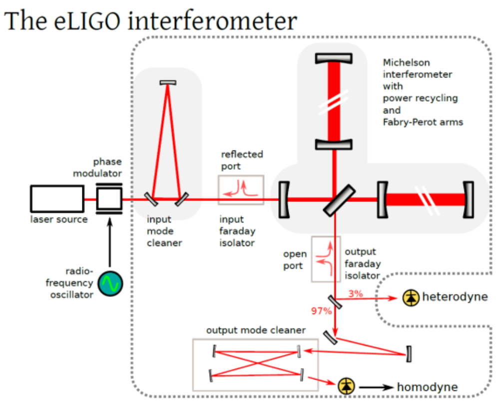
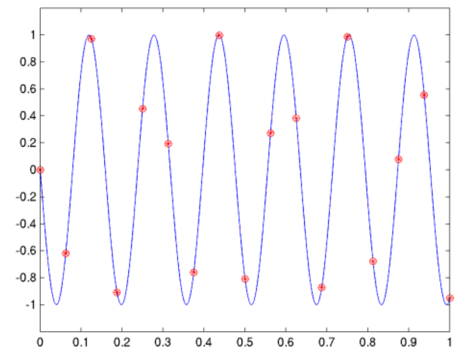
Gravitational Wave Data Analysis – Part 2

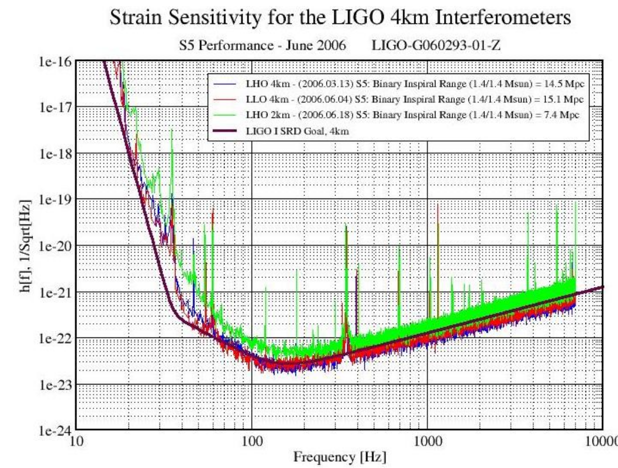
Peter Shawhan

PHYS 879
May 8, 2014

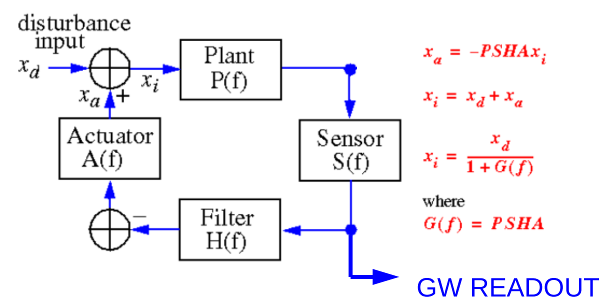
Any Questions from Tuesday?

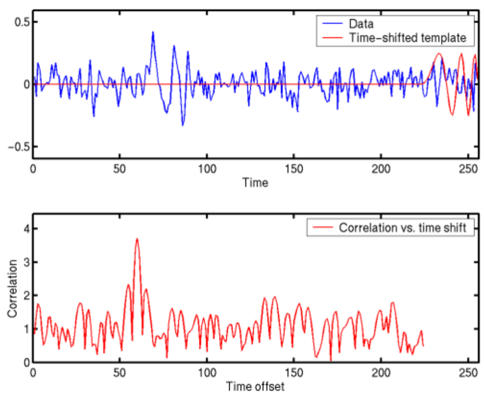


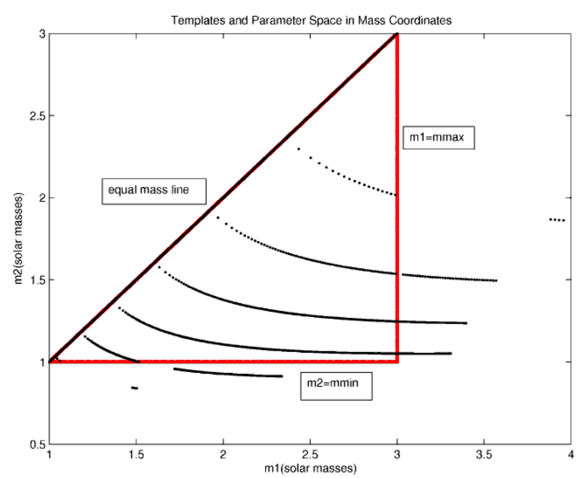


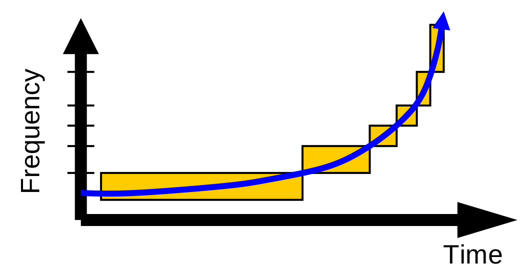


$$C(t) = 4\int_0^\infty \frac{\tilde{s}(f)\ \tilde{h}^*(f)}{S_n(f)}\ e^{2\pi i f t}\ df$$

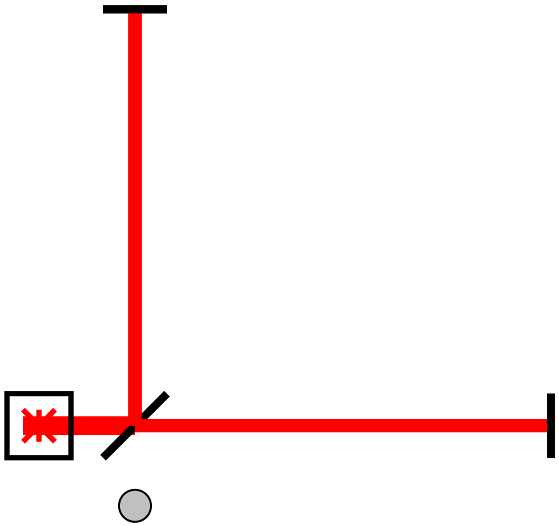








Direction Dependence of Detector Response



Also called “antenna pattern” or
“pattern function” (Maggiore sec. 7.2)

Remember, a laser interferometer records the *difference* in arm lengths

No real sensitivity to common-mode changes

Response to a signal depends on wave’s polarization and arrival direction

$$s_{out}(t) = F_+(\theta, \phi)\, h_+(t) + F_\times(\theta, \phi)\, h_\times(t)$$

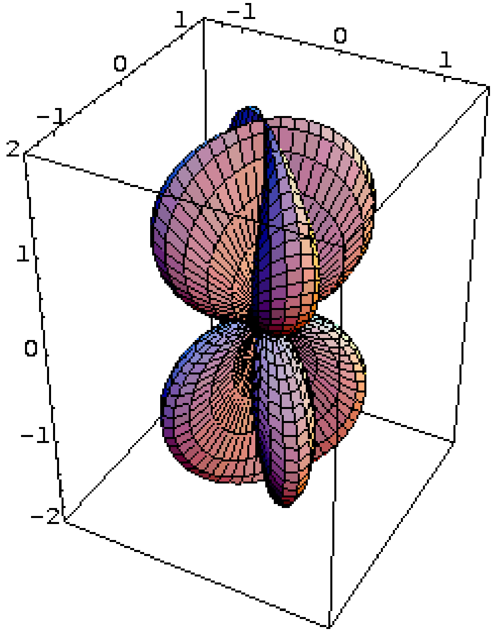
In a convenient choice of basis,

$$F_+(\theta, \phi) = \frac{1}{2}(1 + \cos^2 \theta) \cos 2\phi$$

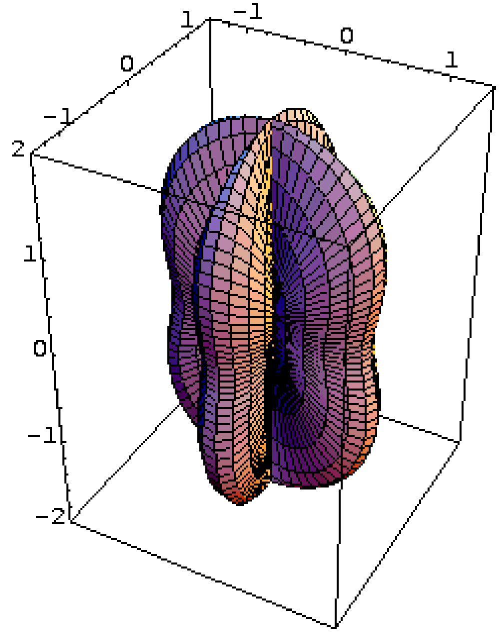
$$F_\times(\theta, \phi) = \cos \theta \sin 2\phi$$

Antenna Pattern of a Laser Interferometer

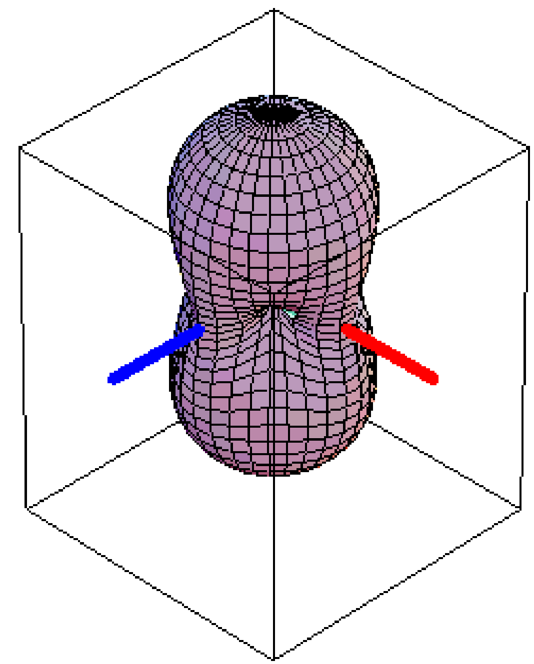
“×” polarization



“+” polarization



RMS sensitivity



A broad antenna pattern

⇒ **More like a radio receiver than a telescope**

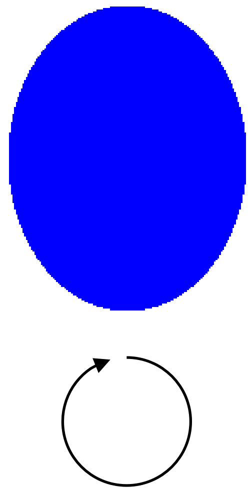
GWs from Spinning Neutron Stars

If not axisymmetric, will emit gravitational waves

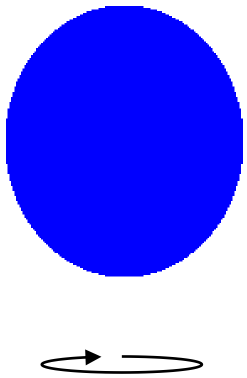
Polarization content depends on spin axis orientation

