Key ideas on how inspiral-merger-ringdown waveforms are built within the effective-one-body formalism

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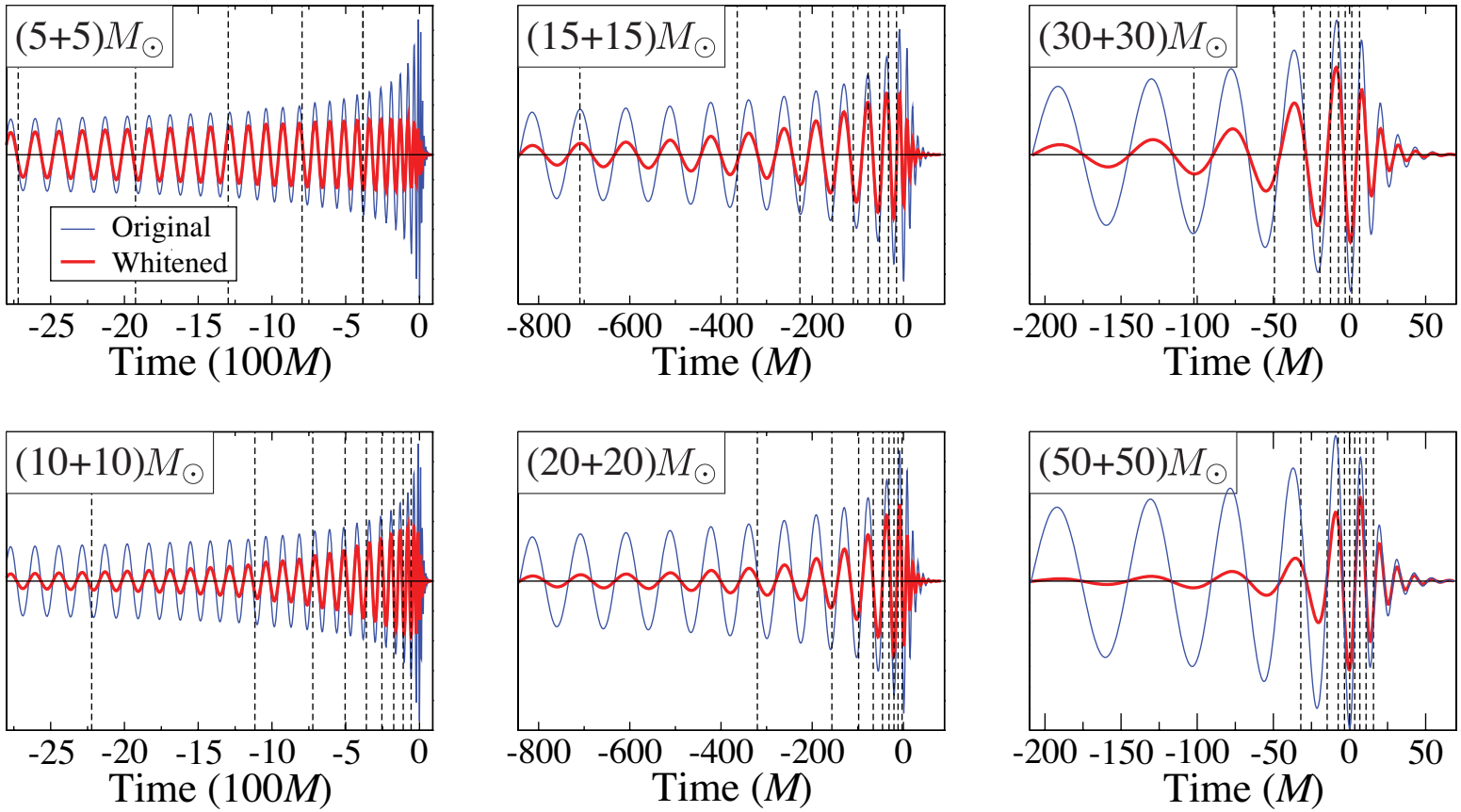
The significance of merger and ringdown signals for LIGO/Virgo

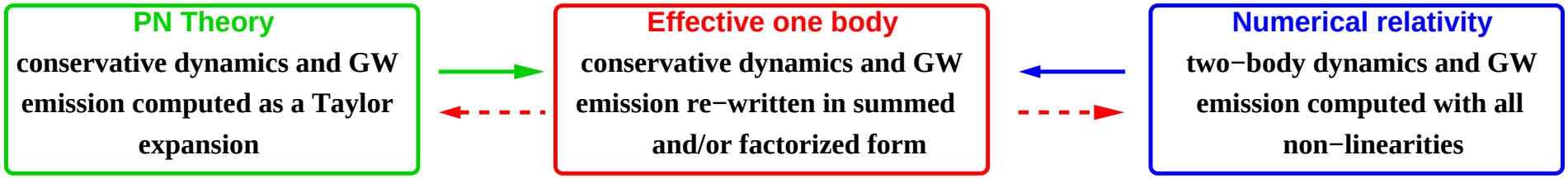
[Pan, AB, Pretorius & NASA-Goddard 07]



Combining post-Newtonian, perturbation theory and numerical-relativity results: the effective-one-body (EOB) approach

- **EOB approach introduced before the NR breakthrough** [AB & Damour 99, 00]

  many papers since then!



