Homework: week1

Solving exercises is the most effective way of learning physics. Although only one third of the final grades for this course will be based on the homeworks, you should take them very seriously.

Recommended readings:

1. A. Abramovici et al. *LIGO: the Laser Interferometer Gravitational-wave Observatory*, Science **256** (1992) 325.
2. C. Cutler and K.S. Thorne, *An overview of gravitational-wave sources*, `gr-qc/0204090` [Proceedings of GR16, 2002]
3. A. Buonanno, *Gravitational waves*, `arXiv:0709.4682` [Proceedings of Les Houches Summer School, 2006]

Assignment to be turned in at the beginning of the class on Thursday, January 31 by students registered to the course:

- State what of the above readings you have done
- Work the three exercises below

Exercises:

1. **Multipolar expansion of the gravitational field** (2 points)

   In the lecture (see also p 8 of lecture's slides) the gravitational field is written in a multipolar expansion. Show that it is dimensionless. Assuming that the gravitational field is proportional to G/r (where r is the distance between the observer and the source, and G is the Newton constant), and that each term of the expansion depends only on c and on derivatives of the multipole mass-moments I_L and current-moments J_L, show that the multipolar expansion is unique.

2. **Orders of magnitude (from Kip Thorne Caltech lectures)** (3 points)

   Use the leading, non-zero term in the multipolar expansion of the gravitational field, i.e., the quadrupolar term $G\,\ddot{I}_2/(c^4\,r)$, to *estimate* the amplitude of gravitational waves produced from the following earth-based events:

   - A meteorite having diameter of 2 km and hitting the ground at a speed of 25 km/sec;
   - A big chunk of piezoelectric material driven to oscillation at a frequency of 100 MHz.

   To derive the above results take into account that gravitational waves exist only in the so-called wave zone, i.e., at a distance from the source which is at least equal to the reduced wavelength.

3. **Strengths of gravitational waves for some typical astrophysical sources** (5 points)

   Use the leading, non-zero term in the multipolar expansion of the gravitational field, i.e., the quadrupolar term $G\,\ddot{I}_2/(c^4\,r)$, to evaluate how the gravitational-field h depends on the frequency for the following astrophysical sources:

   - A rotating body, e.g., a pulsar, of characteristic scale R;
   - A binary moving along a circular orbit for which the relation between the radial separation and the orbital angular frequency is described by the Newton law.

Estimate at which distance the source should be in order to produce an $h \sim 10^{-21}$ on the Earth. In case (a) assume that the deviation from sphericity is 10^{-4}, the inertia moment is $10^{45}\,\mathrm{g\,cm}^2$, and give results for a gravitational-wave frequency of 10^2 Hz and 10^3 Hz. In case (b) give the results for a gravitational-wave frequency of 100 Hz and binaries having total mass $(1.4+1.4)M_\odot$, $(10+10)M_\odot$ (LIGO/Virgo sources), and for a gravitational-wave frequency of 10^{-3} Hz and a binary having total mass $(10^6+10^6)M_\odot$ (LISA source).