Common assumption: ellipsoid with distinct transverse axes

Along spin axis:



From side:



Continuous GW Signals at Earth

Start with a sinusoidal signal with spin-down term(s)

Polarization content depends on orientation/inclination of spin axis

Amplitude modulation

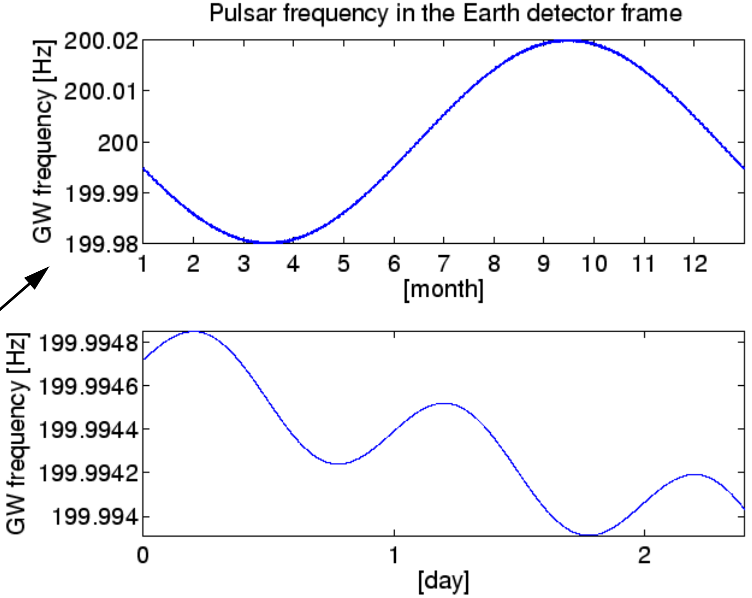
Polarization proj. onto detector changes over a sidereal day

Doppler shift

$$\frac{\Delta f}{f} = \frac{\mathbf{v} \cdot \mathbf{n}}{c}$$

Annual variation: up to ~10^{-4}

Daily variation: up to ~10^{-6}



GW signals from binary systems are more complicated !

Additional Doppler shift due to orbital motion of neutron star

Varying gravitational redshift if orbit is elliptical

Shapiro time delay if GW passes near companion

Search Methods for CW signals

Full signal model has many parameters

Different computational challenges → Different approaches

Candidates

Several cases to consider:

- Sky position and spin frequency known accurately
- Sky position and spin frequency known fairly well
- Sky position known, but frequency and/or binary orbit parameters unknown
- Search for unknown sources in favored sky regions
- Search for unknown sources over the whole sky

Radio pulsars
X-ray pulsars

LMXBs

Globular clusters
Galactic center
Supernova remnants

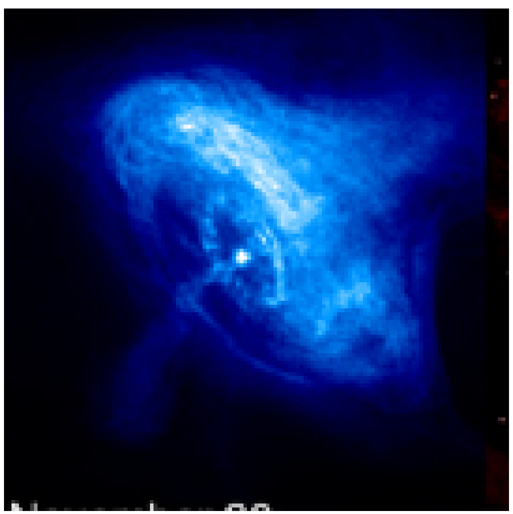
Unseen isolated neutron stars

Search for Gravitational Waves from Known Pulsars

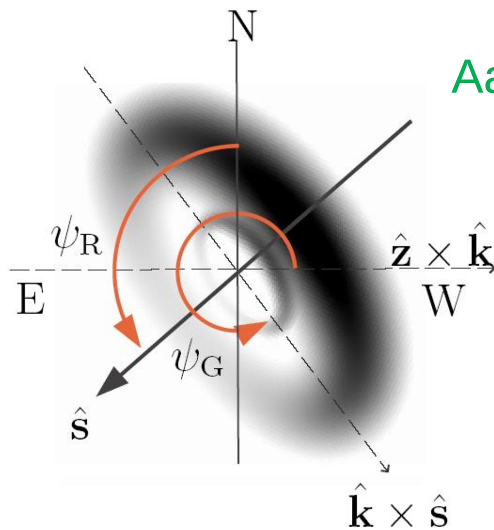
Method: **heterodyne time-domain data using the known spin phase of the pulsar**

- Requires precise timing data from radio or X-ray observations
- Include binary systems in search when orbits known accurately
- Exclude pulsars with significant timing uncertainties
- Special treatment for the Crab and other pulsars with glitches, timing noise

Chandra image



Model



Aasi et al., ApJ **785** (2014)

$h_0 < 1.6 \times 10^{-25}$

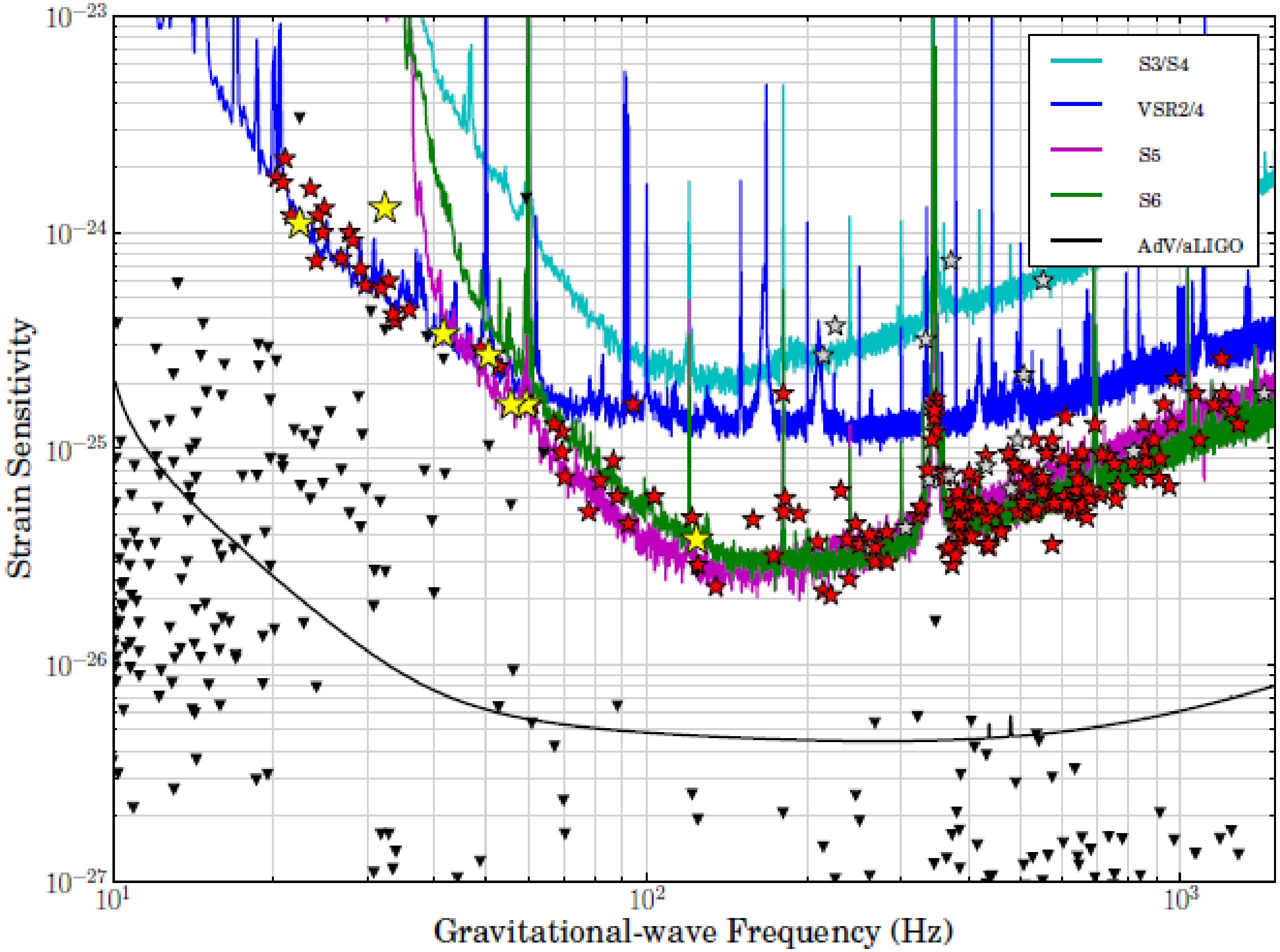
Implies that GW emission accounts for < 2% of total spin-down power

Known Pulsars in LIGO/Virgo Band

Fully coherent analysis at twice the spin frequency

Relies on having good radio (or X-ray) timing, ideally during the period of GW data collection

Aasi et al., ApJ **785** (2014)



Wide Parameter Space Searches

Method: **matched filtering with a bank of templates**

Parameters:

Sky position

Spin axis inclination and azimuthal angle

Frequency, spindown, initial phase

Binary orbit parameters (if in a binary system)

Can use a detection statistic, $\mathcal{F}$, which analytically maximizes over spin axis inclination & azimuthal angle and initial phase

Even so, computing cost scales as ~T^6

Detection threshold also must increase with number of templates

Check for signal consistency in multiple detectors

Problem: huge number of templates needed

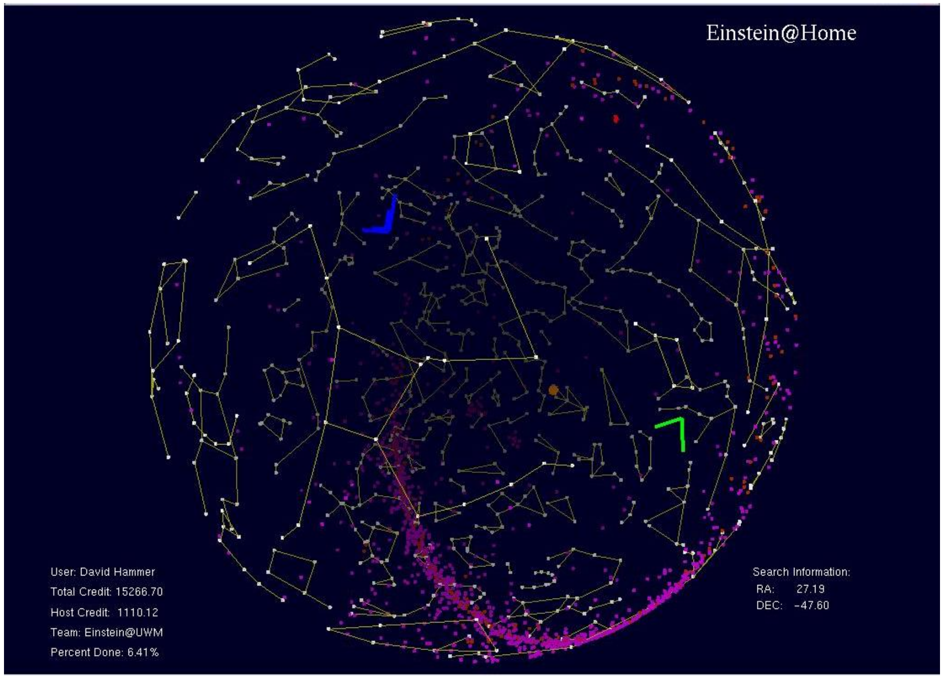
Even using clever semi-coherent analysis methods

Getting by with a Little Help from Our Friends

Public distributed computing project: Einstein@Home

einsteinathome.org

Small bits of data distributed for processing; results collected, verified, and post-processed



Searching for CW signals in LIGO+Virgo data

Also searching for millisecond pulsars in data from Arecibo, Parkes, and the Fermi satellite

So far ~360,000 users, currently providing ~1100 Tflops

Burst Search Philosophy

We're listening to the whole sky – who knows what's out there?