- **The EOB formalism uses the best information available in PN theory, but** *sums* **it in a** *suitable* **way to be able to describe accurately the full evolution: inspiral, merger and ringdown.**

- **The EOB formalism provides us with a** *moment in time* **when to switch from the from the two-body to the one-body description.**

Effective-one-body approach in a nutshell

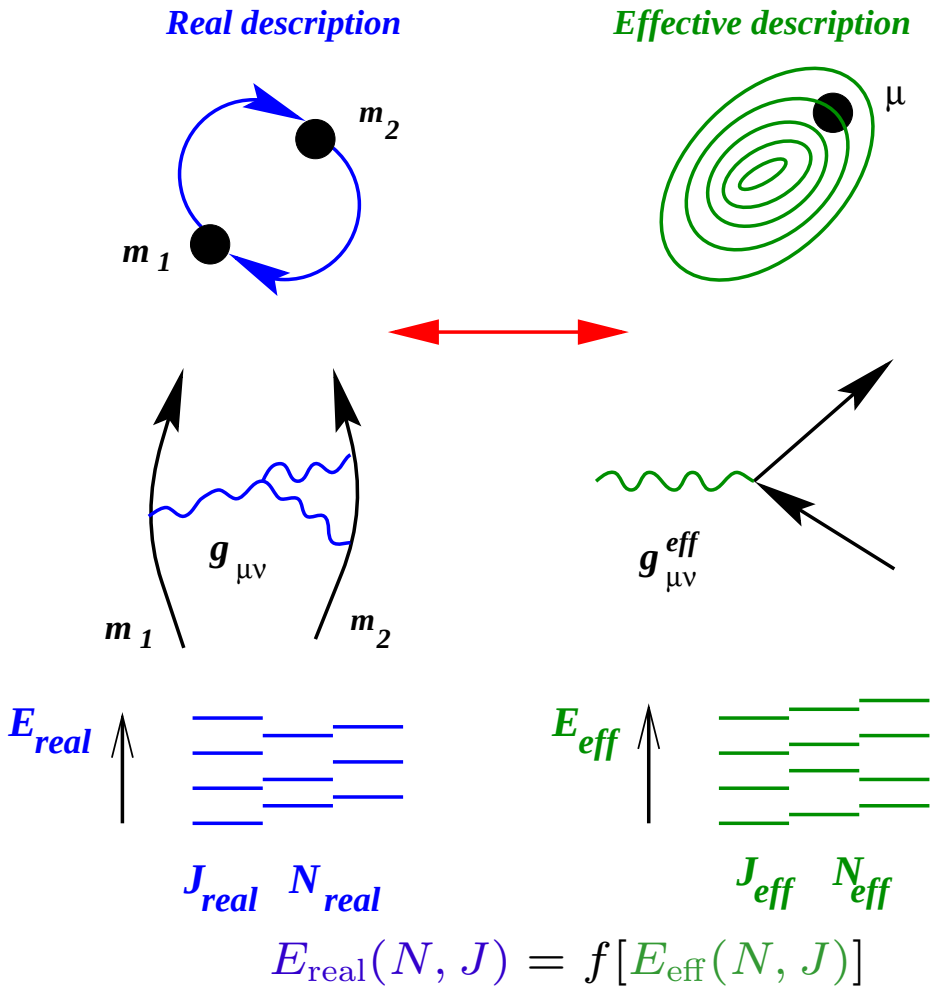
[AB & Damour 99]

$\nu = \mu/M = m_1\, m_2/M^2$

$0 \leq \nu \leq 1/4$

- **Resum so that** $known$ **test-mass limit results are recovered**

- **Resum the PN expansion assuming that the equal-mass limit is a** ν**-deformation of the test-mass limit**



$$E_{\rm real}(N,\,J) = f\,[E_{\rm eff}(N,\,J)]$$

Finding the energy for *comparable-mass black holes*

- *Thinking quantum mechanically*: **the classical Hamiltonian and bound orbits are replaced by the Hamiltonian operator and quantum bound states.**

- **Real description:**

$$E_{\rm real}(N,J) = M\,c^2 - \frac{1}{2}\frac{\mu\,\alpha^2}{N^2}\left[1+\frac{\alpha^2}{c^2}\left(\frac{6}{N\,J}-\frac{1}{4}\,\frac{15-4\nu}{N^2}\right)+\cdots\right], \quad \alpha = G\,M\,\mu$$

- **Effective description:**

$$E_{\rm eff}(N,J) = \mu\,c^2 - \frac{1}{2}\frac{\mu\,\alpha^2}{N^2}\left[1+\frac{\alpha^2}{c^2}\left(\frac{C_{3,1}}{N\,J}+\frac{C_{4,0}}{N^2}\right)+\cdots\right]$$

- **Allow transformation of energy axis:**

$$E^{\rm NR}_{\rm eff} = E^{\rm NR}_{\rm real}\left[1+\alpha_1\,\frac{E^{\rm NR}_{\rm real}}{\mu\,c^2}+\alpha_2\left(\frac{E^{\rm NR}_{\rm real}}{\mu\,c^2}\right)^2+\cdots\right] \quad \Rightarrow \quad \alpha_1=\frac{\nu}{2}, \quad \alpha_2=0$$

