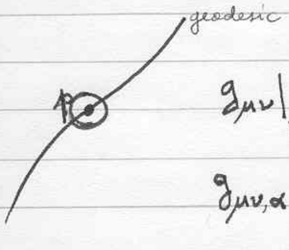
In GR, test particles follow geodesics.

$$\underset{\text{scalar}}{S} = \int \tfrac{1}{2} g_{\mu\nu}(x)\, \dot{x}^\mu \dot{x}^\nu \, d\lambda \qquad , x^\mu(\lambda) \text{ - curve in spacetime}$$

$$\delta S = 0 \longrightarrow \boxed{\frac{d}{d\lambda}\left(g_{\mu\alpha}\dot{x}^\mu\right) - \tfrac{1}{2} g_{\mu\nu,\alpha}\, \dot{x}^\mu \dot{x}^\nu = 0} \qquad (1)$$

← this is only true if λ is afine parameter



$$\left.\begin{aligned} g_{\mu\nu}\big|_p &= \eta_{\mu\nu} \\ g_{\mu\nu,\alpha}\big|_p &= 0 \end{aligned}\right\} \text{ local inertial coordinates}$$

At the point p:

$$(1) \rightarrow g_{\mu\alpha}\ddot{x}^\mu\big|_p = 0 \longrightarrow \eta_{\mu\alpha}\ddot{x}^\mu\big|_p = 0 \longrightarrow \boxed{\ddot{x}^\mu = 0}$$

unaccelerated (local straight lines)

lets reparametrize: $\lambda = \lambda(\xi)$. therefore:

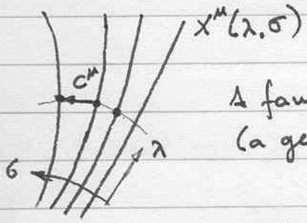
$$0 = \left.\frac{d^2 x^\mu}{d\lambda^2}\right|_p = \frac{d\xi}{d\lambda}\frac{d}{d\xi}\left(\frac{d\xi}{d\lambda}\frac{d}{d\xi}x^\mu(\lambda(\xi))\right) = \frac{d\xi}{d\lambda}\frac{d}{d\xi}\left(\frac{d\xi}{d\lambda}\left.\frac{\partial x^\mu}{\partial\lambda}\right|_{\lambda(\xi)}\frac{d\lambda}{d\xi}\right) =$$

$$= \left(\frac{d\xi}{d\lambda}\right)^2\left(\underbrace{\left.\frac{\partial^2 x^\mu}{\partial\lambda^2}\right|_{\lambda(\xi)}}_{=0}\left(\frac{dt}{d\xi}\right)^2 + \left.\frac{\partial x^\mu}{\partial\lambda}\right|_{\lambda(\xi)}\left(\frac{d^2\lambda}{d\xi^2}\right)\right) \Rightarrow \frac{d^2\lambda}{d\xi^2} = 0, \quad \boxed{\lambda = a\xi + b}$$

so that $\dfrac{d^2 x^\mu}{d\xi^2} = 0$

λ and ξ are affine parameters.

Geodesic Deviation eqn: measures relative acceleration of nearby geodesics.



A family of geodesics (a geodesic for each σ)

$C^\mu = \dfrac{\partial x^\mu}{\partial\sigma}$: connecting vector

$\dot{x}^\mu = V^\mu = \dfrac{\partial x^\mu}{\partial\lambda}$: tangent vector to geodesic

$$\dot{x}^\alpha \nabla_\alpha\left(\dot{x}^\beta \nabla_\beta C^\sigma\right) = - R^\sigma{}_{\varepsilon\mu\nu}\, \dot{x}^\varepsilon \dot{x}^\nu C^\mu =$$

$$= {}^{E}\Phi^\sigma{}_\varepsilon\, C^\varepsilon$$

therefore: $\boxed{{}^{E}\Phi^\sigma{}_\varepsilon = - R^\sigma{}_{\mu\varepsilon\nu}\, \dot{x}^\mu \dot{x}^\nu}$

Einstein tidal tensor.