Models are OK, but don't put *too* much faith in them!

Goal: be able to detect *any* signal

… if it has sufficient power within the sensitive frequency band

… and is "short"

Target Signals for GW Burst Searches

Modeled burst searches

Targets:

- Black hole ringdown
- Neutron star ringdown
- Cosmic string cusp
- Parabolic encounter

Use matched filtering

Issues generally similar to binary inspiral searches

Generic burst searches

Targets:

- Binary black hole merger
- Core collapse supernova
- Signals deviating from model expectations
- Other unexpected or unmodeled sources

Use robust detection methods that do not rely on having a model of the signal

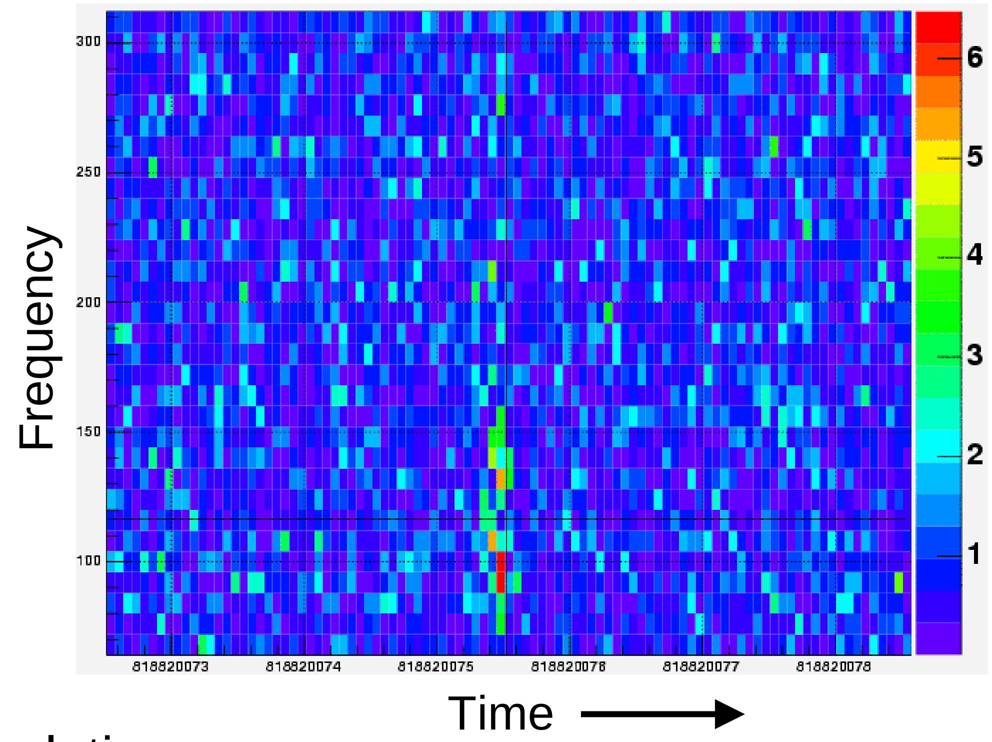
“Excess Power” Burst Search Methods

Decompose data stream into time-frequency pixels

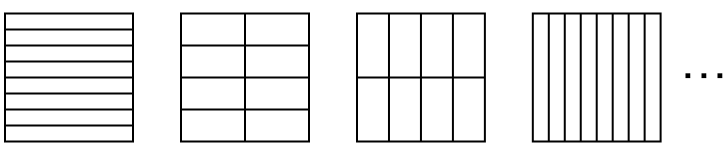
- Fourier components, wavelets, “Q transform”, etc.
- Several implementations of this type of search

Normalize relative to noise **as a function of frequency**

Look for “hot” pixels or clusters of pixels



Can use multiple (Δt,Δf) pixel resolutions



Signal Consistency Tests

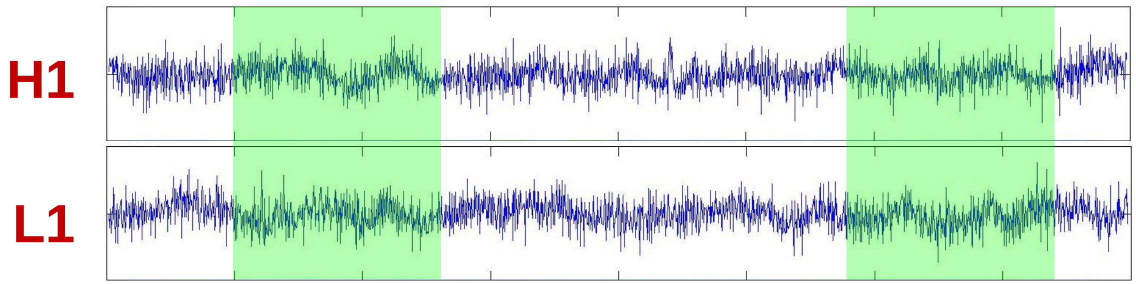
Crucial since a GW burst in a single detector may look just like an instrumental glitch !

Coincidence

Require signals in different detectors to have compatible times, frequencies, amplitudes and/or other waveform properties

Cross-correlation

Look for same signal buried in two data streams



Checks for consistent *shape*, regardless of relative amplitude

Best to integrate over a time interval comparable to the target signal

Coherent Burst Analysis

Each detector measures a linear combination of $h_+(t)$ & $h_\times(t)$ *
with antenna response factors and relative time delay depending on direction of arrival

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} = \begin{bmatrix} F_1^+ & F_1^\times \\ F_2^+ & F_2^\times \\ \vdots & \vdots \\ F_N^+ & F_N^\times \end{bmatrix} \begin{bmatrix} h_+ \\ h_\times \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_N \end{bmatrix}$$

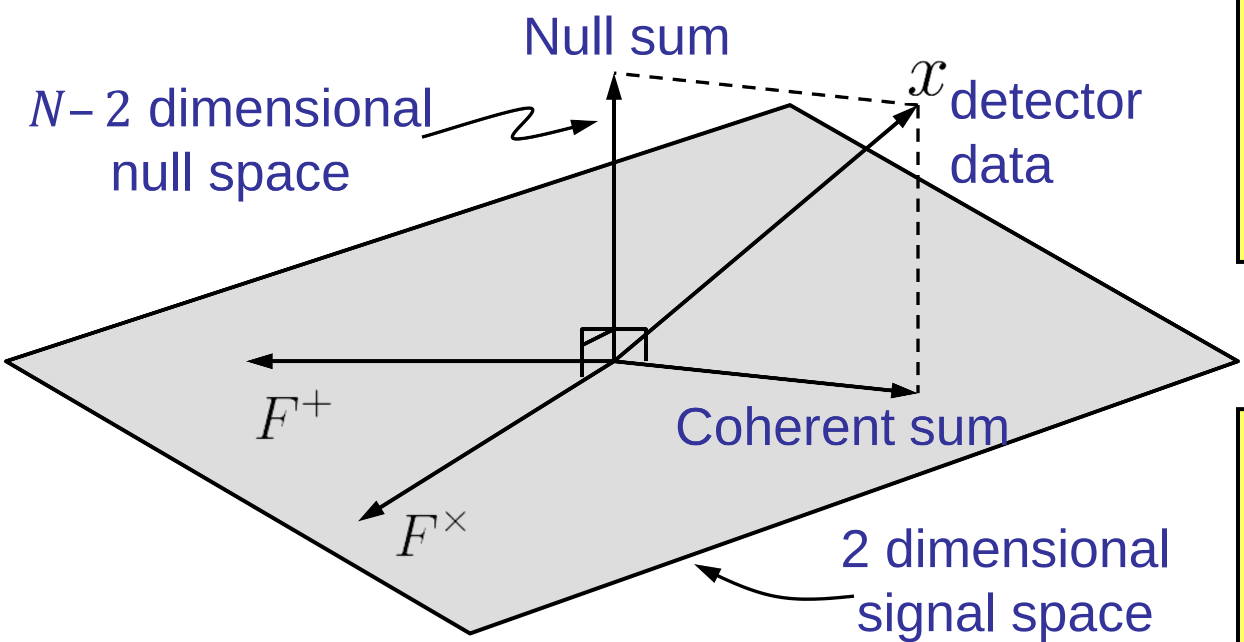
data = response × signal + noise

$\Rightarrow$ Data from 2 sites can uniquely determine $h_+(t)$ and $h_\times(t)$ for an arbitrary signal, *in the absence of noise and if the arrival direction is known*

$\Rightarrow$ Data from 3 or more sites *over-determines* $h_+(t)$ and $h_\times(t)$ if the arrival direction is known

** Assuming that GR is correct !*

Geometric View of Coherent Analysis



Coherent sum:
Find linear combination of detector data that maximizes signal to noise ratio

Null sum:
Linear combination of detector data that has no GW signal—provides consistency test

Treat this as a maximum likelihood problem

Find most likely $h_+(t)$ & $h_\times(t)$, maximizing over arrival directions

Regulator penalizes physically unlikely signal hypotheses

All-Sky Generic GW Burst Search

Analyzed all LIGO and Virgo collected since 2005 when at least two detectors were running

- Total live observation time: 636 days
- LIGO+Virgo coherent analysis
- GEO data often available for investigating possible event candidates

Sensitive to arbitrary GW signals in the range 64–5000 Hz

- Background measured by analyzing data with artificial time shifts
- Event selection thresholds tuned for low false alarm probability