Energy for *comparable-mass bodies*

- **Classical gravity** [AB & Damour 99] (up to 3PN order)

$$E^2_{\rm real} = m_1^2 + m_2^2 + 2m_1\, m_2 \left(\frac{E_{\rm eff}}{\mu}\right)$$

- **Quantum electrodynamics** (eikonal approximation) [Brézin, Itzykson & Zinn-Justin 70]

$$E^2_{\rm real} = m_1^2 + m_2^2 + 2m_1\, m_2 \frac{1}{\sqrt{1+Z^2\,\alpha^2/(n-\epsilon_j)^2}}$$

In summary, here is the *summed* PN conservative dynamics

[AB & Damour 99]

"Real" description

$$H^{\rm PN}_{\rm real} = H_{\rm Newt} + \frac{1}{c^2} H_{\rm 1PN} + \frac{1}{c^4} H_{\rm 2PN} + \cdots$$

"Effective" description

$$H^{\nu}_{\rm eff} = \mu \sqrt{A_\nu(r) \left[1 + \frac{\mathbf{p}^2}{\mu^2} + \left(\frac{1}{B_\nu(r)} - 1\right) \frac{p_r^2}{\mu^2}\right]}$$

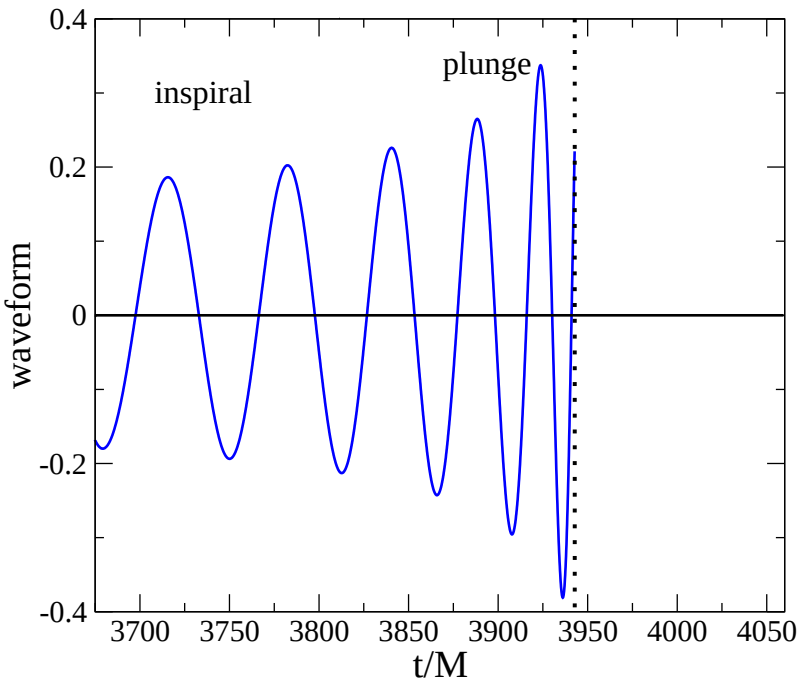
$$H^{\rm EOB}_{\rm real} = M \sqrt{1 + 2\nu \left(\frac{H^{\nu}_{\rm eff}}{\mu} - 1\right)}$$

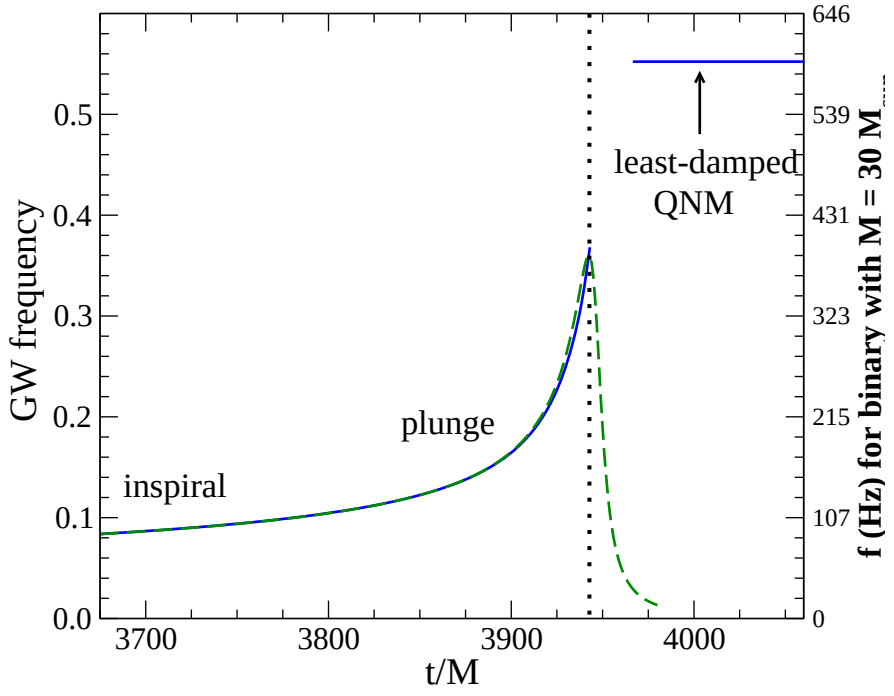
$$ds^2_{\rm eff} = -A_\nu(r)\, dt^2 + B_\nu(r)\, dr^2 + r^2\, d\Omega^2$$

