^a\nabla_a)(V^b\nabla_b)C^n = ({}^E\Phi_V)_b{}^n C^b$$

or

$$\frac{D^2 C}{d\lambda^2} = ({}^E\Phi_V)_b{}^n C^b \quad , \quad ({}^E\Phi_V)_b{}^n = -R_{abm}{}^n V^a V^m .$$

$\rightarrow ({}^E\Phi_V)$ is purelly spacial in the local frame:

$${}^E\Phi_{Vb}{}^n V^b = -R_{abm}{}^n V^a V^m V^b \equiv 0$$

$${}^E\Phi_{Vb}{}^n V_n \equiv 0 .$$

so. it looks like a 4x4 array but only has the degrees of freedom of a 3x3 one!