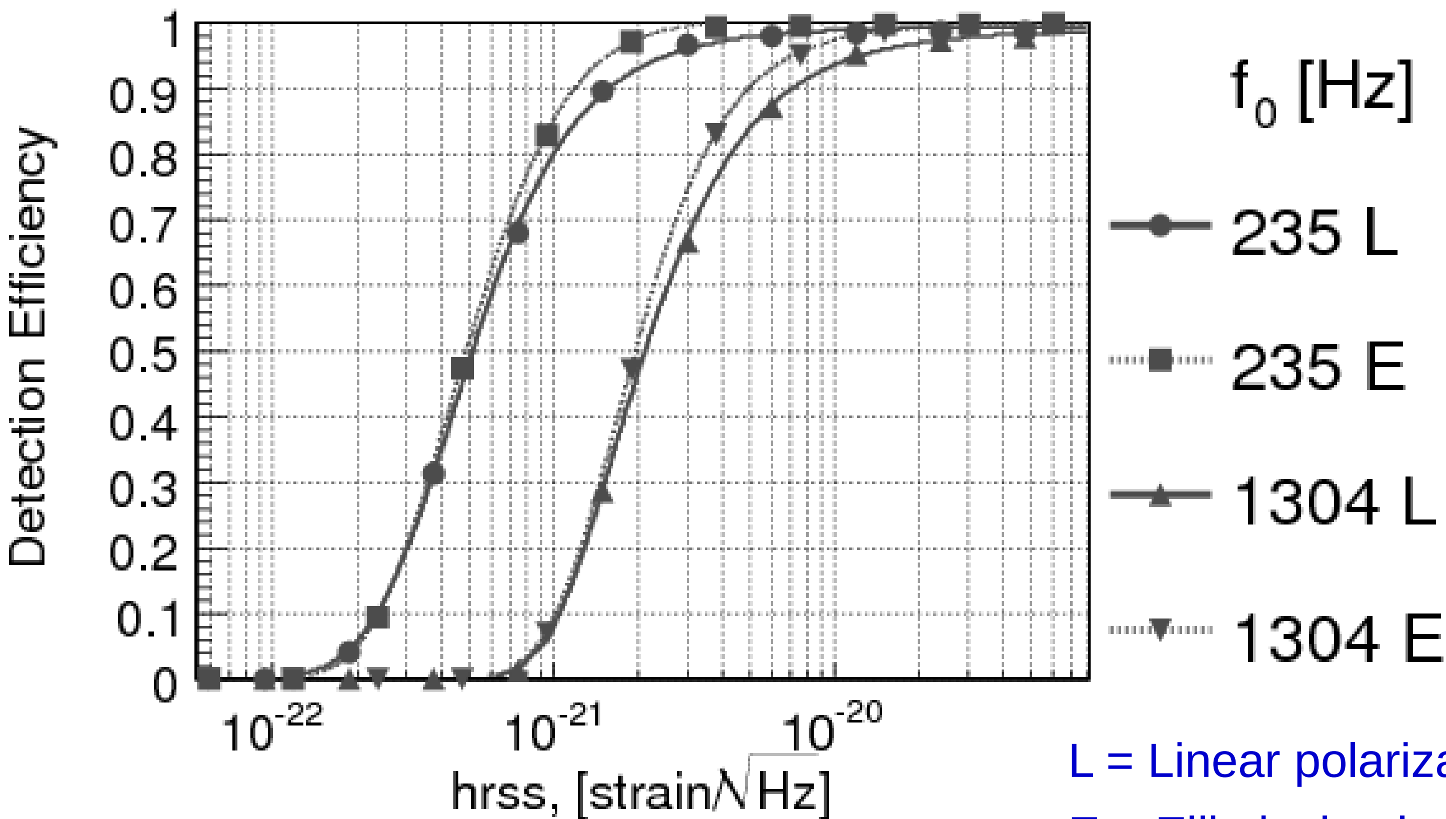
No event survived all selection cuts

We set upper limits on burst rate vs. amplitude for representative waveforms using Monte Carlo

Abadie et al., PRD **85**, 122007 (2012)

Sample Detection Efficiency Curves

For simulated signals with random times and sky positions added to real detector noise

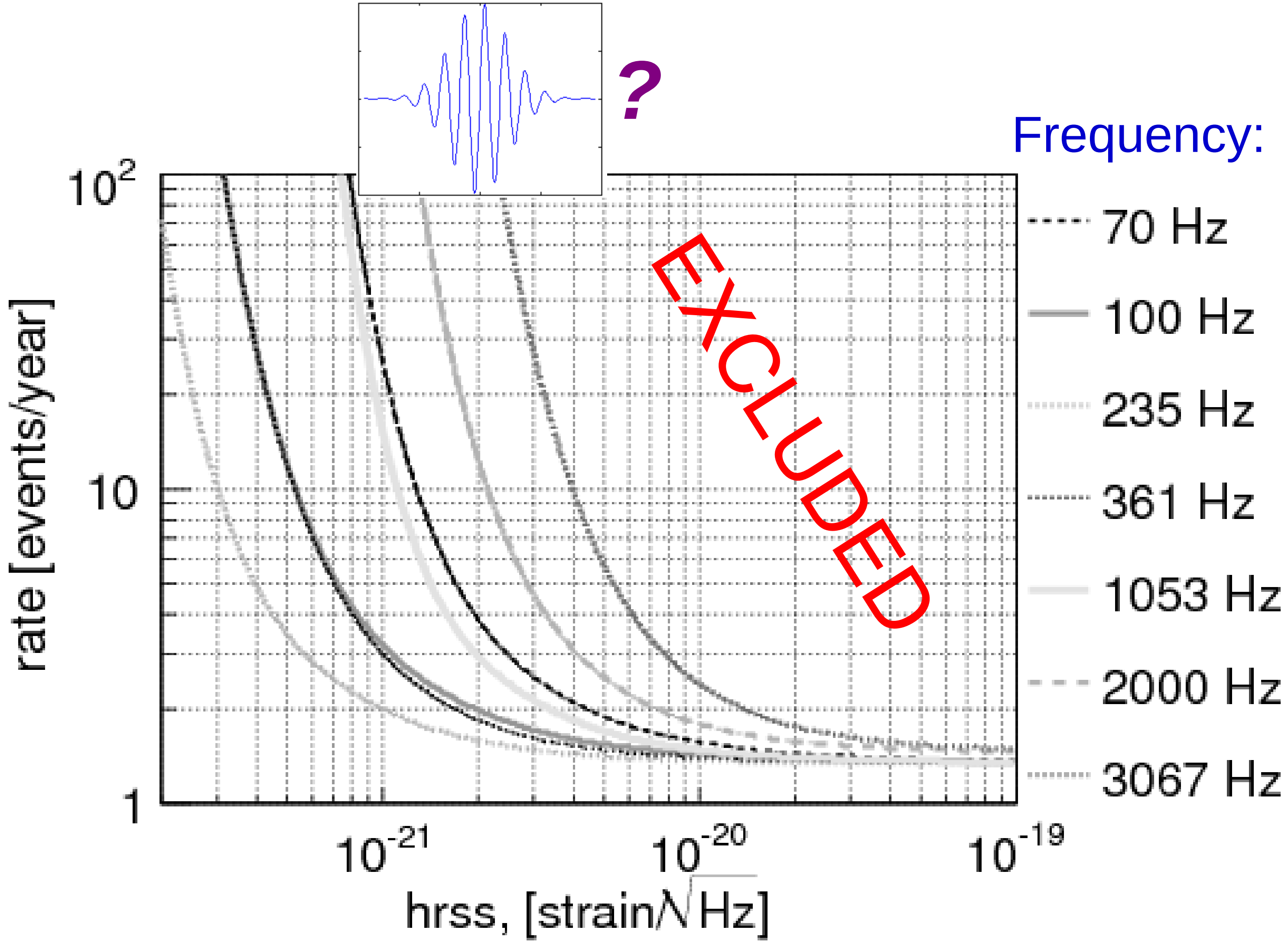


(GW burst amplitude measure)

L = Linear polarization at Earth

E = Elliptical polarization from random inclination of axis of presumed rotating source

Rate Limit vs. Amplitude



How Sensitive are Burst Searches?

Not as sensitive as matched filtering for a known waveform

But not *too* much worse, when the signal duration is short

Typically about a factor of 2

Can relate signal amplitude to energy emission in a general way:

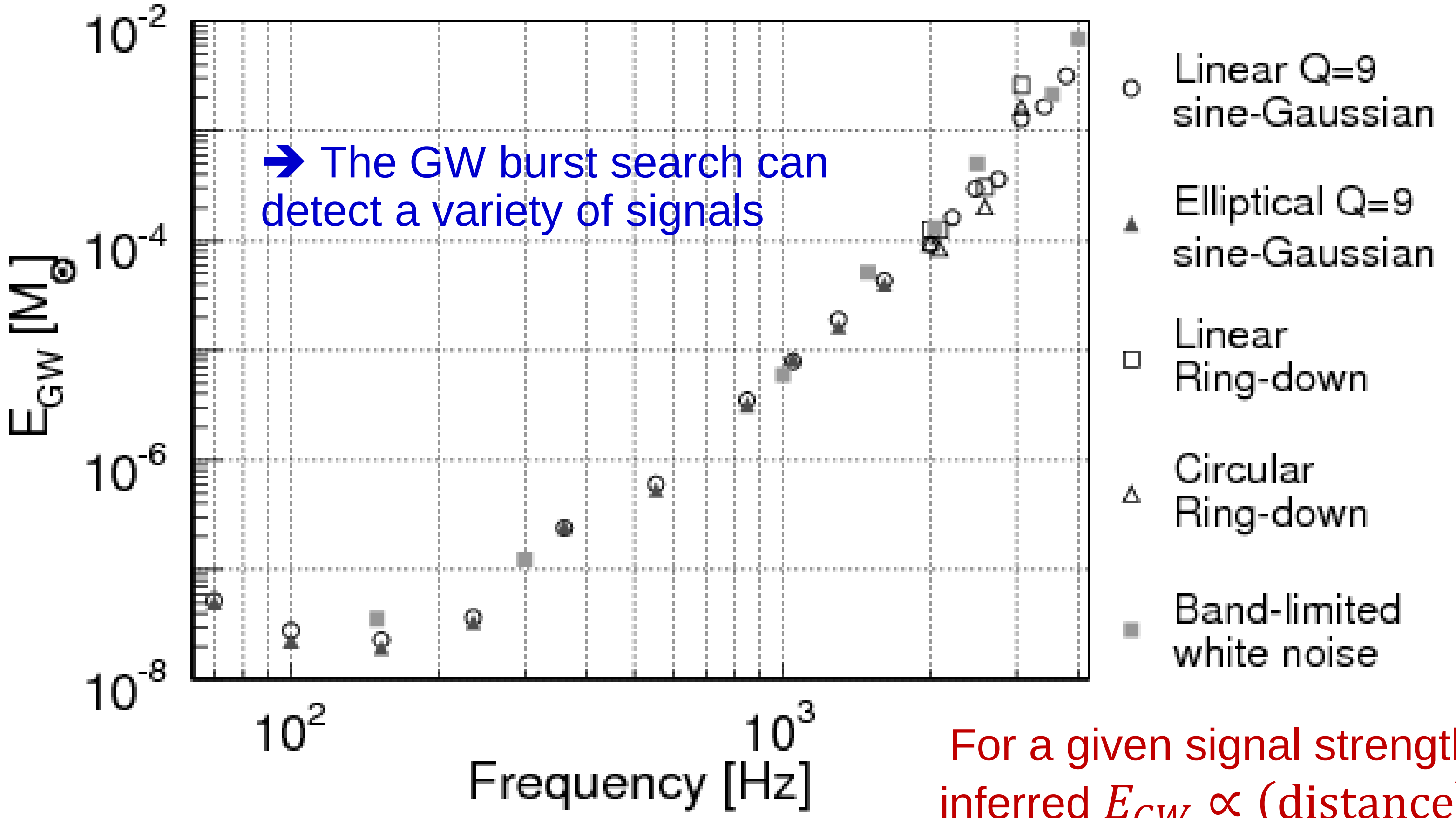
$$E_{\mathrm{GW}} = \frac{r^2 c^3}{4G} (2\pi f_0)^2 h_{\mathrm{rss}}^2$$