- **Dynamic condensed in $A_\nu(r)$ and $B_\nu(r)$**
- **$A_\nu(r)$, which encodes the energetics for circular orbits, is rather *simple***

$$A_\nu(r) = 1 - \frac{2M}{r} + \frac{2M^3\nu}{r^3} + \left(\frac{94}{3} - \frac{41}{32}\pi^2\right) \frac{M^4\,\nu}{r^4} + \frac{a_5(\nu)}{r^5} + \frac{a_6(\nu)}{r^6} + \cdots$$

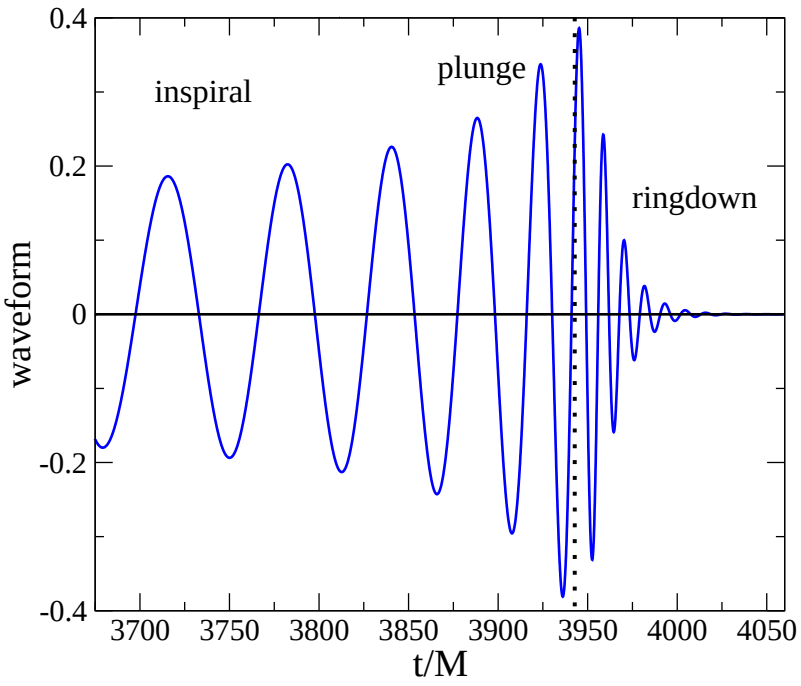
EOB inspiral-plunge waveform

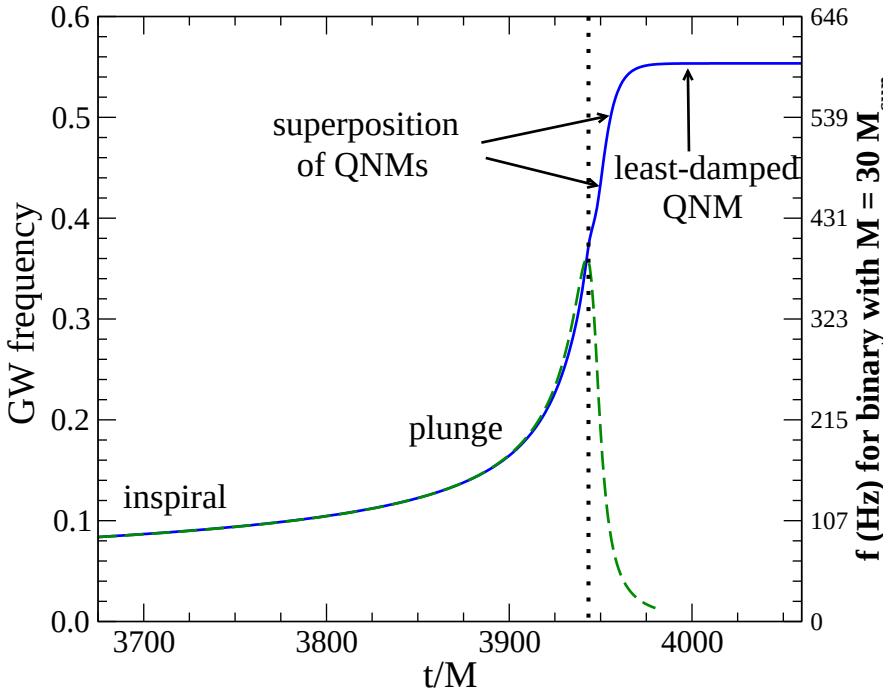




- **The plunge is a smooth continuation of the adiabatic inspiral** [AB & Damour 00]

