HW#1 —Phys879—Spring 2014 Prof. Alessandra Buonanno
Due before class, Tuesday, Feb. 4, 2014 Room PSC 3149, (301)405-1440
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Recommended readings:

1. A. Abramovici et al. *LIGO: the Laser Interferometer Gravitational-wave Observatory*, Science **256** (1992) 325.
2. E.E. Flanagan and S.A. Hughes, *The basics of GW theory*, `arXiv:grqc/0501041`.

Exercises:

1. **Multipolar expansion of the far-zone gravitational field:** [5 pts.]

   In the lecture (see also p 8 of lecture's slides) the gravitational field is written in a multipolar expansion. Show that it is dimensionless. Assuming that the gravitational field is proportional to G/r (where r is the distance between the observer and the source, and G is the Newton constant), and that each term of the expansion depends only on c and on derivatives of the multipole mass-moments I_L and current-moments J_L, show that the multipolar expansion is unique.

2. **Orders of magnitude of gravitational-wave strength for sources on the Earth:** [10 pts.]

   Use the leading, non-zero term in the multipolar expansion of the gravitational field, i.e., the quadrupolar term $G\,\ddot{I}_2/(c^4\,r)$, to *estimate* the amplitude of gravitational waves produced from the following earth-based events:

   - A meteorite having diameter of $2\,\mathrm{km}$ and hitting the ground at a speed of $25\,\mathrm{km/sec}$;
   - A big chunk of piezoelectric material driven to oscillation at a frequency of $100\,\mathrm{MHz}$.

   To derive the above results take into account that gravitational waves exist only in the so-called wave zone, i.e., at a distance from the source which is at least equal to the reduced wavelength.

3. **Strengths of gravitational waves for some typical astrophysical sources:** [15 pts.]

   Use the leading, non-zero term in the multipolar expansion of the gravitational field, i.e., the quadrupolar term $G\,\ddot{I}_2/(c^4\,r)$, to evaluate how the gravitational-field h depends on the frequency for the following astrophysical sources:

   - A rotating body, e.g., a pulsar, of characteristic scale R;
   - A binary moving along a circular orbit for which the relation between the radial separation and the orbital angular frequency is described by the Newton law.

   Estimate at which distance the source should be in order to produce an $h \sim 10^{-21}$ on the Earth. In case (a) assume that the deviation from sphericity is 10^{-4}, the inertia moment is $10^{45}\,\mathrm{g\,cm}^2$, and give results for a gravitational-wave frequency of 10^2 Hz and 10^3 Hz. In case (b) give the results for a gravitational-wave frequency of 100 Hz and binaries having total mass $(1.4+1.4)M_\odot$, $(10+10)M_\odot$ (LIGO/Virgo sources), and for a gravitational-wave frequency of 10^{-3} Hz and a binary having total mass $(10^6+10^6)M_\odot$ (LISA source).