This assumes isotropic emission – unphysical, but fine for a rough estimate

Search Sensitivity in Energy Units

GW energy emission assuming a Galactic source (10 kpc) that could have been detected with 50% efficiency

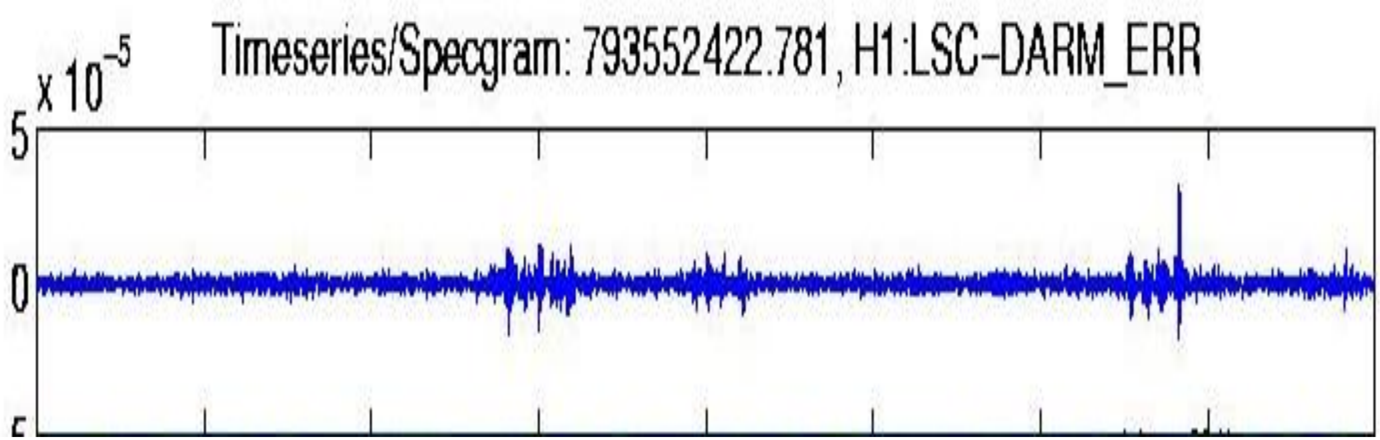
3-detector LIGO+Virgo network data, S6/VSR2+3 run



For a given signal strength, inferred $E_{GW} \propto (\text{distance})^2$

Checking an Apparent Signal

How do we know whether a signal in the data is a real GW?



Available tools:

- Consistency of the signal with a source model (if there *is* a model)
- Coincidence / consistency of signals in multiple GW detectors
- Absence of instrumental problems at the time of the signal
- Validation of instrument response and data analysis software
- Association with a known astrophysical object / event

Consistency with Source Model?

Inspiral: (Matched filter already supposes a source model)
Chi-squared test
Sanity of filter output and/or chi-squared time series

Cont.-wave: Does it show the expected Doppler modulation?
Is it present all the time?

Stochastic: Does the signal have the expected spectrum?
Is it on all the time?

Burst: Is it isolated in time?

These are not all *absolute* requirements, but agreement with the "expected" source model can add confidence

Coincidence / Consistency Tests

Having multiple detectors is extremely valuable

Signals should arrive at consistent times

- LIGO Hanford vs. Livingston: within ±10 ms
- LIGO vs. Virgo: within ±27 ms
- Also get sky position information from having multiple detectors

Signals should have consistent properties

- Same or similar templates, if a matched-filter search
- Consistent frequencies, durations
- Consistent amplitudes (allowing for different orientations)

Background Estimation

Background = expected "detection" rate of false events

Depends on criteria for a "trigger"

e.g. threshold on some measure of signal strength

Any analysis involves a trade-off between sensitivity and background

How can we determine the background?

Simple method: product of average trigger rates in each detector and coincidence time window

More reliable: **Analysis of time-shifted data**

- Choose time shifts longer than maximum light travel time, so any real GW in the data is no longer coincident
- Incorporates the consistency tests used in the actual analysis
- Follows time variability better

Can only get an *estimate* of the background

Using many different time shifts, get high statistics

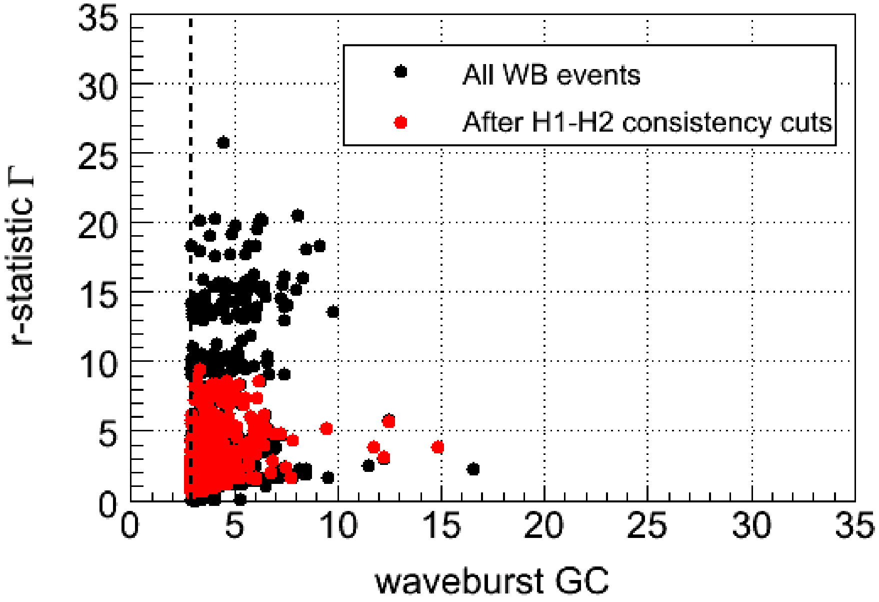
Data Quality

We attempt to catalog various environmental and instrumental conditions, then study relevance using *time-shifted* triggers

Example from LIGO S4 all-sky burst search:

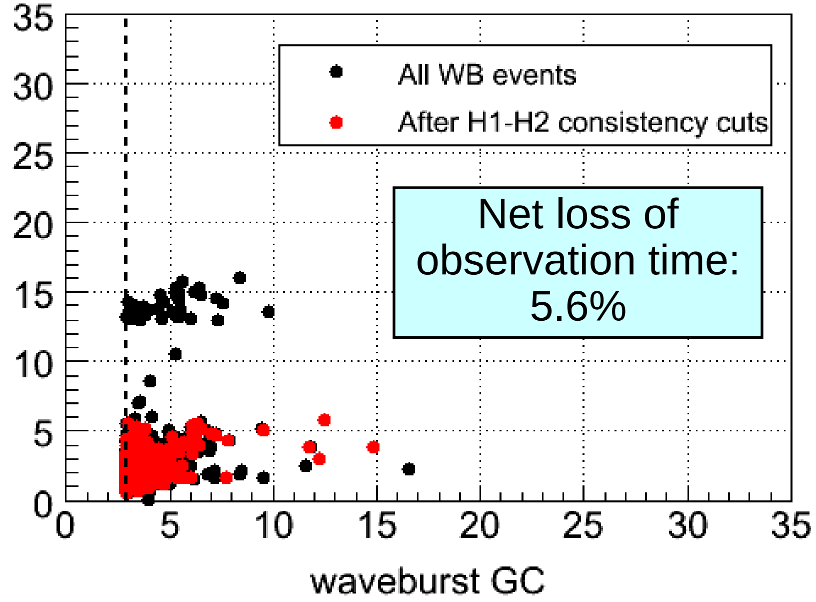
Minimal data quality cuts

Require locked interferometers
Omit hardware injections
Avoid times of ADC overflows



Additional data quality cuts

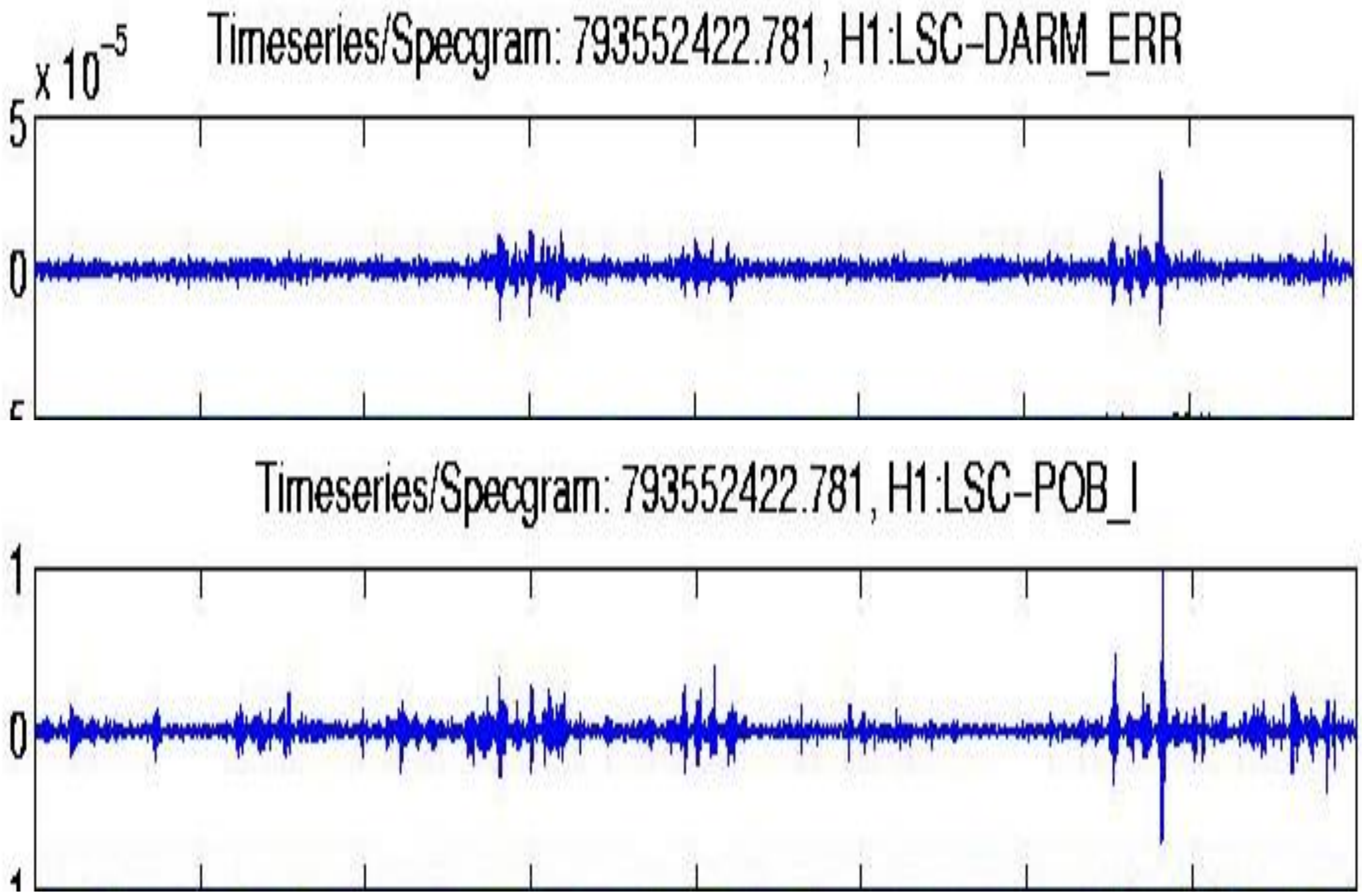
Avoid high seismic noise, wind, jet
Avoid calibration line drop-outs
Avoid times of "dips" in stored light
Omit last 30 sec of each lock