EOB inspiral-merger-ringdown waveforms





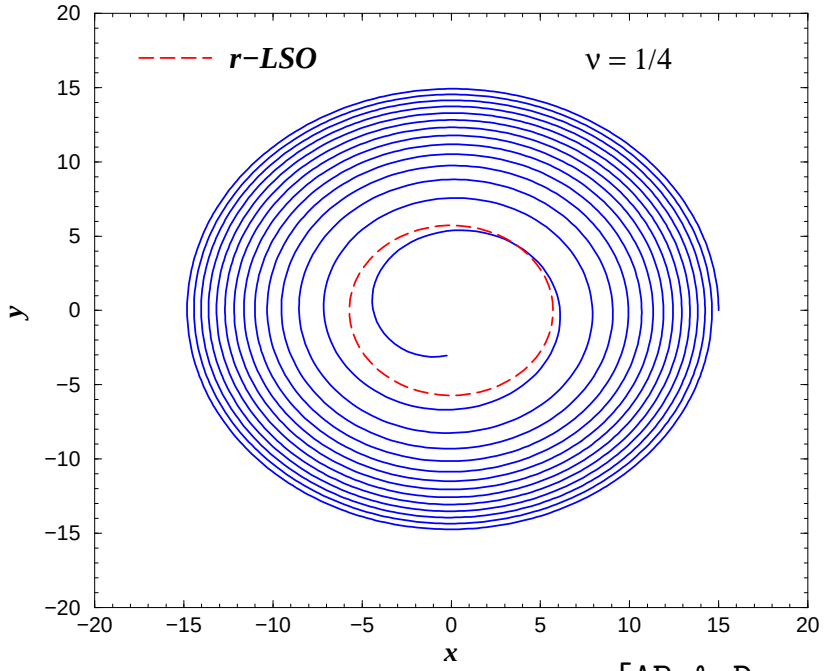
- **Very *short* transition merger–ringdown**
- **Energy quickly released during merger**

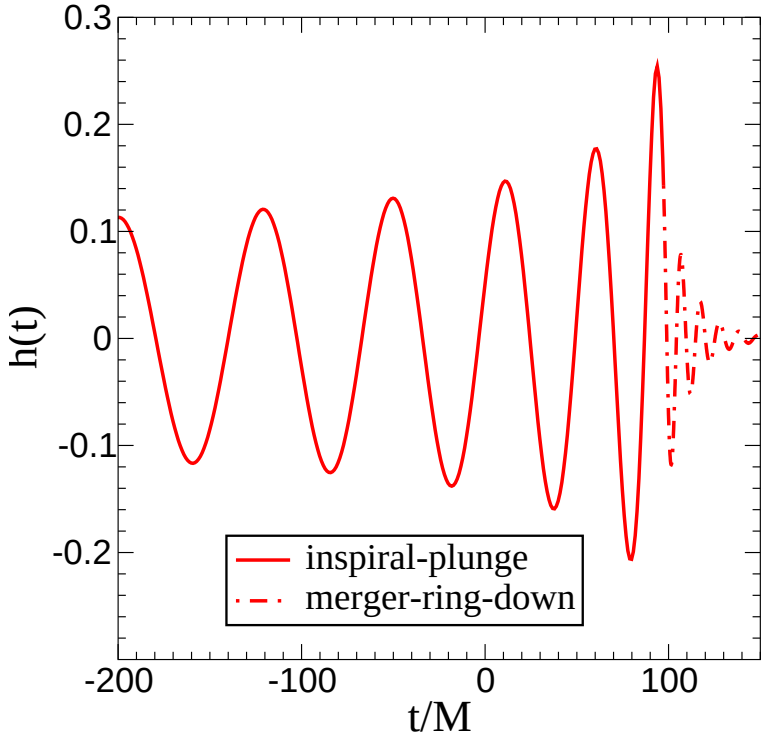
- $E_{\rm rad} \sim 2\%$–$12\%\, M\, c^2$

  $1 M_\odot\, c^2 \sim 10^{54}\,{\rm erg} \sim 10^{56}\,{\rm GeV}$!

Full waveform as predicted by the EOB model

- **The plunge (~ 1.5 GW cycles) is a smooth continuation of the inspiral phase**
- **The transition merger to ringdown was assumed** *very short*
- One single **QNM matched using** $M_{\rm BH} = 0.976\,M$, $a_{\rm BH}/M_{\rm BH} = 0.77$

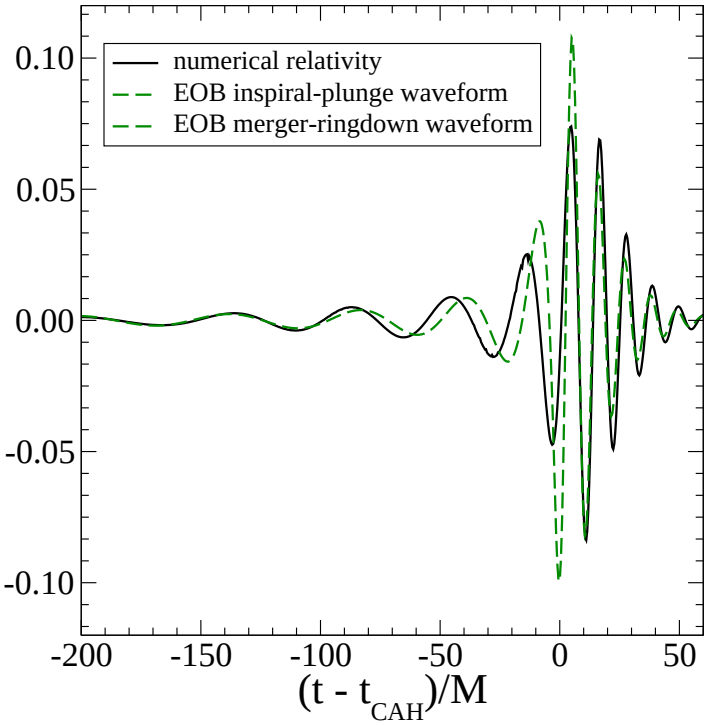




[AB & Damour 99, 00]

