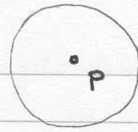
at a neighborhood of p : 

Can we set

$$g_{\mu\nu,\lambda}\Big|_p = \partial_\lambda g_{\mu\nu}\Big|_p = 0 \quad ?$$

$$g'_{\mu\nu,\lambda} = \frac{\partial}{\partial x'^\lambda} g'_{\mu\nu} = \partial'_\lambda\left(\frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}\right) =$$

$$= 2\underbrace{\left(\frac{\partial^2 x^\alpha}{\partial x'^\lambda \partial x'^\mu}\right)}\frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta} + \left(\frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial x^\beta}{\partial x'^\nu}\right)\partial'_\lambda g_{\alpha\beta}$$

Yes. Have exactlly enougth freedom in choosing $\dfrac{\partial^2 x}{\partial x' \partial x'}$.

$\alpha \leftarrow 4$

$(\mu\nu) \leftarrow 10$

40 free parameters at p.

and there are 40 comp. of $g'_{\mu\nu,\lambda}$.

Can we set: $g_{\underset{10}{\mu\nu},\underset{10}{\lambda\sigma}}\Big|_p = 0$?

$\partial'_\sigma\left[g'_{\mu\nu,\rho}\right] = \dots =$ look the 3^{rd} derivatives:

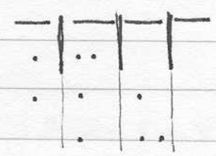
$$\frac{\partial x^\alpha}{\partial x'^\sigma \partial x'^\lambda \partial x'^\mu} \longrightarrow 4$$

$$\longrightarrow (\sigma\lambda\mu) \Rightarrow \frac{(d-1+3)!}{(d-1)!\,3!} = \frac{d(d+1)(d+2)}{3!}$$

No.
Comparing:

D	$g_{\mu\nu,\lambda\sigma}$	$\frac{\partial^3 x}{\partial x \partial x \partial x}$	diff
#	$\frac{d^2(d+1)^2}{4}$	$\frac{d^2(d+1)(d+2)}{3!}$	$\frac{d^2(d^2-1)}{12}$
1	1	1	0
2	9	8	1
3	36	30	6
4	100	80	20
10	55^2		825

4 dim = 4 slots

3 indices = 3 dots $\rightarrow$ 

# of dispositions of 3 bars and 3 dots $= \dfrac{(3+3)!}{3!\,3!} = \dfrac{6!}{36} = \dfrac{5\cdot 4\cdot 6}{6} = 20$

$$\frac{d^2(d+1)^2}{4} - \frac{d^2(d+1)(d+2)}{6} = d^2(d+1)\left[\frac{d+1}{4} - \frac{d+2}{6}\right] =$$

$$= \frac{d^2(d+1)(d-1)}{12} = \frac{d^2(d^2-1)}{12}$$

Riemann curvature tensor:

$$R_{\underset{6}{\mu\nu}\,\underset{6}{\alpha\beta}} - \frac{6(6+1)}{2} = 21$$

$$R_{[\mu\nu\alpha]\beta} = 0 \Longrightarrow \boxed{\frac{d^2(d^2-1)}{12}} \text{ comp.} \quad 20 \text{ in } 4d.$$