Non-Stationary Noise / Glitches

GW channel

Beam splitter pick-off

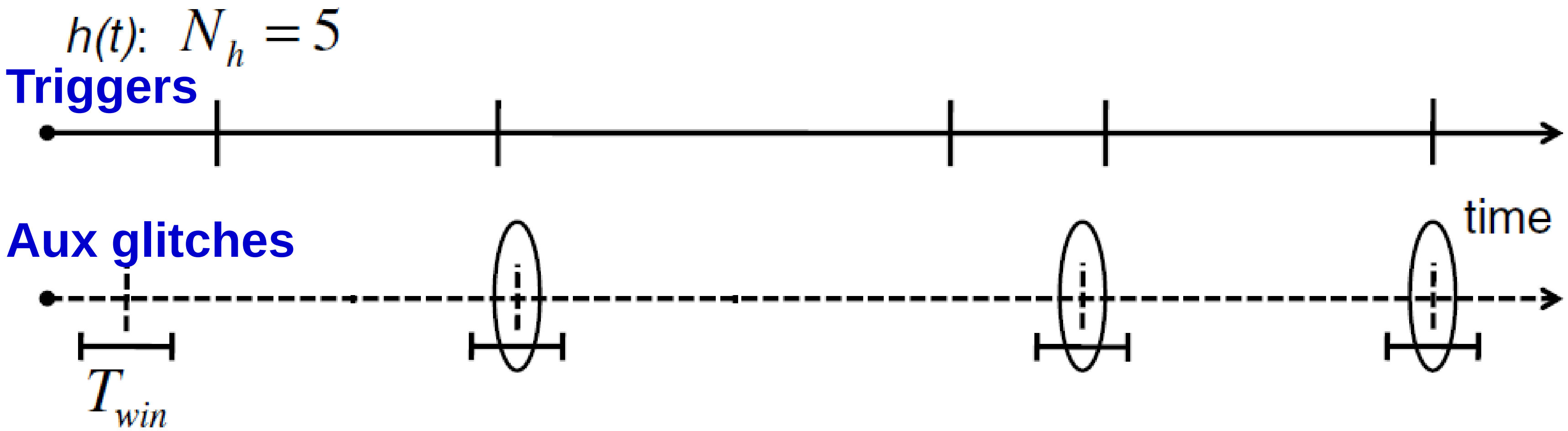


Vetoes

If there is a significant glitch in a selected auxiliary channel, then veto any trigger found at the same time in the GW channel

Only want to do this for relevant auxiliary channels

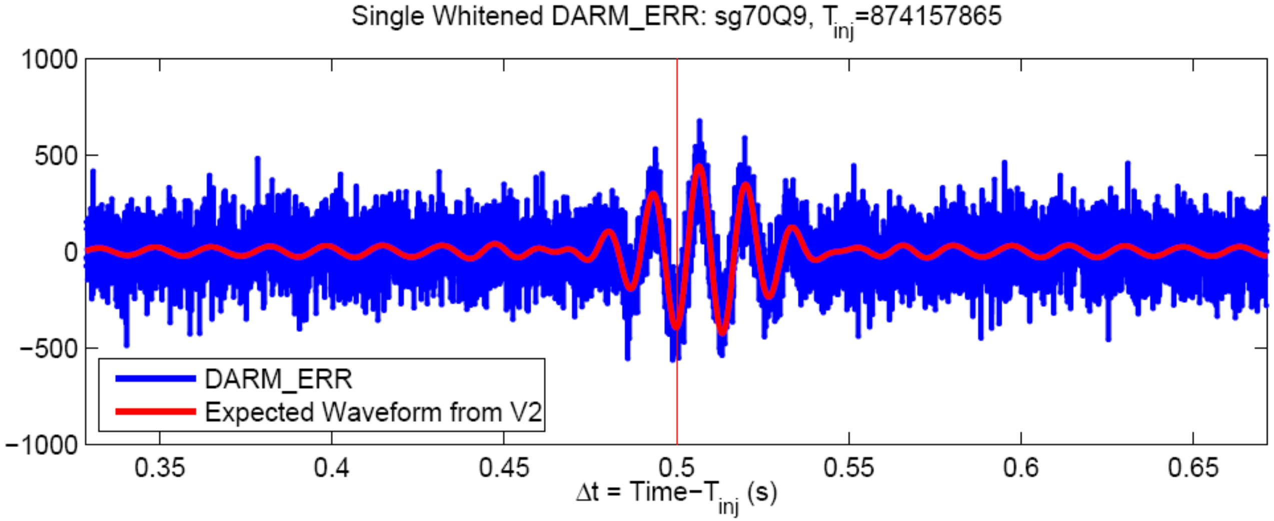
- Ideally, with known physical coupling mechanism to GW channel
- Or, established statistically with single-detector triggers or time-shifted coincidence triggers from GW channel



Validating the Detector: Hardware Signal Injections

Shake the mirrors to mimic a GW signal

Can inject an arbitrary waveform



Also used to inject simulated pulsar signals continuously

Goals:

End-to-end test of interferometer and data analysis software

Checks calibration

Useful for veto “safety” checks

Connections with Other Observations

Many (most?) sources of gravitational waves are expected to release energy in other forms too

⇒ Search for GW bursts or inspirals associated with astrophysical events/objects observed by other means

Targets:

Gamma-ray bursts (GRBs)

Magnetar (SGR/AXP) flares

Supernovae

Anomalous optical transients

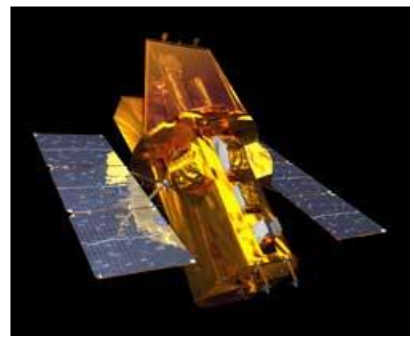
Pulsar spin-frequency glitches

LMXB X-ray intensity variations

High-energy neutrinos

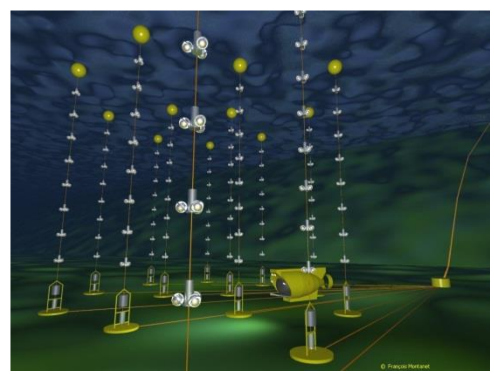
Low-energy neutrinos

Radio bursts



Swift

Palomar 48" Schmidt – Now the Ochsin telescope



ANTARES

Green Bank

Multi-Messenger Advantages

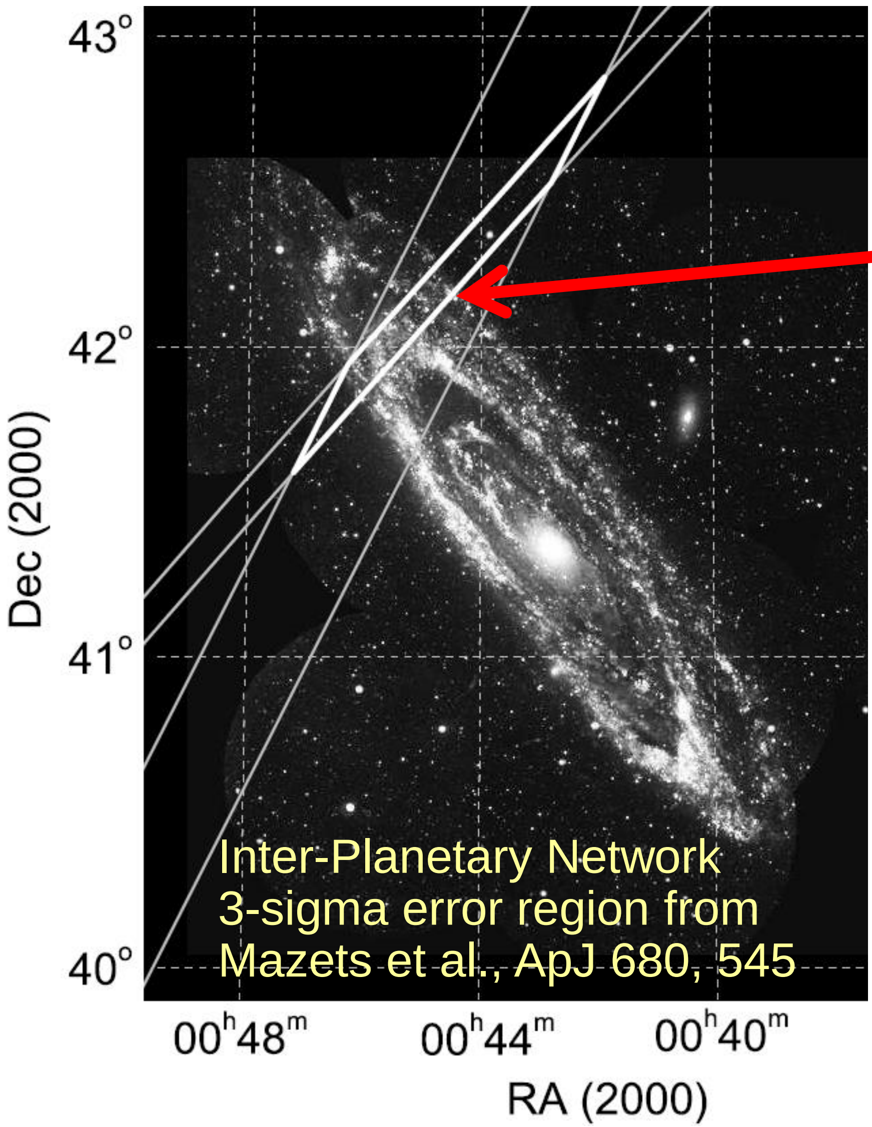
If an event has already been detected, then GW searches:

- know *when* to look at the data
- know *where* in the sky to look
- may know *what kind* of GW signal to search for
- may know the distance to the source

As a result,

- Background is suppressed, so a weaker GW signal could be confidently detected
- The extra information from the combined observations will reveal more about the astrophysics of the source
- *Non*-detection of a GW signal can still provide useful information

GRB 070201



Short, hard gamma-ray burst

Leading model for short GRBs: merger involving a neutron star

Position was consistent with being in M31 (Andromeda galaxy)

Both LIGO Hanford detectors were operating

- Searched for inspiral & burst signals

No plausible GW signal found ➔ very unlikely to be a merger in M31

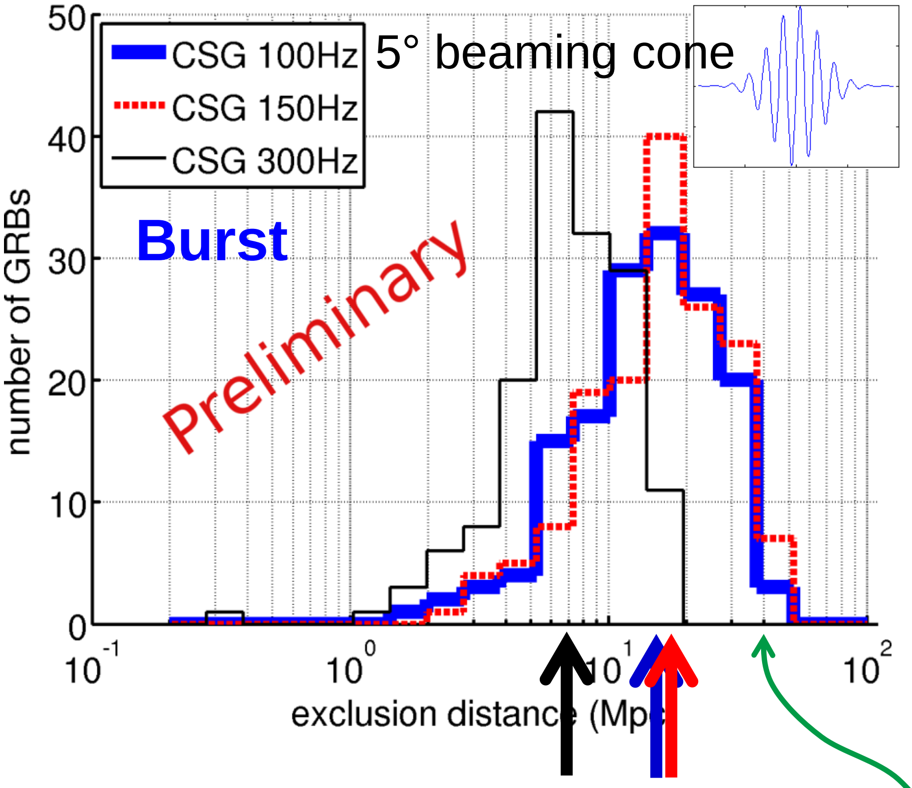
Abbott et al., ApJ 681, 1419 (2008)

Similar analysis done for GRB 051103

Abadie et al., arXiv:1201.4413

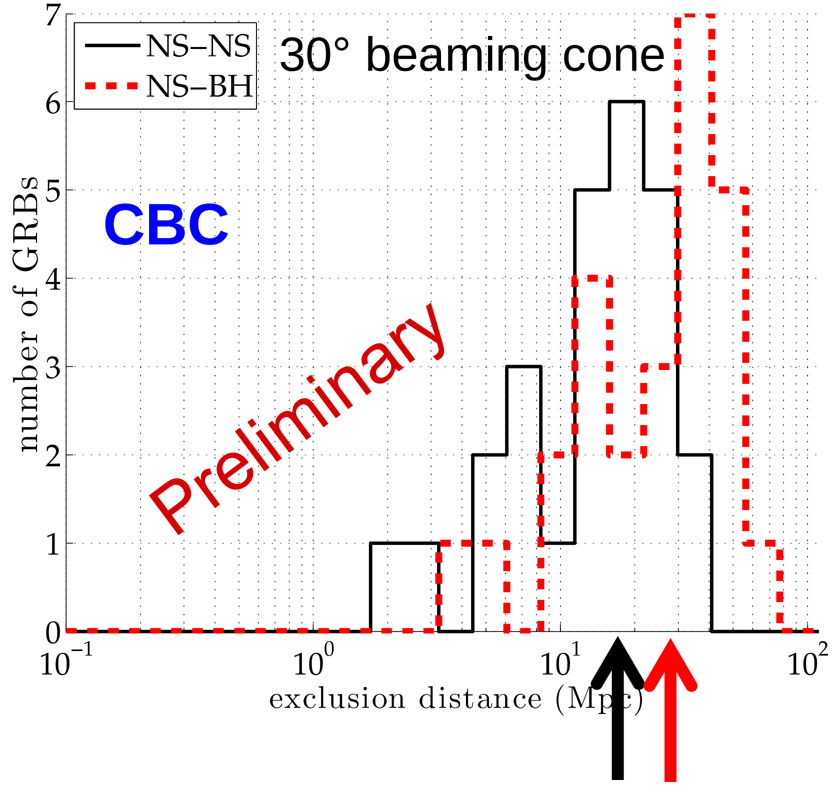
GRB Progenitor Exclusion Distances

Assuming sine-Gaussian waveform with optimistic but possible $E_{GW} = 0.01\, M_{\odot} c^2$



Median distances: 7, 15,17 Mpc

Assuming coalescence of NS-NS or NS-BH binary

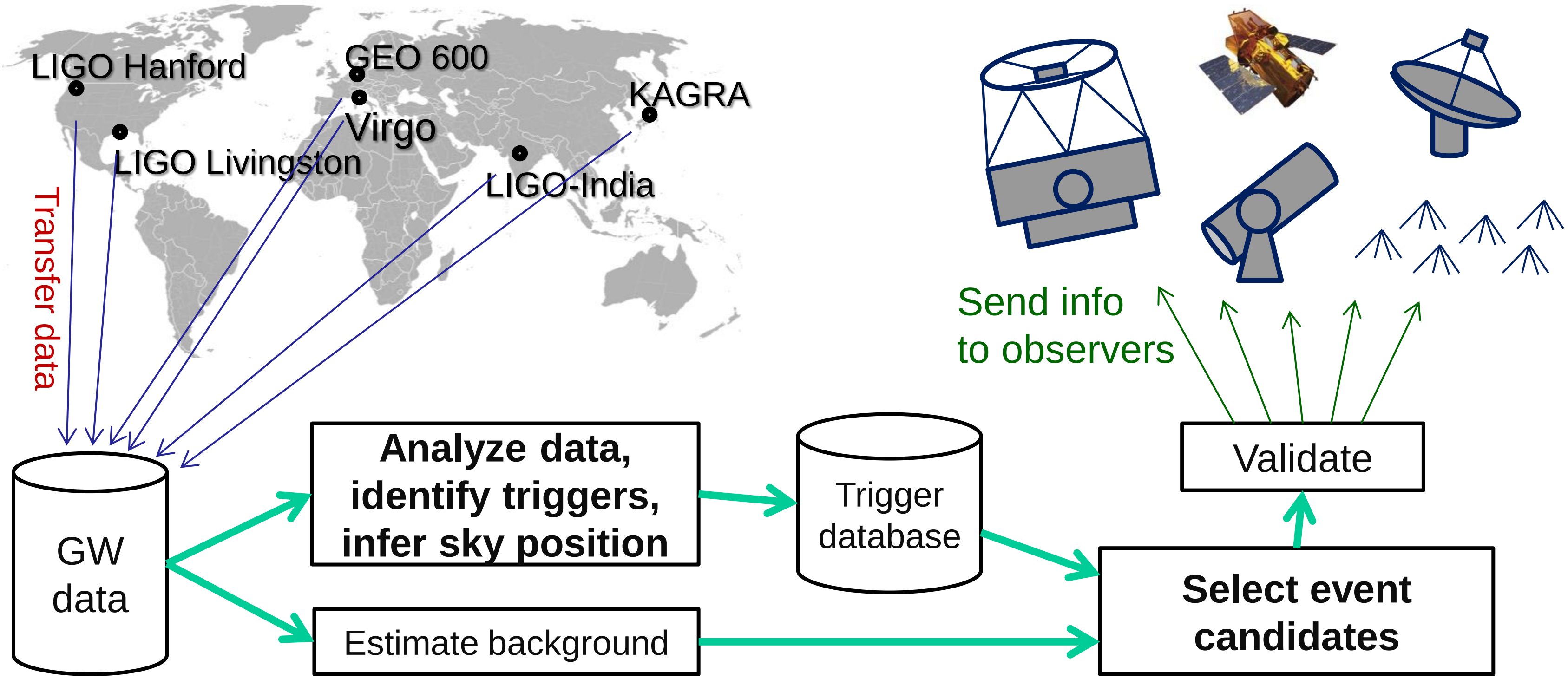


Median distances: 17, 29 Mpc

Distance to GRB 980425 / SN 1998bw

e.g. Kulkarni et al., Nature 395, 663

Rapid Alerts for Follow-up Observations



Swift: NASA E/PO, Sonoma State U., Aurore Simonnet

Goal: Catch a counterpart that would have been missed (or detected only later)

Missed GRB, orphan afterglow from off-axis or “failed” GRB, kilonova, …

➔ Confirm event, localize accurately, compare GW & EM emissions

Properties of a Stochastic GW Signal

Random signal from sum of unresolved sources

From the early universe, or from astrophysical sources since then

Looks basically like extra noise in each detector !