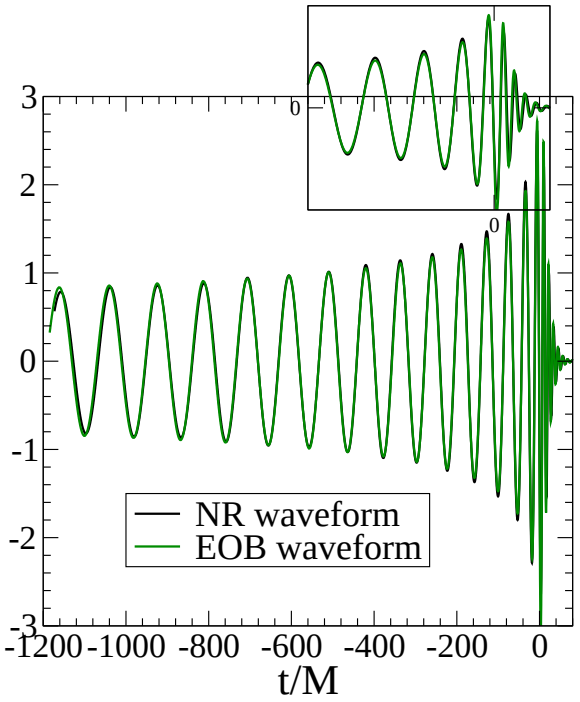
First comparisons/calibrations between NR and EOB model

[AB, Cook & Pretorius 06]



Uncalibrated **EOB model at 3PN order**

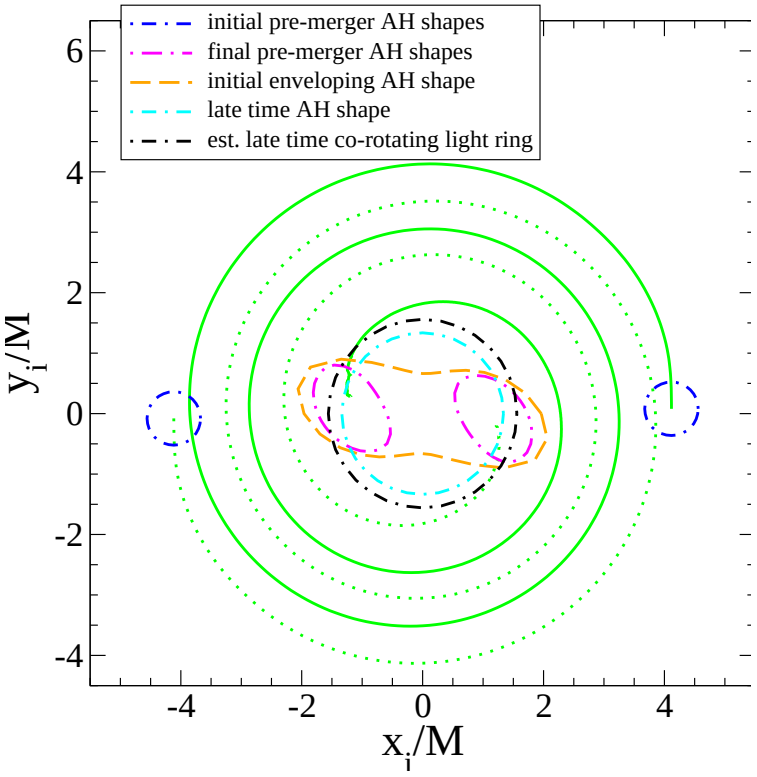
[AB, Pan & NASA-Goddard 07]