Usual assumptions about the signal:

Stationary

Gaussian

Unpolarized

If cosmological:

Power-law frequency dependence, probably (e.g. f^{-3})

Isotropic

If astrophysical:

Modeled power spectrum

May be anisotropic

How to Search for a Stochastic Signal

Use cross-correlation between GW data streams

- No time delay for all-sky isotropic search – will affect correlation
- For anisotropic ("radiometer") search, fix time delay between streams

Include a filter function in the cross-correlation

$$Y := \int_{-T/2}^{T/2} dt_1 \int_{-T/2}^{T/2} dt_2 \, x_1(t_1) x_2(t_2) K(t_1 - t_2)$$

↓

$$Y = \int_{-\infty}^{\infty} df \int_{-\infty}^{\infty} df' \, \delta_T(f - f') \widetilde{x}_1^*(f) \widetilde{x}_2(f') \widetilde{K}(f')$$

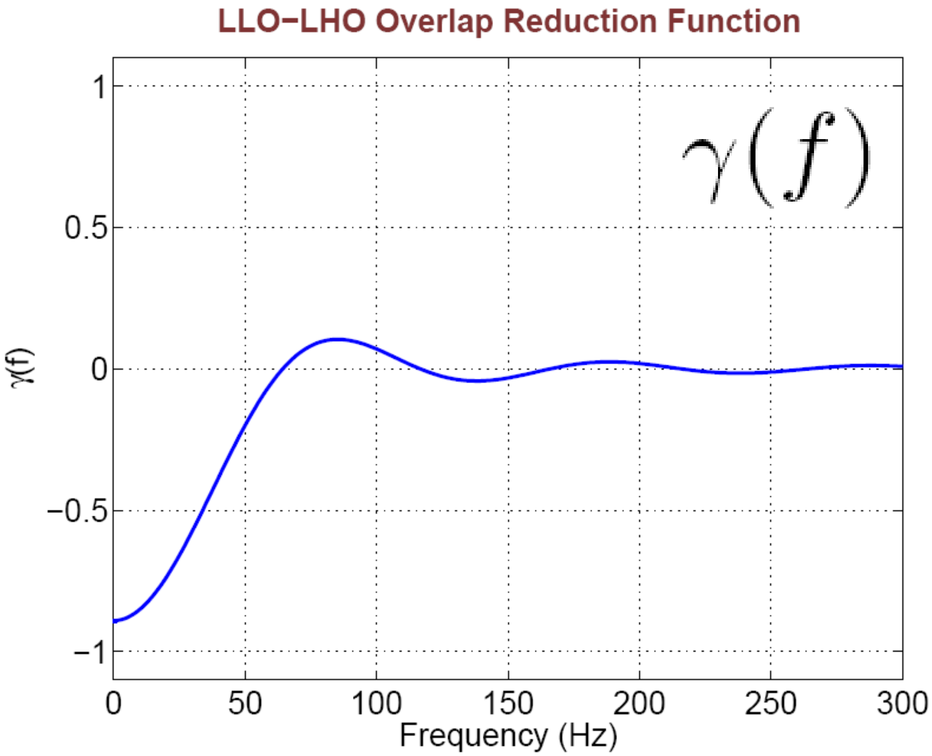
Filter function optimizes the detection statistic, accounting for two effects:

- Power spectrum of the signal being searched for
- Expected correlation between detectors, which depends on frequency due to their separation

Overlap Reduction Function
for isotropic search

Calculate expected correlation as a function of frequency

e.g. for the two LIGO observatories:



Optimal Filter for Stochastic Search

Choose:

Power spectrum of the GW signal

$$\widetilde{K}(f) = \frac{\gamma(f)P_{\rm gw}(f)}{P_{n_1}(f)P_{n_2}(f)}$$

Noise power spectra

Then signal-to-noise ratio is:

$$\left(\frac{S}{N}\right)_{\rm opt} = \sqrt{T}\left[\int_{-\infty}^{\infty} df\, \frac{\gamma^2(f)P_{\rm gw}(f)}{P_{n_1}(f)P_{n_2}(f)}\right]^{1/2}$$

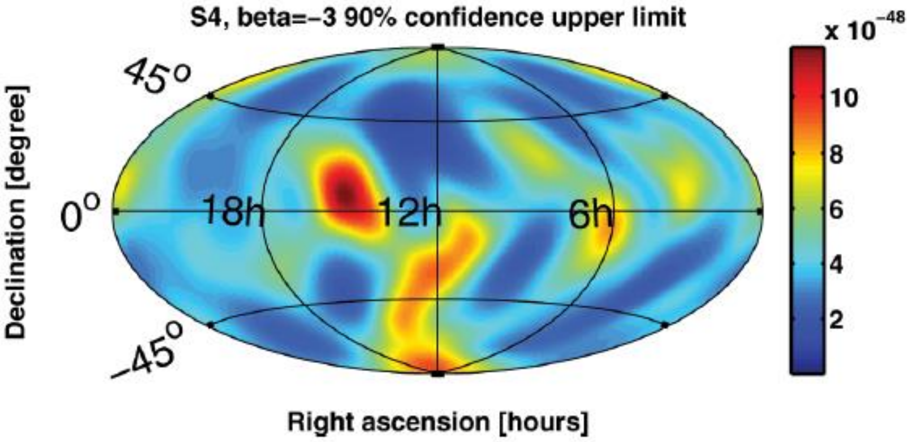
Interpret isotropic search in terms of the energy density of gravitational waves, relative to the critical energy density needed to close the universe:

$$\Omega_{\rm GW}(f) = \frac{f}{\rho_c}\frac{{\rm d}\rho_{\rm GW}}{{\rm d}f} \quad \Rightarrow \quad P_{\rm gw}(f) = \frac{3H_0^2}{20\pi^2}\frac{\Omega_{\rm gw}(f)}{f^3}$$

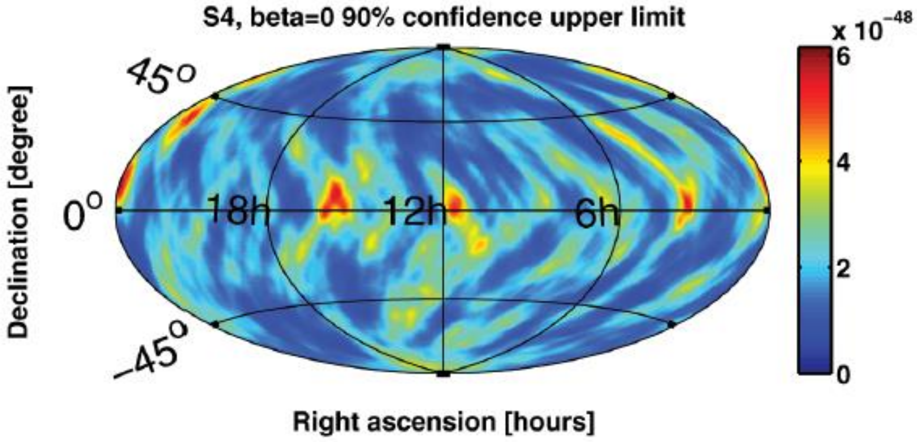
Directional Stochastic Searches

Cross-correlation with different filtering

“Radiometer” search, optimized for finding point sources:



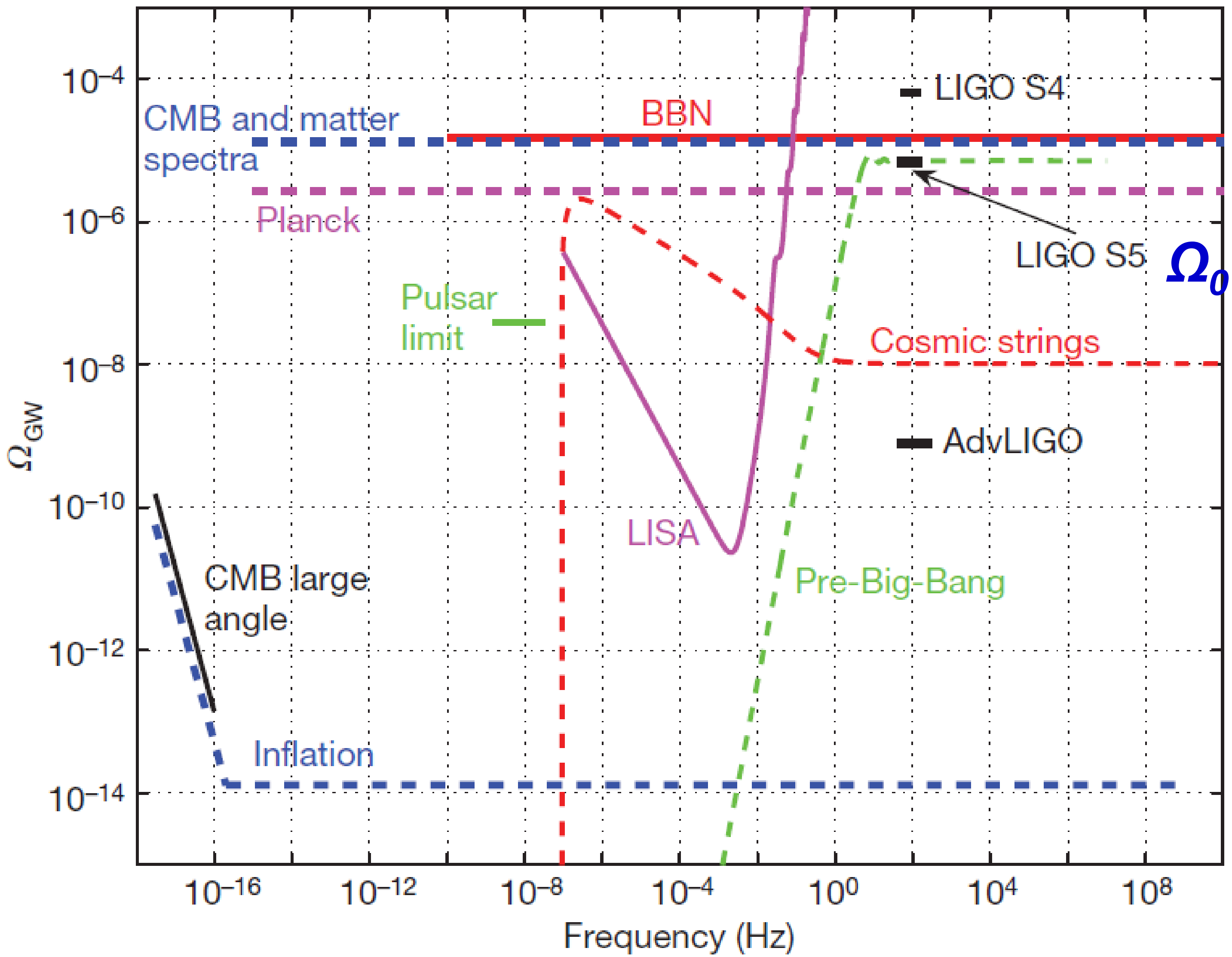
Assuming constant GW
energy spectrum



Assuming constant GW
strain spectrum

Another kind of directional search:
spherical harmonic decomposition of the sky

Isotropic Stochastic Search Results



LSC+Virgo, Nature **460**, 990 (2009)

$\Omega_0 < 6.9\times10^{-6}$ assuming flat GW energy spectrum