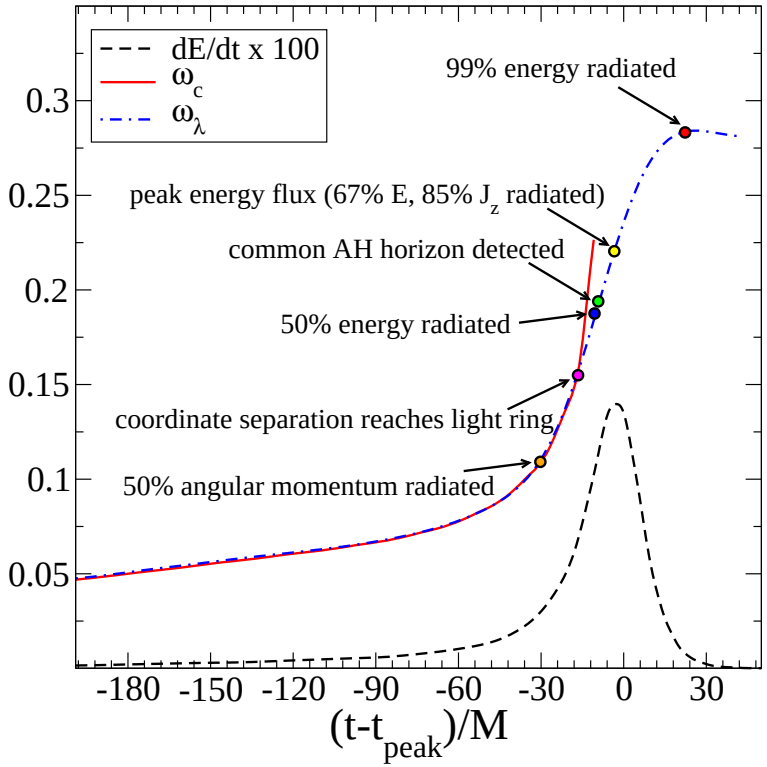


Calibrated **EOB model at 4PN order**

The (plunge and) merger

[AB, Cook & Pretorius 06]





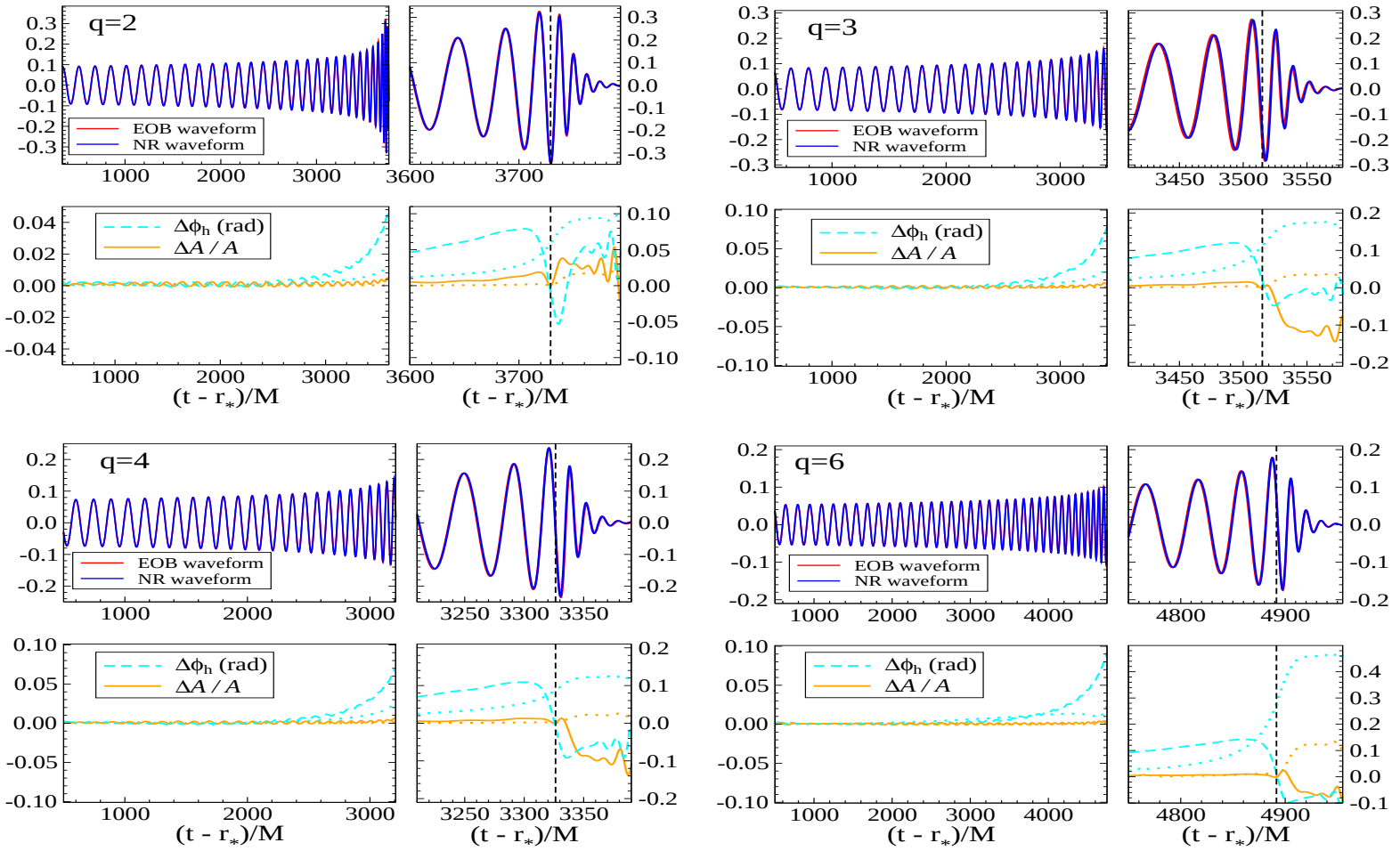
- *Short* **transition merger–ringdown**

[AB, Cook & Pretorius 06]

- **Energy and angular-momentum quickly released during merger**

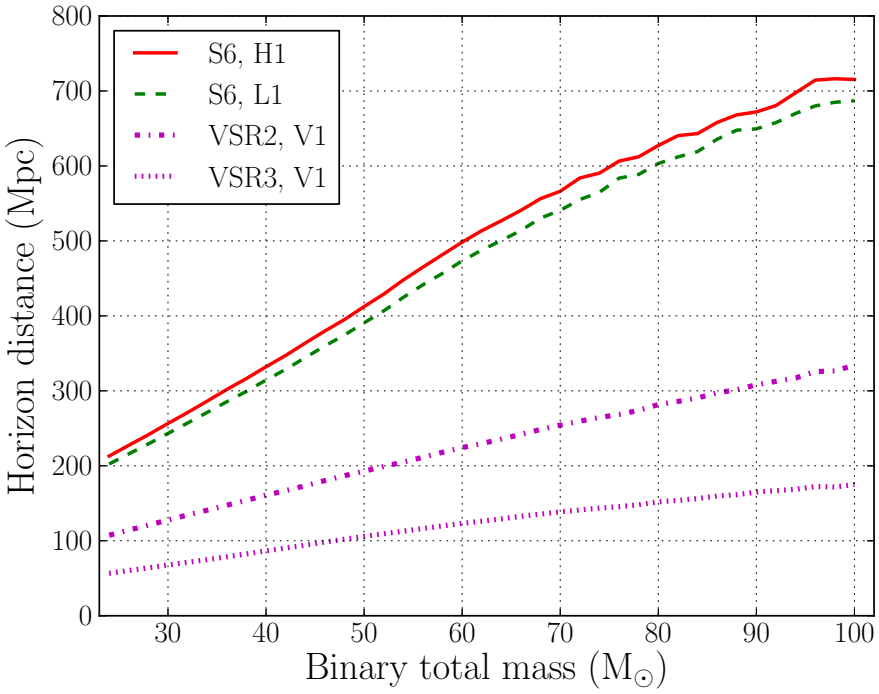
Calibrating highly accurate waveforms for several mass-ratio binaries

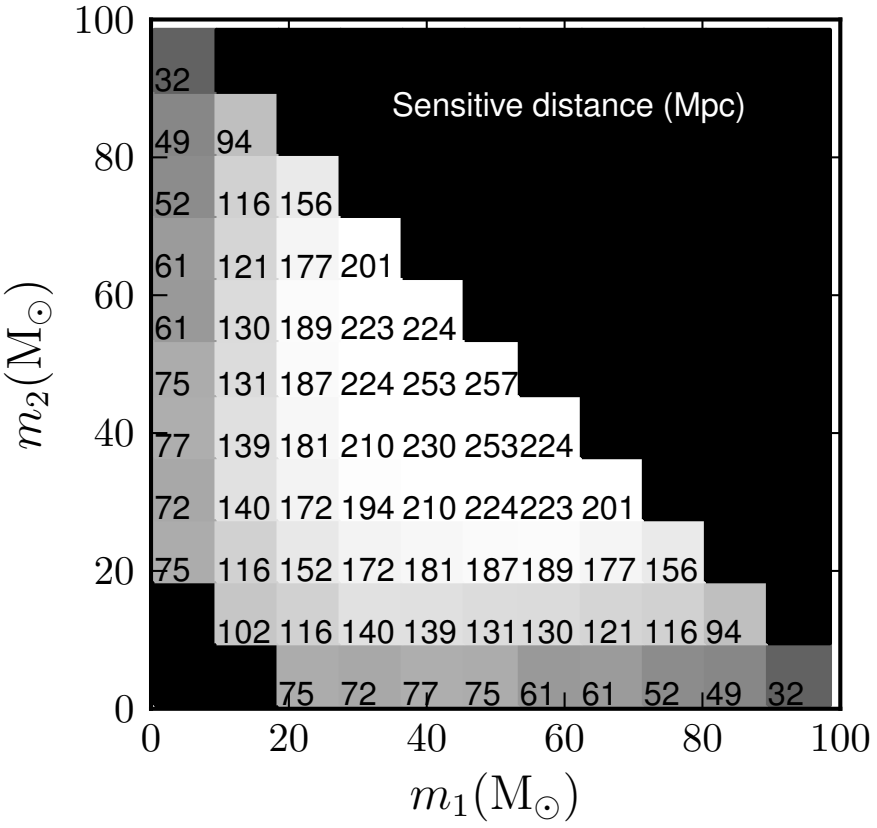
[Pan, AB, Boyle, Buchman, Kidder, Pfeiffer & Scheel 11]



The EOB waveforms were used in LIGO searches high-mass compact binaries

[Aasi et al. 12 (The LSC/Virgo Collaboration)]





Calibrating waveforms of spinning, non-precessing black holes

- **EOB models with spins** [Damour 01, Damour, Jaranowski & Schaefer 08, Barausse & AB 09,11]
- **The PN Hamiltonian of two BHs of masses $m_{1,2}$ and spins $\mathbf{S}_{1,2}$ is mapped into the effective Hamiltonian of a spinning test-particle of mass μ and spin $\mathbf{S}^*$ moving in a deformed-Kerr spacetime with mass M and spin $\mathbf{S}_{\rm Kerr}$.**

[Pan et al. 09; Taracchini, Pan, AB, Barausse, Chu, Boyle, Pfeiffer & Scheel 12]